



### *The 83rd Workshop on General Algebra* & the 27th Conference of Young Algebraists

# ABSTRACTS

Novi Sad, Serbia, March 15–18, 2012

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We gratefully acknowledge the support of the Secretariat of Science and Technological Development of the Autonomous Province of Vojvodina.

Typeset by Igor Dolinka.

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### INVITED TALKS

### **Robust algorithms for CSPs**

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A polynomial algorithm for a constraint satisfaction problem (CSP) is called robust if it outputs an almost satisfying assignment on an almost satisfiable instance. More precisely, there is a function g(e) which approaches 0 as e goes to zero and g(0) = 0such that the algorithm outputs an assignment satisfying (1 - g(e))-fraction of the constraints given a (1 - e)-satisfiable instance.

It is known that NP-complete CSPs do not have robust algorithms. Actually, for some e > 0, it is NP-hard to find an assignment for a satisfiable instance of 3-*SAT* satisfying (1 - e)-fraction of the constraints, and this fact is equivalent to the famous PCP theorem. But also some tractable CSPs do not have robust algorithms, for instance, the problem of solving systems of linear equations over a finite field. An example of CSP which admits a robust algorithm is 2-coloring, the algorithm is based on certain semidefinite programming relaxation.

Guruswami and Zhou conjectured that a CSP has a robust algorithm iff the corresponding decision problem can be solved by local consistency checking algorithm (assuming  $P \neq NP$ ). We give an affirmative answer. The proof is based on the universal algebraic approach which is finding its way to the area of approximation algorithms.

This is a joint work with MARCIN KOZIK (Krakow).

### **Topological Birkhoff**

### MANUEL BODIRSKY

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One of the most fundamental contributions of Garrett Birkhoff is the HSP theorem, which implies that a finite algebra *B* satisfies all equations that hold in a finite algebra *A* of the same signature if and only if *B* is a homomorphic image of a subalgebra of a finite power of *A*. On the other hand, if *A* is infinite, then in general one needs to take an *infinite* power in order to obtain a representation of *B* in terms of *A*, even if *B* is finite.

We show that by considering the natural topology on the functions of *A* and *B* in addition to the equations that hold between them, one can do with finite powers even for many interesting infinite algebras *A*. More precisely, we prove that if *A* and *B* are at most countable algebras which are oligomorphic, then the mapping which sends each function from *A* to the corresponding function in *B* preserves equations and is *continuous* if and only if *B* is a homomorphic image of a subalgebra of a *finite* power of *A*.

Our result has the following consequences in model theory and in theoretical computer science: two  $\omega$ -categorical structures are primitive positive bi-interpretable if and only if their topological polymorphism clones are isomorphic. In particular, the complexity of the constraint satisfaction problem of an  $\omega$ -categorical structure  $\Gamma$  only depends on the topological polymorphism clone of  $\Gamma$ .

Joint work with MICHAEL PINSKER.

### Algebra, graphs and dualities

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Graphs and digraphs have long provided a testing ground for algebraic techniques and conjectures associated with the study of the complexity of the CSP (Constraint Satisfaction Problem). Recently it was shown that in fact many interesting properties and problems of CSPs of arbitrary relational structures can in fact be restricted to an equivalent properties or problem of CSPs of digraphs, making this the natural setting to study these properties. We will survey some results on the algebraic methods applied to (di)graphs, with a particular emphasis on the ones that characterise certain types of duality. A CSP for a structure *B* has a duality of some type if the existence of a homomorphism from a given structure *A* to *B* is equivalent to the non-existence of a homomorphism to *A* from a structure belonging to a certain nice class.

### Lattice polynomial functions and their use in qualitative decision making

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Aggregation refers to processes of merging or fusing several values, scores, etc., into a single meaningful one. Such processes are achieved by so-called aggregation functions, whose importance has become more and more apparent in an increasing number of areas not only of mathematics or physics, but especially of applied fields such as engineering, computer science, economics and social sciences. In particular, aggregation functions have attracted much attention in decision sciences since they provide an elegant and powerful formalism to model preference. These facts explain the rapid growth of aggregation theory which proposes, analyzes, and characterizes aggregation function classes.

Traditionally, aggregation functions are regarded as mappings  $A: I^n \to I$ , where I is a real interval (e.g., I = [0, 1]), which are nondecreasing and satisfy the boundary conditions  $A(\wedge I^n) = \wedge I$  and  $A(\vee I^n) = \vee I$ . Typical examples include arithmetic and geometric means, and so-called Choquet integrals. Such examples of aggregation functions, despite producing values which are rather representative of their arguments, rely heavily on the rich arithmetic structure of the real numbers. Thus they are of little use over domains where no structure other than an order is assumed, e.g., qualitative scales such as

{very bad, bad, satisfactory, good, very good}.

In such situations, the most widely used aggregation functions are the so-called Sugeno integrals, which can be thought of as certain lattice polynomial functions, namely, those which are idempotent.

This observation will be the starting point of our talk, in which we shall present a study of these lattice functions rooted in aggregation theory and motivated by their application in qualitative decision making. We shall start by presenting characterizations of lattice polynomial functions in terms of necessary and sufficient conditions which have natural interpretations in aggregation theory. Then we shall consider certain extensions of lattice polynomial functions which play an important role in decision making, in particular, in preference modeling, and present their characterizations accordingly. As we shall see, these results pave the way towards an axiomatic treatment of qualitative decision making.

Some of the results we will discuss were obtained in collaboration with D. DUBOIS, J.-L. MARICHAL, H. PRADE, A. RICO, and T. WALDHAUSER.

### Automorphism groups of ordered sets and the Bergman property

### MANFRED DROSTE

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In this survey, we will present various permutation groups with the Bergman property. Here, a group G is said to have the Bergman property, if for any generating subset *E* of *G*, already some bounded power of  $E \cup E^{-1} \cup \{1\}$  covers *G*. This property arose in a recent interesting paper of Bergman where it was derived for the infinite symmetric groups. Groups which were, soon after Bergman's paper, shown to have the Bergman property include automorphism groups of various kinds of homogeneous spaces. Such groups include the homeomorphism groups of the rationals, the irrationals, or Cantor's set, measure automorphism groups of the reals or of the unit interval, and groups of non-singular or ergodic transformations of the reals. We will concentrate on automorphism groups of ordered sets. The groups of all order automorphisms of the rationals or of the reals have the Bergman property. Also, the order automorphism groups of any weakly 2-transitive countable tree and of the universal homogeneous countable distributive lattice were recently shown to have the Bergman property. However, e.g. groups of bounded order automorphisms of the rationals do not have the Bergman property. The problem arises to find further examples as well as general criteria for classes of groups (or transformation semigroups) acting on structures with the Bergman property. For which of your favorite algebraic structures does the automorphism group (or transformation semigroup) have the Bergman property?

Joint work with R. GÖBEL, C. HOLLAND and G. ULBRICH, resp. with J. TRUSS.

### Free idempotent generated semigroups over the full linear monoid

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Many semigroups that arise in nature are idempotent generated. For instance, J. A. Erdos (1967) proved that every non-invertible matrix of the full linear monoid  $M_n(F)$  of all  $n \times n$  matrices over a field F is expressible as a product of idempotent matrices (in fact, this is true more generally for matrices taken over an arbitrary division ring Q). The set of idempotents E of an arbitrary semigroup has the structure of a so called biordered set. These structures were studied in detail in work of Nambooripad (1979) and Easdown (1985). The free idempotent generated semigroup IG(E) is the universal object in the category of all idempotent generated semigroups with biordered set of idempotents E. A question that has been of interest in the literature is: which groups can arise as maximal subgroups of free idempotent generated semigroups? Early results on this problem led to a conjecture that all such groups must be free. The first counterexample to this conjecture was given by Brittenham, Margolis and Meakin (2009), where it was shown that the free abelian group of rank 2 can arise. Gray and Ruškuc (2012) then went on to show that every group arises in this context. In contrast, less is known about the the structure of the maximal subgroups of free idempotent generated semigroups on naturally occurring biordered sets. Recently, using topological tools, Brittenham, Margolis and Meakin (2010) have shown that the rank 1 component of the free idempotent generated semigroup of the biordered set of  $M_n(Q)$  has maximal subgroup isomorphic to the multiplicative subgroup of Q. In this talk I will present some recent joint work with IGOR DOLINKA (University of Novi Sad) in which we extend this result, showing that general linear groups arise as maximal subgroups in higher rank components.

## Mal'cev conditions derived from hereditary properties (and other algebraic malfunctions)

### MARCIN KOZIK

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I will present a number of results, with origin in CSP, connecting structural properties of algebras with Mal'cev conditions (e.g. describing congruence meet semidistributive varieties, varieties with few subalgebras of powers etc.). I will discuss the regrettable lack of theory unifying these results.

### Almost all finite semigroups are finitely related

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An algebraic structure is finitely related if its clone of term functions is equal to the clone of operations preserving a single finitary relation. We investigate this concept for algebras in general and show in particular that the following finite semigroups are finitely related: 3-nilpotent monoids, regular bands, semigroups with only one idempotent, and Clifford semigroups. This extends some recent results by Davey-Jackson-Pitkethly-Szabó on semigroups and by Aichinger-Mayr-McKenzie on groups. We also present the first known example of a finite semigroup that is not finitely related.

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For an algebraic structure *A* denote by d(A) the smallest size of a generating set for *A*, and let  $\mathbf{d}(A) = (d(A), d(A^2), d(A^3), \ldots)$  (direct powers of *A*). Thus, for example, for the cyclic group  $C_5$ , the symmetric group  $S_5$  and the alternating group  $A_5$  we have

$$\mathbf{d}(C_5) = (1,2,3,4,\ldots)$$
 (dimensions of vector spaces)  

$$\mathbf{d}(S_5) = (2,2,3,4,\ldots)$$
 (a nice undergraduate exercise)  

$$\mathbf{d}(A_5) = (\underbrace{2,2,\ldots,2}_{19},\underbrace{3,3,\ldots,3}_{1649},4,4,\ldots)$$
 (a lovely old result of Hall)

In this talk, I will not be concerned so much with the exact values, but instead with the type of behaviour. So,  $\mathbf{d}(C_5)$  and  $\mathbf{d}(S_5)$  are (eventually) linear, while  $\mathbf{d}(A_5)$  (seems to be) logarithmic. I will discuss the following issues:

- Are the above isolated examples, or part of a pattern?
- What pattern?
- Is this pattern specific for groups, or is there a variant for 'group like' (= 'classical') algebraic structures rings, modules, associative algebras and Lie algebras?
- Or is this perhaps something to do with associativity semigroups?
- Does size matter finite vs. infinite?
- How about more modern structures: lattices, tournaments, Steiner triple systems, universal algebras?
- What are interesting open questions?

No advanced algebraic background is required to follow the talk.

### Homogeneous multipartite graphs

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I shall outline the classification of the countable homogeneous multipartite graphs (joint work with JENKINSON and SEIDEL) and the generalization to the case in which colours are allowed on the edges (joint work with LOCKETT). The monochromatic case extends from the bipartite classification (which is given in a paper of Goldstern, Grossberg and Kojman) successively to the tripartite, quadripartite and general cases. A key configuration which can be omitted is called an 'omission quartet', comprising a family of four tripartite graphs which interact in a specified manner, and resulting from the two major lemmas needed to complete the classification, the 'Non-monic realization theorem', and the 'Non-complication theorem', one sees that omission quartets form essentially the only obstruction to a clean description of all cases. Where colours are allowed on edges, similar methods can be applied; but this is considerably more involved, and so far the situation is only properly understood up to about 4 parts.

### CONTRIBUTED SHORT TALKS

### On the direct composition of nilpotent expanded groups

### ERHARD AICHINGER

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For expanded groups, we have two notions that generalize the group theoretic concept of nilpotence. One notion is the notion of nilpotence coming from commutator theory; the second one is the concept of supernilpotence. We call a finite expanded group **A** *supernilpotent* if  $\log(|\mathbf{F}_{\mathcal{V}(\mathbf{A})}(n)|)$  is bounded from above by a polynomial in *n*; this property can be expressed by the non-existence of certain functions in Pol(**A**).

From a result by Kearnes it follows that every finite supernilpotent expanded group is a direct product of algebras of prime power order. We present a proof of this result that allows to generalize Kearnes's result to infinite expanded groups with congruence lattice of finite height.

Supported by the Austrian Science Fund (FWF) in the project P24077.

### Stably weakly coreflective subcategories of the category of acts over a monoid

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In 1960 Bass proved that every (right) module over a ring has a projective cover if and only if the ring satisfies the descending chain condition on principal (left) ideals. In 1976 Fountain proved that every (right) act over a monoid has a projective cover if and only if the monoid satisfies the descending chain condition on principal (left) ideals and every act satisfies the ascending chain condition on cyclic subacts.

In 1981 Enochs showed that every module has a projective cover if and only if the class of projective modules is a stably weakly coreflective subcategory of the class of all modules. He then defined a flat cover using this alternative definition and conjectured that every module has a flat cover. This conjecture was finally proved independently by Enochs, and Bican and El Bashir in 2001.

We show that every act over a monoid satisfying the ascending chain condition on cyclic subacts has a (strongly) flat cover. In their related recent work Khosravi et al. [1] used a different definition of cover and we will briefly explain the relationship with their results. This is joint work with JIM RENSHAW (University of Southampton).

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### On linear groups of degree 2 over a finite-dimensional algebra over a finite field

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The topic of the talk is motivated by the study of the subgroup structure of linear groups over commutative semilocal rings that have finite residue fields. Recall that a commutative ring is called semilocal if it has a finite number of maximal ideals. As an example of such a ring, we can consider the direct sum of a finite family of fields. A residue field of a semilocal ring is a quotient of the ring by any of its maximal ideals. Here we prove the following result which is the first step to the generalization of the well known L. E. Dickson's theorem about linear groups of degree 2 over finite fields [1].

**Theorem.** Let *k* be a prime field of order p > 3 and *K* an associative *k*-algebra generated over *k* by an element  $\alpha$ . Suppose that  $\alpha$  is a root of a polynomial which is a product of two distinct polynomials irreducible over *k*. If  $\lambda$  is an invertible element of *K* and *G* is the subgroup of the special linear group SL(2, K) generated by the matrices  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ ,  $\begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$ , then *G* coincides with the group  $SL(2, k[\lambda])$ . Therefore, *G* is isomorphic either to the direct product of two groups each of which is isomorphic to the special linear group of degree 2 over a finite field or to the special linear group of degree 2 over a finite field or to the special linear to the direct sum of two finite fields or to a single finite field.

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### Palindromic defect and highly potential words

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The *palindromic defect* of a finite word w has been introduced by Brlek et al. as the difference between the length of w increased by one and the number of palindromic factors of w (by an earlier result of Droubay, Justin and Pirillo, this difference is always non-negative). A natural extension of this definition to infinite words has also been introduced.

In this talk we present a construction of a class of infinite words, called *highly potential words* because of their seemingly high potential of being a good supply of examples and counterexamples regarding various problems on words, particularly the ones related to the palindromic defect and related notions. One of the most interesting properties of highly potential words is the fact that they are all aperiodic words of a finite positive defect, having the set of factors closed under reversal; words satisfying this combination of conditions have been sought after in some recent works, but not a single example is found so far.

### Decomposing distributive lattices up to polynomial equivalence using RST

#### MIKE BEHRISCH Institute of Algebra University of Technology, DRESDEN (GER) mike.behrisch@mailbox.tu-dresden.de

Relational Structure Theory (RST) is a localisation theory for finite algebras, inspired by ideas from Tame Congruence Theory and R. McKenzie's characterisation result, [McK96], on categorical equivalence of varieties. The theory, originally introduced by K. Kearnes and Á. Szendrei in [Kea01] and further studied in [Beh09], generalises the concept of neighbourhood known from Tame Congruence Theory (TCT) to images of idempotent term operations and defines induced algebras on such subsets. While the main focus of classical TCT is limited to the congruence lattice of algebras, RST associates the full relational clone of invariant relations with finite algebras. These relational structures can be restricted to neighbourhoods in a canonical way, corresponding to the induced algebras. Using a product-retract construction, it is possible to re-obtain the relational dual of a given finite algebra from sufficiently many restricted structures, and therefore to reconstruct the original algebra up to term equivalence. Collections of neighbourhoods allowing for this kind of localisation process are called covers. It is one of the main results of RST that finite algebras have got a unique nonrefinable cover up to isomorphism, which enables an-in some sense-most effective decomposition.

In the talk we present a case study for polynomial expansions of (finite) distributive lattices, that is, all nullary constant operations are added to the fundamental operations of the algebra. In this context, the generally hard problem to describe all neighbourhoods and nonrefinable covers is feasible. We show that the set of neighbourhoods equals that of all intervals, and we single out the doubly irreducible ones among them as candidates for nonrefinable covers. Finally, we outline how the latter can be found using information on isomorphy of interval sublattices.

This is a joint work with F. M. SCHNEIDER (Technische Universität Dresden).

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An "easy" problem by McNulty

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In 2010, McNulty introduced a list of "A Dozen Easy problems", which contained new and revived challenges for the Universal Algebra community. One question asks for a classification of dualizable finite automatic algebras. In this talk we will give an introduction to automatic algebras and the problem in question, present various partial solutions (both positive and negative), and comment on the general difficulty of the task.

This is a joint work with B. A. DAVEY (La Trobe University), J. G. PITKETHLY (La Trobe University), and R. WILLARD (University of Waterloo).

### CSP dichotomy for special oriented trees

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An oriented tree is *special* if it can be constructed from an oriented tree of height 1 by replacing every edge by a minimal path of some fixed height. Using recent results on the algebraic approach to CSP we will prove the CSP dichotomy for special trees with maximum degree at most 3. Every such tree is either NP-complete or has bounded width. Moreover, in the latter case every absorption-free subalgebra of its polymorphism algebra has totally symmetric idempotent operations of all arities. We will discuss possible generalizations.

### Monoids of powers in varieties of *f*-idempotent groupoids

### VESNA CELAKOSKA-JORDANOVA

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Powers in a variety  $\mathcal{V}$  of groupoids are defined as elements of a free groupoid  $E_{\mathcal{V}} = (E_{\mathcal{V}}, \cdot)$  in  $\mathcal{V}$ , with one-element generating set  $\{e\}$ . A new operation " $\circ$ " (superposition of powers) is defined and the monoid  $(E_{\mathcal{V}}, \circ, e)$  of groupoid powers in  $\mathcal{V}$  is obtained. If  $\mathcal{V}_c$  is the variety of commutative groupoids in  $\mathcal{V}$ , then the corresponding monoid of powers in  $\mathcal{V}_c$  is denoted by  $E_{\mathcal{V}_c} = (E_{\mathcal{V}_c}, \circ, e)$ . We consider the varieties of: idempotent groupoids, right idempotent groupoids, left and right idempotent groupoids and *f*-idempotent groupoids, where *f* is an irreducible groupoid power with the length at least 3. The monoids of powers in these varieties of groupoids are constructed and the following two questions are considered: 1) Is the monoid  $(E_{\mathcal{V}}, \circ, e)$  free? 2) Are the monoids  $(E_{\mathcal{V}}, \circ, e)$  and  $(E_{\mathcal{V}_c}, \circ, e)$  isomorphic?

### The asymptotic number of ways to intersect two composition series

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Assume that  $\vec{H} = \{1 = H_0 \triangleleft H_1 \triangleleft \cdots \triangleleft H_n = G\}$  and  $\vec{K} = \{1 = K_0 \triangleleft K_1 \triangleleft \cdots \triangleleft K_n = G\}$  are composition series of a group *G*. Let  $(\{H_i \cap K_j : i, j \in \{0, \dots, n\}\}; \subseteq)$  be denoted by  $CSL_G(\vec{H}, \vec{K})$ . It is a partially ordered set. Actually,  $CSL_G(\vec{H}, \vec{K})$  is a lower semimodular lattice of length *n*. We call it a *composition series lattice*; this is where the notation CSL comes from. The number of (isomorphism classes of) lattices  $CSL_G(\vec{H}, \vec{K})$  of length *n* is denoted by f(n). Our goal is to determine its asymptotic behavior as follows.

THEOREM. f(n) is asymptotically n!/2. That is,  $f(n)/n! \rightarrow 1/2$  as  $n \rightarrow \infty$ .

The proof is based on three different areas. From group theory, we need a 1939 result of H. Wielandt implying that  $CSL_G(\vec{H}, \vec{K})$  is really a lattice. From lattice theory, we need a recent description of these lattices by permutations, due to G. Czédli and E. T. Schmidt. Finally, since different permutations may determine isomorphic lattices, we need a combinatorial argument to conclude the proof; this part is due to G. Czédli, L. Ozsvárt, and B. Udvari.

This is a joint work with E. TAMÁS SCHMIDT (Budapest University of Technology and Economics), LÁSZLÓ OZSVÁRT (University of Szeged), and BALÁZS UDVARI (University of Szeged).

### CSP dichotomy and H-bipartite digraphs

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In this talk we will investigate the reduction of a general CSP with a finite template to the homomorphism problem for finite digraphs. This reduction in its original form is due to T. Feder and M. Vardi. As it turns out, this construction can be refined in a way that reduces the original CSP to the one for a very particular class of digraphs: the h-bipartite digraphs. While it is known in general that digraph CSPs cannot exhibit the full range of polymorphism properties found over arbitrary CSPs, the discrepancy is actually rather small: the majority of interesting equational properties carry over directly to digraphs. As a consequence, many open problems arising in the algebraic study of CSPs are equivalent to corresponding problems restricted to the class of digraphs. In particular, we can give an essentially algebraic proof that there are finite loopless digraphs whose homomorphism problem is in P but are neither of bounded width nor do they have edge polymorphisms of any arity. Another consequence is that the algebraic dichotomy conjecture is equivalent to its restriction to digraphs.

This is a joint work with J. BULÍN (Charles University), M. JACKSON and T. NIVEN (La Trobe University).

### On symmetry groups of Boolean and other functions

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The talk starts with some results of [CloK91], [Hor94], [Kis98], [Wnu80] about symmetry (invariance) groups of *n*-ary functions on a *k*-element set (in particular, of Boolean functions).

Let  $f : \{0, 1, ..., k - 1\}^n \to \{0, 1, ..., k - 1\}$ . We say that f is invariant under the permutation  $\sigma \in S_n$  and write  $\sigma \vdash f$ , if for all  $(x_1, ..., x_n) \in \{0, 1, ..., k - 1\}^n$ ,  $f(x_1, ..., x_n) = f(x_{\sigma 1}, ..., x_{\sigma n})$ . The relation  $\vdash$  induces a Galois connection between permutations and functions.

We characterize the Galois-closed groups in the cases  $k \in \{n - 1, n - 2\}$ . Moreover, some results about computer calculations about the Galois closed groups in case  $k < n \le 7$  will be given.

This is a joint work with GÉZA MAKAY (University of Szeged) and REINHARD PÖSCHEL (Technische Universität Dresden).

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### Equational theory of left divisible left distributive groupoids

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The fact that the following varieties of groupoids coincide:

- the variety generated by groups with the conjugacy operation,
- the variety generated by left distributive idempotent left quasigroups,
- the variety generated by left cancellative left distributive idempotent groupoids,
- the variety generated by left divisible left distributive idempotent groupoids,

was consecutively proved by Joyce, Kepka and Larue.

There exists a similar result without idempotency by Kepka and Dehornoy stating that

- the variety generated by groups with the half-conjugacy operation,
- the variety generated by left distributive left quasigroups,
- the variety generated by left cancellative left distributive left idempotent groupoids

are equal. It is thus an open question whether the variety generated by left divisible left distributive groupoids falls in this list too. We give a partial result showing that any left divisible left distributive groupoids, with the property that  $a \mapsto a^2$  is a surjective mapping, lies in the variety generated by left distributive left quasigroups.

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In 1980, G. Bijev and K. Todorov introduced the concept of coregular semigroups in their correspondent paper. Every element *s* in such a special semigroup satisfies the relation  $s^3 = s$ . Our main concern is the description of coregular subsemigroups of the full transformation semigroup  $T_n$ . Therefore, we focus for now on the maximal coregular subsemigroups of the ideals  $K(n, r) = \{\alpha \in T_n : |Im(\alpha)| \le r\}$ . These ideals are not coregular on their own – with exception of the ideal K(n, 1)of constant transformations. As a first result we represent a characterization of all maximal coregular subsemigroups in case of K(n, 2).

### Idempotent tropical matrices and their (tropical) geometry

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Tropical mathematics is the mathematics of the real numbers, together with the operations of addition and maximum. This mathematical structure, called the tropical semifield, shares many of the key properties of a field, with addition playing the role of multiplication, and maximum the role of addition. Let us consider the  $n \times n$ matrices with entries in the tropical semifield. These tropical matrices can be added together and multiplied just as in classical mathematics, using the new tropical operations of addition and maximum in place of multiplication and addition.

Recent research has shed new light on the algebraic, combinatorial and geometric structure of idempotent tropical matrices (that is, those matrices satisfying  $E^2 = E$ , tropically). In this talk I will present some consequences of this work.

This is a joint work with MARK KAMBITES (University of Manchester).

### Multiplication and geometry of tropical matrices

MARK KAMBITES School of Mathematics University of MANCHESTER (GBR) Mark.Kambites@manchester.ac.uk

I shall report on a programme of research aiming to understand matrix semigroups over the tropical (max-plus) semiring. Their structure turns out to be intimately connected with the geometry of tropical convexity; indeed, every algebraic property of the full  $n \times n$  tropical matrix semigroup seems to manifest itself in some beautiful geometric phenomenon involving tropical polytopes. I shall try to explain (in a way accessible to a broad audience) how these connections arise, and the insight they give in both semigroup theory and tropical geometry.

Various parts of the research described are joint work with people including CHRIS-TOPHER HOLLINGS, ZUR IZHAKIAN and MARIANNE JOHNSON.

### Piecewise testable languages and the word problem

### KAMILLA KÁTAI-URBÁN

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Piecewise testable languages are widely studied area in the theory of automata. We analyze the algebraic properties of *k*-piecewise testable languages via their syntactic monoids. A normal form of the words is presented for k = 2 and 3. Moreover an asymptotic formula is given for the logarithm of the number of words for arbitrary *k*.

This is a joint work with PÉTER PÁL PACH, GABRIELLA PLUHÁR, CSABA SZABÓ (Eötvös Loránd University) and ANDRÁS PONGRÁCZ (Central European University).

# Essential variables of cooperations and the consequences for operations

### SEBASTIAN KERKHOFF

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When studying functions of multiple variables, it is natural to distinguish between those variables that have an effect on the function's value, called essential variables, and those variables that do not influence the function's value, called nonessential (or fictitious) variables. In the past decades, questions related to the notion of essential variables such as the determination of the so-called arity sequence (also called  $p_n$ -sequence) or the arity gap were investigated for operations of various kinds, e.g., for homomorphisms between structures, for term functions of an algebra, or for the functions of a given clone.

In the talk, we propose a different approach where we generalize the essentiality of variables and all the corresponding notions into a category-theoretic setting and investigate them for cooperations (also called dual operations) in a rather abstract way. Although the notion of essential variables of cooperations is simply the dualized notion of essential variables of operations, we will see that cooperations offer a different view on essential variables and that this view makes several problems much easier to solve than in the usual scenario.

Although this general investigation might be considered as an interesting task in itself, we will show that it is also beneficial for those that are mainly interested in operations like those mentioned above. In fact, by using the principle of duality, we will outline how the results for cooperations can be used to obtain new results for questions related to essential variables in the classical scenario, that is, for operations on sets.

# New maximal subsemigroups of the semigroup of all transformations on a countable set

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The maximal subsemigroups of the full transformation semigroup T(X) for a finite set X are well known. Maximal subsemigroups of T(X) with particular properties were also determined for this semigroup. The situation is quite different if X is infinite. First, Lutz Heindorf [2] and Michael Pinsker [3] have determined the maximal semigroups of T(X) containing the symmetric group if X is countable infinite, and if X uncountable infinite, respectively. Recently, J. East, J. D. Mitchell, and Y. Péresse [1] have characterized maximal subsemigroups of T(X) containing particular subgroups (particular stabilizers) of the symmetric group. We want to proceed the study of the maximal subsemigroups of T(X) if X is countable. So, we consider subsemigroups  $W \leq T(X)$  such that there is an  $\alpha \in T(X) \setminus W$  which is a generator of T(X) modulo W. In our main theorem, we characterize the maximal subsemigroups of T(X) containing such a subsemigroups of T(X). As a consequence of this theorem, we obtain all maximal subsemigroups of T(X) containing the set  $T(X) \setminus A$  for any (of the five) maximal subsemigroups A of T(X) containing the symmetric group.

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# Categorical equivalence of semilattices and lattices

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A variety of algebras is considered to be a category here; the objects are the algebras in the variety and the morphisms are the homomorphisms between them. Two algebras *A* and *B* are called categorically equivalent, if there is a categorical equivalence between the varieties they generate that sends *A* to *B*.

In [1], Zádori explored categorical equivalence of finite groups by proving that two categorically equivalent finite groups are weakly isomorphic.

We have studied the categorical equivalence of other algebras of the same type. We proved the following two propositions:

- (1) Semilattices are categorically equivalent if and only if they are isomorphic.
- (2) Lattices with the smallest (or the largest) element, in particular, finite lattices, are categorically equivalent if and only if they are isomorphic.

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## Linear complexity of extensions of Fermat quotients

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The linear complexity is a quality measure for sequences over finite fields used in cryptography. The sequence of Fermat quotients guarantees a high linear complexity. We generalize the notion of Fermat quotients and determine results about the linear complexity of the resulting sequences. The first step is to define extended Fermat quotients for polynomials in  $\mathbb{Z}[X]$ , and in the second step we define higher order extended Fermat quotients to achieve a larger period length. This research is part of a student's thesis project supervised by ARNE WINTERHOF, RICAM Linz.

# Generalized entropy in algebras with neutral element and in inverse semigroups

#### ERKKO LEHTONEN

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An algebra  $\mathbf{A} = (A; F)$  is said to have the *generalized entropic property* if for every *n*-ary  $f \in F$  and every *m*-ary  $g \in F$ , there exist *m*-ary term operations  $t_1, \ldots, t_n$  of  $\mathbf{A}$  such that  $\mathbf{A}$  satisfies the identity

$$g(f(x_{11}, x_{21}, \dots, x_{n1}), \dots, f(x_{1m}, x_{2m}, \dots, x_{nm})) \approx f(t_1(x_{11}, x_{12}, \dots, x_{1m}), \dots, t_n(x_{n1}, x_{n2}, \dots, x_{nm})).$$

We investigate the relationships between the generalized entropic property and the commutativity of the fundamental operations of an algebra. In particular, we show that an algebra with a neutral element has the generalized entropic property if and only if it is derived from a commutative monoid, and an inverse semigroup has the generalized entropic property if and only if it is commutative.

This is a joint work with AGATA PILITOWSKA (Warsaw University of Technology).

## Centralizing monoids on a three-element set

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Let *A* be a non-empty set and  $\mathcal{O}_A$  be the set of (multi-variable) functions defined on *A*. For a subset *S* of  $\mathcal{O}_A$  the *centralizer* of *S*, denoted by *S*<sup>\*</sup>, is the set of functions in  $\mathcal{O}_A$  which commute with all members of *S*. A monoid *M* of unary functions on *A* is a *centralizing monoid* if it is the unary part of the centralizer *S*<sup>\*</sup> for some subset *S* of  $\mathcal{O}_A$ . Equivalently, *M* is a centralizing monoid if *M* is the unary part of *M*<sup>\*\*</sup>.

In this talk, we consider the case where *A* is a three-element set, i.e.,  $A = \{0, 1, 2\}$ . We determine all centralizing monoids on *A*. The total number of centralizing monoids on *A* is 192, which are divided into 48 conjugate classes. In the course of discussion, Kuznetsov criterion is applied to judge, negatively, that certain monoid is not a centralizing monoid while a set of functions called "witness" is found to determine, positively, that certain monoid is a centralizing monoid.

This is a joint work with I. G. ROSENBERG (Montreal, Canada).

# The discrete logarithm problem in semisimple group algebras

# AYAN MAHALANOBIS

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Public key cryptography is the backbone of our modern world of secure communications. In public key cryptography, the discrete logarithm problem provides us with a secure cryptographic primitive, on which many cryptosystems like the ElGamal cryptosystem are built.

The discrete logarithm problem can be described in any cyclic group, or rather in a cyclic subgroup of any group. We will talk about the discrete logarithm problem in the group of non-singular circulant matrices. This discrete logarithm problem is interesting in its own right, as it is unique and different from other discrete logarithm problems used in practice.

We will show that a cryptosystem built on this discrete logarithm problem will be fast and secure. This approach of looking at the discrete logarithm problem in circulant matrices is the first step towards understanding the discrete logarithm problem in a semisimple group algebra.

## CSP dichotomy on small structures

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Given a finite relational structure (template), the fixed-template Constraint Satisfaction Problem asks whether there exists a homomorphism from a similar finite relational structure (which is the input of the problem) into the template. Dichotomy Conjecture about complexity of the Constraint Satisfaction Problem states that each fixed-template Constraint Satisfaction Problem is either tractable or NP-complete (depending on which template is fixed). We review the recent results on the CSP Dichotomy Conjecture which attempt to solve it in small cases. The highlight of our lecture is the result that the Dichotomy Conjecture holds for all templates having at most four elements, but we will also briefly review other recent results on small templates, as well as another application of the main technique used in our proof.

This is a joint work with BOJAN BAŠIĆ, MIKLÓS MARÓTI, SLAVKO MOCONJA and PREDRAG TANOVIĆ.

# Properties of the automorphism group and a probabilistic construction of a class of countable labeled structures

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For a class of countably infinite ultrahomogeneous structures that generalize edgecolored graphs, and that we refer to as *labeled structures*, we provide a probabilistic construction. As a special case, this construction yields an elementary probabilistic construction of the rational Urysohn space. Also, we give fairly general criteria for the automorphism group of such structures to have the small index property and strong uncountable cofinality, thus generalizing some results of Solecki, Rosendal, and several other authors.

This is a joint work with IGOR DOLINKA (University of Novi Sad).

# Inverse semigroups: some open problems

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This talk will discuss several open problems in the theory of inverse semigroups.

## VALENTINA MIOVSKA

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The (m + k, m)-bands are (m + k, m)-semigroups which satisfy five identities. Here, a generalization of (m + k, m)-bands is made by replacing identity

$$\begin{bmatrix} m+k\\ x \end{bmatrix} = \overset{m}{x}$$

with the identity

$$\begin{bmatrix} a & a & a & a \\ a & x & a & x & a \end{bmatrix}_{i} = \begin{bmatrix} a & a & a & a & a \\ a & x & a & x & a & a \end{bmatrix}_{j},$$

for a fixed element *a* and  $i, j \in \mathbb{N}_m$ . Structural description and characterization of generalized (m + k, m)-bands are given. Free generalized (m + k, m)-bands are also described.

This is a joint work with DONČO DIMOVSKI (Faculty of Natural Sciences and Mathematics, Ss. Cyril and Methodius University, Skopje).

## Many-sorted and single-sorted algebras

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A pure heterogeneous algebra is an algebra with either all empty or all non-empty sorts. J.M. Barr proved that the class of all pure heterogeneous algebras in a variety is equivalent to a variety of single-sorted algebras. The identities defining the single-sorted variety, or even the type of the variety, were not given explicitly.

In this talk we will present a detailed, fully type-based general method for translating the class of all pure, many-sorted algebras of a given constant-free type into an equivalent variety of single-sorted algebras of defined, constant-free type. In terms of identities, this single-sorted variety has a complicated basis. We show that it is possible to characterize the single-sorted variety by simple identities and quasi-identities involving only a small number of variables.

This is joint work with JONATHAN D.H. SMITH (Iowa State University, Ames, Iowa, U.S.A.) and ANNA B. ROMANOWSKA (Warsaw University of Technology).

# How many higher commutator operations can we define on the congruence lattice of a given Mal'cev algebra?

#### Nebojša Mudrisnki

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Higher commutator operations on the congruence lattice of an algebra have been introduced by A. Bulatov as a generalization of the binary commutator operation. They are a sequence of operations (one operation for each natural number n, n > 1). On Mal'cev algebras they satisfy certain properties. The question is:

How many sequences of operations on the congruence lattice of a given Mal'cev algebra do exist that satisfy these properties?

We describe the cases when there are finite, countable, or uncountable many of them.

This is joint research with E. AICHINGER (JKU Linz, Austria).

Universal homogeneous constraint structures and the hom-equivalence classes of weakly oligomorphic structures

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Weak oligomorphy is a natural weakening of the notion of oligomorphy. A countable relational structure is weakly oligomorphic iff its endomorphism monoid has of every arity only finitely many invariant relations. Clearly, every oligomorphic structure is weakly oligomorphic, but the reverse does not hold, in general. While the Ryll-Nardzewski theorem says that any countable oligomorphic structure is  $\aleph_0$ -categorical (i.e., it is up to isomorphic structures that have the same positive existential theory, are hom-equivalent. This observation motivates to study hom-equivalence classes of weakly oligomorphic structures.

In this talk, we will report on recent results regarding the hom-equivalence classes of weakly oligomorphic relational structures equipped with the embedding quasi order. In particular, we will describe its extremal elements. The emphasis will be on largest elements — i.e. on universal structures. To this end we will introduce constraint relational structures, and, using a category-theoretic version of Fraïssé's Theorem due to Droste and Göbel, we will show that there exist universal homogeneous constraint relational structures. These structures give raise to universal elements in hom-equivalence classes of weakly oligomorphic structures. Moreover, when the signature is finite, these structures turn out to be  $\aleph_0$ -categorical. While the hom-equivalence class of a weakly oligomorphic homomorphism homogeneous structure always contains a smallest element that is homogeneous, oligomorphic, and a core, we have (somewhat surprisingly) that the largest elements of such hom-equivalence classes will not be homogeneous. We conclude the talk by pointing out the small index property for the automorphism groups of certain universal homogeneous constraint structure.

This is a joint work with MAJA PECH (University of Novi Sad).

# Galois connections between group actions and functions – some results and problems

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Let  $\Gamma$  be a group acting on a set A and let  $f \in K^A$  for some set K (e.g.  $K = \{0, 1\}$ ). For  $\sigma \in \Gamma$  and  $f \in K^A$ , the relation

$$\sigma \vdash f : \iff \forall x \in A : f(x^{\sigma}) = f(x)$$

induces a Galois connection between subsets of  $\Gamma$  and subsets of  $K^A$ . Galois closed subsets of  $\Gamma$  are special subgroups. These Galois closed subgroups are characterized in general as well as for special group actions  $(A, \Gamma)$ . Some application and problems are mentioned.

This is a joint work with E. FRIESE (Rostock University, Germany).

# Unary polynomial functions on a class of finite groups

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We describe unary polynomial functions on groups *G* that are semidirect products of a finite elementary abelian group of exponent *p* and a cyclic group of prime order *q*,  $p \neq q$ .

This is a joint work with KALLE KAARLI (University of Tartu).

## Generalized convexity and closure conditions

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Convex subsets of affine spaces over the field  $\mathbb{R}$  of real numbers may be described as so-called barycentric algebras. We will discuss possible extensions of the geometric and algebraic definitions of convex sets to the case of convex subsets of affine spaces over more general rings, in particular over principal ideal subdomains of  $\mathbb{R}$ . We will discuss some of the consequences of the new definitions, in particular those concerning the concept of algebraic closure and its relation to topological closure.

The results form part of a larger project being undertaken by GÁBOR CZÉDLI (Univerity of Szeged) and myself.

## A relational localisation theory for topological algebras

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In 2001, inspired by "classical" Tame Congruence Theory, Ágnes Szendrei and Keith Kearnes developed a powerful relational localisation theory for finite algebras that studies structures by decomposing their respective relational counterparts. Our basic idea is to topologise the established concepts, replace finiteness arguments by reasoning in terms of approximation and develop a relational localisation theory which applies to arbitrary topological algebras. In fact, it turns out that this approach allows us to explore a topological algebra up to equivalence of the generated topological quasivariety. Moreover, we will illustrate how compactness properties of the underlying space or the generated operational clone, resp., make an impact on the introduced localisation process. Finally, some classes of examples, such as topological lattices, groups and continuous monoid actions, will be discussed.

# The minimal clones generated by semiprojections on a four-element set

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The set of minimal clones on a given finite set is known to be finite and by Rosenberg's classification theorem there are only five types. Although the description given there is necessary it is not sufficient. Especially the minimal clones generated by semiprojections are not exactly characterised by this, and for sets with at least four elements these minimal clones are not fully known.

We determined all minimal clones generated by semiprojections on a four-element set, whereby the list of all 6030 minimal clones on a four-element set is finished.

Weak congruence representability of lattices and related Delta-suitability of their elements

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A symmetric and transitive relation compatible with an algebra is said to be a weak congruence of the algebra. All the weak congruences of an algebra form an algebraic lattice under inclusion, which is called the weak congruence lattice. We treat the problem of the representability of algebraic lattices by the weak congruence lattices of algebras. To make this problem nontrivial, we set in advance an element different from zero in a given lattice which shall, if possible, be represented by the diagonal relation of an algebra representing the lattice. Any lattice having such an element, representable by the diagonal relation, is said to be nontrivially representable. Any such element is said to be  $\Delta$ -suitable.

We present some generalizations of the known results ([1, 2, 3, 4]), giving necessary conditions for an element of an algebraic lattice to be  $\Delta$ -suitable, [5]. These results are a tool for some brand new investigations of so-called derived representability. Namely, we investigate whether the representability of a lattice, or a set of lattices, under some conditions, implies the representability of other lattices. We present some results on the topic, including the representability of an ideal, or a filter, or some other sublattices or suborders of a representable lattice, [6]. Using a set of representable lattices we form a lattice similar to the direct product of the set, which is proved to be representable.

We also prove that the direct product of a representable lattice and an arbitrary algebraic lattice, slightly extended, is representable. Using this we come to a generalization of a known result, given in [7], giving a sufficient condition for a lattice to be representable. We also present a recent result proving that any atomic Boolean algebra is nontrivially representable.

This is a joint work with ANDREJA TEPAVČEVIĆ and BRANIMIR ŠEŠELJA (University of Novi Sad).

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# Almost structural completeness; an algebraic approach

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In logic the notion of structural completeness has received considerable attention for many years. By translating it to algebra, a quasivariety is structurally complete if it is generated by its free algebras. It appears that many logics (quasivarieties), like S5 or  $MV_n$  fails structural completeness for a rather immaterial reason. Therefore the adjusted notion was introduced: almost structural completeness. We characterize almost structurally complete quasivarieties in general, and also provide an applicable criterion for it.

This is a joint work with WOJCIECH DZIK (Silesian University, Katowice).

# Semigroups of *n*-ary operations on finite sets

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On the set of *n*-ary operations  $O^n(A)$  on finite set *A*, we define a binary operation + by f + g := f(g, ..., g) (composition of operations). The operation + is associative giving us a semigroup  $(O^n(A); +)$ . We study the semigroup-theoretical aspects of this structure and show its relationship to the full transformation semigroup.

## Predrag Tanović

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Let *T* be a countable, complete first-order theory having infinite models and such that every uncountable model of *T* is  $\aleph_0$ -saturated. Some 30 years ago David Kueker conjectured that *T* must be categorical in some infinite power. Hrushovski proved it for stable theories, for theories that interpret a linear ordering and for theories with Skolem functions. Using different techniques we reduce the conjecture to the following cases:

- (1) T = Th(M) where *M* is an *almost minimal* structure (meaning that  $M = acl(\emptyset)$  and that there is a unique non-algebraic 1-type).
- (2) *T* has infinitely many constants but does not have *the strict order property* (no definable ordering on *n*-tuples has infinite chains).

Local monotonicities and lattice derivatives of Boolean and pseudo-Boolean functions

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We propose local versions of monotonicity for Boolean and pseudo-Boolean functions: a function is said to be p-locally monotone if none of its partial derivatives changes in sign on tuples that differ in less than p positions. This parameterized notion provides a hierarchy of monotonicities. These local monotonicities are tightly related to lattice counterparts of classical partial derivatives via the notion of permutable derivatives. More precisely, p-locally monotone functions have ppermutable lattice derivatives and, in the case of symmetric functions, these two notions coincide. We provide further results relating local monotonicities and lattice derivatives, and present a classification of p-locally monotone functions, as well as of functions having p-permutable derivatives, in terms of certain forbidden "sections", i.e., functions which can be obtained by substituting constants for variables.

This is a joint work with MIGUEL COUCEIRO and JEAN-LUC MARICHAL (University of Luxembourg).

# **Representing lattices by lattices of subclasses**

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An abstract class of algebraic systems of a given signature is a (*finitary*) *prevariety*, if it is closed under taking substructures and (finite) Cartesian products.

We investigate the structure of lattices of subclasses of different types; among those are relative sub[quasi]variety lattices as well as relative [finitary] subprevariety lattices. In this talk, we focus on representing lattices by lattices of relatively axiomatizable classes and those of [finitary] prevarieties, also mentioning some general algebraic properties of those lattices.

In particular, we prove that the lattice of meet subsemilattices of an arbitrary meet semilattice with unit is isomorphic to the lattice of finitary subprevarieties of a prevariety, while the lattice of complete meet subsemilattices of an algebraic lattice is isomorphic to the lattice of subprevarieties of a quasivariety.

This is a joint work with MARINA V. SEMENOVA (Sobolev Institute of Mathematics, Siberian Branch RAS, Novosibirsk).

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