



UNIVERSITY OF NOVI SAD
FACULTY OF SCIENCES
Department of Mathematics and Informatics



AAA94 + NSAC 2017
The 94th Workshop on General Algebra
in conjunction with the
5th Novi Sad Algebraic Conference

ABSTRACTS

Novi Sad, Serbia, 15-18 June 2017

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We gratefully acknowledge the support of The Secretariat of Higher Education and Scientific Research of the Autonomous Province of Vojvodina.

Typeset by Igor Dolinka.
Printed by: *Futura*, Petrovaradin.



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PLENARY TALKS





A dichotomy for first-order reducts of unary structures

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Many natural decision problems can be formulated as constraint satisfaction problems for reducts A of finitely bounded homogeneous structures. This class of problems is a large generalisation of the class of CSPs over finite domains. In this talk I present a general polynomial-time reduction from such infinite-domain CSPs to finite-domain CSPs. We use this reduction to obtain new powerful polynomial-time tractability conditions that can be expressed in terms of the topological polymorphism clone of A . Moreover, we study the subclass C of CSPs for structures A that are first-order definable over a structure with a unary language. Also this class C properly extends the class of all finite-domain CSPs. We show that for the class C the general tractability conjecture of Pinsker and myself for reducts of finitely bounded homogeneous structures is equivalent to the finite-domain tractability conjecture.

Joint work with ANTOINE MOTTET.

The complexity of the Constraint Satisfaction Problem

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In this talk we present an algorithm solving constraint satisfaction problems that have been conjectured to be polynomial time solvable, and introduce the techniques to show the algorithm is sound.





From weak congruence lattices to circles and an online game

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The talk outlines how I got from weak congruence lattices to circles in the plane and to an online game. In order to do so, some results on *congruence lattices* and similar structures related to a *single* algebra, usually to a *lattice*, will be surveyed.

My experience with structures formed by congruence-like relations of a single algebra (rather than those of a *class* of algebras) started in *Novi Sad* ten years ago; see [Czédli, B. Šešelja, and A. Tepavčević, *AU* 58, 349–355 (2008)]. We proved that all subgroups of a finite group G are normal iff the diagonal relation $\{(x, x) : x \in G\}$ is a join-semidistributive element in the lattice of *weak congruences* of G .

Congruence lattices of *lattices* have been studied intensively for decades. In a 2007 paper, G. Grätzer and E. Knapp opened a new direction by putting planar semimodular lattices into the focus of interest. The talk outlines some results on these lattices and their congruence lattices. Congruence lattices of other lattices and posets of principal congruences of lattices are also included in the talk.

There are two by-products of the investigations mentioned above; both are outside Algebra and both can easily be explained even for non-mathematicians. First, we characterize *circles* in the plane with the help of a resilient rubber loop [Czédli 2016, arxiv:1611.09331]. Second, an online computer game [Czédli and Makay 2016, arxiv:1607.06809] based on lattice congruences is available at the authors' websites.





The homomorphism lattice induced by a finite algebra

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Each finite algebra \mathbf{A} induces a lattice $\mathbf{L}_\mathbf{A}$ via the quasi-order \rightarrow on the finite members of the variety generated by \mathbf{A} , where $\mathbf{B} \rightarrow \mathbf{C}$ if there exists a homomorphism from \mathbf{B} to \mathbf{C} . We introduce the question: ‘Which lattices arise as the homomorphism lattice $\mathbf{L}_\mathbf{A}$ induced by a finite algebra \mathbf{A} ?’ Our main result is that each finite distributive lattice arises as $\mathbf{L}_\mathbf{Q}$, for some quasi-primal algebra \mathbf{Q} . We also obtain representations of some other classes of lattices as homomorphism lattices, including all finite partition lattices, all finite subspace lattices and all lattices of the form $\mathbf{L} \oplus \mathbf{1}$, where \mathbf{L} is an interval in the subgroup lattice of a finite group.

Critical points for congruence lattices

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For a class \mathcal{K} of algebras we consider $\text{Con } \mathcal{K}$, the class of all lattices isomorphic to congruence lattices of algebras from \mathcal{K} . The task of describing $\text{Con } \mathcal{K}$ has proved extremely difficult, even for the most common classes of algebras, like groups or lattices. Therefore, we concentrate on a less ambitious aim of distinguishing $\text{Con } \mathcal{V}$ and $\text{Con } \mathcal{W}$ for (usually locally finite and congruence-distributive) varieties \mathcal{V} and \mathcal{W} . We define the critical point $\text{Crit}(\mathcal{V}; \mathcal{W})$ as the smallest cardinality of L_c for $L \in \text{Con } \mathcal{V} \setminus \text{Con } \mathcal{W}$. (Here L_c is the set of all compact elements L .)

We present an overview of results and methods, with an emphasis on the case when the critical point is infinite. The two main methods are based on the concept of diagram lifting, and on topological properties of dual spaces of congruence lattices.

An important result of P. Gillibert says that, under some reasonable restriction, the critical point cannot exceed \aleph_2 . We present pairs of finitely generated, congruence-distributive varieties with critical points \aleph_0 , \aleph_1



and \aleph_2 , and show some methods how to construct such examples. Recently, M. Ploščica proved that for varieties with Compact intersection property, the critical point cannot exceed \aleph_1 (again, under some finiteness conditions). The proof of this result provides an interesting link between liftability of diagrams and topological properties of the dual spaces.

Congruence lattices of diagram monoids

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It is well known that the lattice of normal subgroups (and hence congruences) of the symmetric group S_n ($n > 4$) is a three element chain. Mal'cev in 1952 showed that the congruence lattice of the full transformation monoid T_n is also a chain, even though its length depends (linearly) on n . Analogous results have been proved for the partial transformation monoid, symmetric inverse monoid, monoid of order preserving transformations of $\{1, \dots, n\}$ and the full matrix monoid over a division ring. In this talk I will discuss the corresponding question for the partition monoid, and some of its distinguished submonoids, such as the Brauer monoid, planar partition monoid, and Jones monoid. It will turn out that the lattices in question are no longer chains, and I will present a theoretical framework within which their structure (as well as that of the earlier, classical cases) can be explained.

This is joint work with JAMES EAST (Western Sydney), JAMES D. MITCHELL and MICHAEL TORPEY (St Andrews).

Diameter 3

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We will discuss a Ramsey proof for a certain class of finite metric spaces with distances in the set $\{0, 1, 2, 3\}$.

Quandles and universal algebra

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Quandles are algebraic structures arising naturally in knot theory and elsewhere. A standard approach to quandles is to consider some of the naturally associated groups, such as the multiplication group (generated by translations), or the associated group (freely generated subject to conjugation relations induced by the quandle operation). Recently, with Bonatto, Jedlicka, Pilitowska and Zamojska-Dzienio, we had a moderate success applying universal algebraic methods in quandle theory, and describing abstract universal algebraic concepts within the group theoretical framework.

In the lecture, I will give a brief overview of the group theoretic approach, and then report on our projects, based on the observation that, in quandles, the abstract notion of abelianness and centrality is closely related to abelianness and semiregularity of certain subgroups of the multiplication group.



Cube terms and the Subpower Membership Problem

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How hard is it to decide if a tuple belongs to a compatible relation of a finite algebra? In more detail, suppose that \mathbf{A} is a finite algebra, $\mathbf{R} \leq \mathbf{A}^n$ is the compatible relation (or subpower) of \mathbf{A} generated by a_1, \dots, a_k , and suppose that $b \in \mathbf{A}^n$. The problem is to decide if b belongs to \mathbf{R} . We will discuss the relationship between the time complexity of this problem and the strength of linear Mal'tsev conditions satisfied by \mathbf{A} .

Local finiteness for Green's relations in semigroup varieties

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The lattice of varieties of semigroups is studied with respect to the following concepts: a variety \mathcal{V} is said to be locally \mathcal{H} -finite, where \mathcal{H} stands for any of the five Green's relations, if every finitely generated semigroup from \mathcal{V} has only finitely many \mathcal{H} -classes.

The talk is based on a joint work with PEDRO SILVA and FILIPA SOARES.

The proof of the Constraint Satisfaction Problem

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Formally, the *Constraint Satisfaction Problem (CSP)* is defined as a triple $\langle \mathbf{X}, \mathbf{D}, \mathbf{C} \rangle$, where

- $\mathbf{X} = \{x_1, \dots, x_n\}$ is a set of variables,
- $\mathbf{D} = \{D_1, \dots, D_n\}$ is a set of the respective domains,





- $\mathbf{C} = \{C_1, \dots, C_m\}$ is a set of constraints,

where each variable x_i can take on values in the nonempty domain D_i , every constraint $C_j \in \mathbf{C}$ is a pair (t_j, ρ_j) where t_j is a tuple of variables of length m_j , called the *constraint scope*, and ρ_j is an m_j -ary relation on the corresponding domains, called the *constraint relation*.

Many natural combinatorial problems can be expressed as constraint satisfaction problems. This class of problems is known to be NP-complete in general, but certain restrictions on the form of the constraints can ensure tractability. The standard way to parameterize interesting subclasses of the constraint satisfaction problem is via finite constraint languages. The main problem is to classify those subclasses that are solvable in polynomial time and those that are NP-complete. It was conjectured that if a core of a constraint language has a weak near unanimity polymorphism then the corresponding constraint satisfaction problem is tractable, otherwise it is NP-complete.

In the talk we present an algorithm that solves Constraint Satisfaction Problem in polynomial time for constraint languages having a weak near unanimity polymorphism, which proves the remaining part of the conjecture.

We will need few definitions to formulate the main ideas of the algorithm. A CSP instance is called *cycle-consistent* if for every i and $a \in D_i$, any path starting and ending with x_i in Θ connects a and a . A CSP instance Θ is called *linked* if for every i and $a, b \in D_i$ there exists a path in Θ that connects a and b .

A CSP instance Θ is called *irreducible* if for any subinstance $\Theta' \subseteq \Theta$ and any set of variables \mathbf{X}' the projection of Θ' onto \mathbf{X}' is linked or subdirect.

The algorithm is based on the following three ideas. First, in polynomial time we can check whether our instance is cycle-consistent and irreducible. If it is not cycle-consistent or irreducible, then we can reduce the domain of at least one variable.

Second, if the instance is cycle-consistent and irreducible, then restricting any domain to a binary absorbing subuniverse cannot remove the only solution. The same can be done for two more cases: for a proper center, for an equivalence class of polynomially complete congruence.



Third, if there is no a binary absorbing subuniverse, a center, and a polynomially complete congruence, then for every nontrivial domain there exists a proper congruence σ on this domain such that the weak near-unanimity operation factorized by σ is equal to $t \cdot (x_1 + \dots + x_m)$, where t is an integer, $+$ is an operation of the Abelian group. In this case, we combine two methods. We solve the factorized instance, which is a system of linear equations. For some solutions of the factorized instance we check whether we have the corresponding general solution by solving constraint satisfaction problem on a smaller domain.



SHORT TALKS







A general algebraic motivation for loops

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In this talk we try to explain how loops might be helpful in finding the solution of an old problem in general algebra: is there a largest congruence modular variety?

Mal'cev clones on finite sets

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Given a finite algebra \mathbf{A} with a Mal'cev term, we consider the lattice of clones containing $\text{Clo}(\mathbf{A})$. This lattice is at most countable and satisfies the descending chain condition. We provide some information on some of its intervals, such as the interval between the polynomial and the congruence preserving functions. For many algebras, we can characterize when this interval is finite and when the congruence preserving functions are finitely generated.

We present results that have been obtained in joint work with G. HORVÁTH, M. LAZIĆ, and N. MUDRINSKI.

On the variety of strict pseudosemilattices

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The variety **SPS** of all strict pseudosemilattices is the smallest variety of pseudosemilattices which is not associative. It is generated by the four element pseudosemilattice of idempotents of the five element idempotent generated combinatorial completely 0-simple semigroup A_2 . The





following results will be discussed: (i) an identity basis for **SPS** is presented, (ii) **SPS** is shown to be inherently non-finitely based, (iii) **SPS** is shown to have no irredundant identity basis. The main ingredient of the proofs, which will be also presented, is a new graph theoretical model of the free pseudosemilattice on a set.

This is joint work with L. OLIVEIRA.

On classes of algebras axiomatizable by a collection of anti-identities and disjunctive identities

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An “anti-identity” is a universal formula such that its boolean part is a disjunction of negated equalities. Similarly, a “disjunctive identity” is a universal formula such that its boolean part is a disjunction of equalities. Fields are an example of a class of algebraic systems that can be defined by a theory containing only formulas of these forms.

In this talk we will study some problems related to these classes of algebras. In particular, we will characterize them by a theorem analogous to the HSP theorem. Unfortunately, it seems like we need ultraproducts to accomplish this, in contrast to Birkhoff’s result. This is, nevertheless, consistent with the fact that the corresponding characterization for classes axiomatizable by a collection of anti-identities uses ultraproducts.

Cores of oligomorphic clones revisited

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Manuel Bodirsky proved that every countably categorical structure is homomorphically equivalent to a unique model-complete core, that is,



a structure whose automorphism group is dense in the endomorphism monoid. Although this fact is essentially a claim about transformation monoids, Bodirsky's proof is heavily model-theoretic. As a part of a joint work with M. KOMPATSCHER, M. OLŠÁK, T. V. PHAM, and M. PINSKER we have found a shorter algebraic proof.

A problem on squares and its application to the square packing problem

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We treat the following problem: given an $n \times n$ square $ABCD$, determine the minimum number of points, denoted by $\text{punct}(n)$, that need to be chosen inside the square $ABCD$ such that there does not exist a unit square inside the square $ABCD$ containing none of the chosen points in its interior. In other words, we are interested to know how to most efficiently “destroy” a square-shaped object of side length n , where “destroying” is achieved by puncturing as few as possible small holes, and the square is considered “destroyed” if no unpunctured square piece of unit side length can be salvaged. We show that $\text{punct}(n) = n^2$ when $n \leq 7$, and give an upper bound for $\text{punct}(n)$ that is asymptotically equal to $\frac{2}{\sqrt{3}}n^2$, which we believe is asymptotically tight. We then generalize our reasoning in order to obtain a similar upper bound when $ABCD$ is a rectangle, as well as an upper bound for $\text{punct}(x)$ when x is not necessarily an integer. Finally, we show that our results have an application to the problem of packing a given number of unit squares in the smallest possible square; it turns out that our results present a general “framework” based on which we are able to reprove many results on the mentioned problem (originally obtained independently of each other) and also obtain a new result on packing 61 unit square.

This is a joint work with ANNA SLIVKOVÁ (Novi Sad).



Centralizing monoids with majority witnesses on four-element domains

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Extending previous work by Goldstern, Machida and Rosenberg, we attempt to describe all unary parts of centralizer clones (centralizing monoids) determined by majority operations on a four-element set. As opposed to the three-element case studied by Machida et al., they are too many in number to present them all, but nevertheless we shall give an overview of the different cases into which this question can be broken down.

Infinitely many reducts of homogeneous structures

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It is shown that the countably infinite dimensional pointed vector space (the vector space equipped with a constant) over a finite field has infinitely many first-order reducts. This implies that the countable homogeneous Boolean algebra has infinitely many reducts. Our construction over the 2-element field is related to Reed-Muller codes.

Taylor and Mal'tsev quandles

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Quandles are idempotent left-distributive left-quasigroups and they arise in different areas of mathematics as knot theory, the study of braided vector spaces and the classification of pointed Hopf algebras.





Because of *left-distributivity*, a lot of different properties of a quandle can be translated in group-theoretical properties of its *displacements group* defined as follows:

$$Dis(X) = \langle L_x L_y^{-1}, x, y \in X \rangle.$$

As a further example of the importance of the displacements group in quandle theory, DAVID STANOVSKÝ and I developed a commutator theory for quandles, which is based on the existence of a Galois connection between the congruence lattice of a quandle and the lattice of some normal subgroups of the left-multiplication groups contained in the displacements group.

From a universal algebraic point of view, it is interesting to give a characterization of *Taylor* quandles (quandles admitting a Taylor term), in order to capture quandles satisfying any non-trivial *Mal'tsev conditions*. In the finite case, the general characterization of Taylor algebras turns out to be a meaningful and quite natural condition for quandles.

As a further outcome, we proved that Mal'tsev quandles (quandles admitting a Mal'tsev term) and Taylor quandles coincide.



\mathcal{R} -cross-sections of the semigroup of order-preserving transformations of a finite chain

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Let ρ be an equivalence relation on a semigroup S . A subsemigroup S' of S called a ρ -*cross-section* of S if S' contains exactly one representative of each ρ -class. Of course, given S and ρ , a ρ -cross-section need not exist, and if it exists, it need not be unique. A natural research direction is to study the existence of cross-sections of Green's relations in classical transformation semigroups and to classify these cross-sections when they exist, cf. [1, Chapter 12]. For the semigroup \mathcal{T}_n of all full transformations of the set $[n] = \{1, 2, \dots, n\}$ (written on the right), the corresponding results can be summarized as follows:





Green's relation \mathcal{H}	\mathcal{H}	\mathcal{R}	$\mathcal{J} = \mathcal{D}$	\mathcal{L}
\mathcal{H} -cross-sections	exist only for $n = 1, 2$, unique [1]	exist, unique up to isomorphism [1]	exist, no description is known	exist, not unique, even up to isomorphism [2,3]

A transformation $\alpha \in \mathcal{T}_n$ is called *order-preserving* provided that $x \leq y$ implies $x\alpha \leq y\alpha$ for all $x, y \in [n]$. The set of all order-preserving transformations forms a well-studied (cf. [1,Chapter 14]) regular subsemigroup of \mathcal{T}_n denoted by \mathcal{O}_n . We address the question of studying cross-sections of Green's relations in \mathcal{O}_n .

The semigroup \mathcal{O}_n is \mathcal{H} -trivial whence its only \mathcal{H} -cross-section is \mathcal{O}_n itself. From the description of \mathcal{L} -cross-sections of \mathcal{T}_n (cf. [2,3]), it can be seen that each of these cross-sections respects a certain linear order on $[n]$, and thus, gives rise to an \mathcal{L} -cross-section in \mathcal{O}_n . Conversely, each \mathcal{L} -cross-section in \mathcal{O}_n serves also as an \mathcal{L} -cross-section in \mathcal{T}_n . The situation with \mathcal{R} -cross-sections of \mathcal{O}_n turns out to be much more involved as they do not reduce to \mathcal{R} -cross-sections of \mathcal{T}_n . Nevertheless, we have managed to give a complete classification of all \mathcal{R} -cross-sections of \mathcal{O}_n in terms of certain binary trees.

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Spirals

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Which minimal clones (that is, clones having no nontrivial proper reduct) on a finite set are Taylor algebras? As it turns out, there are three types of such algebras: idempotent reducts of vector spaces over a prime



field, minimal majority algebras, and a new type of algebra generalizing 2-semilattices which I call minimal spirals. Spirals are defined as algebras with a single binary commutative idempotent operation, such that any subalgebra which is generated by two elements either has size two or maps surjectively onto the free semilattice on two generators. The collection of spirals is closed under taking products, subalgebras, and “locally finite quotients”, so the finite spirals form a pseudovariety. I’ll give some examples of minimal spirals and prove some of their basic properties.

On groupoids of relations with primitive-positive operations

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A set of binary relations closed with respect to some collection of operations on relations forms an algebra that is called an *algebra of relations*. A. Tarski was the first to treat algebras of relations from the point of view of universal algebra [1]. For any set Ω of operations on binary relations, let $R\{\Omega\}$ ($R\{\Omega, \subseteq\}$) denote the class of all algebras (partially ordered algebras) isomorphic to ones whose elements are binary relations and whose operations are members of Ω . Let $Var\{\Omega\}$ ($Var\{\Omega, \subseteq\}$) be the variety and $(Qvar\{\Omega\})$ ($(Qvar\{\Omega, \subseteq\})$) be the quasivariety generated by $R\{\Omega\}$ ($R\{\Omega, \subseteq\}$).

One of the most important classes of operations on relations is the class of *primitive-positive* operations [2] (in other terminology – Diophantine operations [3,4]). An operation on relations is called primitive positive, if it can be defined by a formula containing in its prenex normal form only existential quantifiers and conjunctions. Equational and quasiequational theories of algebras of relations with primitive positive operations are described in [3,4,5].

We will consider algebras of relations with one binary operation, i.e., groupoids of relations. The motivation of these investigations and some results can be found in [6,7,8].



Let us focus our attention on the following binary primitive-positive operation on relations: $\rho * \sigma = \{(x, y) \in X \times X : (\exists z)(x, x) \in \rho \wedge (x, z) \in \sigma\}$, where ρ and σ are relations on X .

Theorem 1. *The class $R\{*\}$ forms a variety. A groupoid (A, \cdot) belongs to the variety $R\{*\}$ if and only if it satisfies the identities:*

- (1) $x(xy) = xy$, (2) $(xy)y = xy$, (3) $x^2y = xy$, (4) $xy^2 = yx^2$,
 (5) $(xy)z = (xz)y$, (6) $x(yz) = y(xz)$, (7) $(xy^2)z = x(y^2z)$.

Theorem 2. *The class $R\{*, \subseteq\}$ forms a quasivariety. The quasivariety $Qvar\{*, \subseteq\}$ does not form a variety.*

A partially ordered groupoid (A, \cdot, \leq) belongs to the variety $Var\{*, \subseteq\}$ if and only if it satisfies the identities (1)-(7) and the identity (8) $xy \leq x^2$.

A partially ordered groupoid (A, \cdot, \leq) belongs to the class $R\{*, \subseteq\}$ if and only if it satisfies the identities (1)-(8) and the quasiidentity (9) $x \leq y^2 \rightarrow x \leq yx$.

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Finite coverability property

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A class of algebras \mathcal{K} has the finite embeddability property (FEP) if every finite partial subalgebra of any algebra from \mathcal{K} can be embedded into a finite member of \mathcal{K} . It is well known to be a key notion when dealing with the word problem and it is also often used in logic. A slightly different approach is obtained after the following generalization. An algebra \mathcal{A} satisfies the generalized finite embeddability property (GFEP) for a class \mathcal{K} of algebras of the same type if every finite partial subalgebra of \mathcal{A} can be embedded into an algebra from \mathcal{K} . For algebras of a finite type it can be proved that \mathcal{A} satisfies GFEP for \mathcal{K} if and only if $\mathcal{A} \in ISP_U(\mathcal{K})$. The aim of the talk is to present an analogous representation of the class $HSP_U(\mathcal{K})$. Although the problem may seem quite similar, it emerged that dealing with the corresponding condition called FCP is not as straightforward as it may appear.



Clonoids and Promise CSP

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Let \mathbb{A}, \mathbb{B} be a fixed pair of relational structures in the same language. The Promise Constraint Satisfaction Problem, $PCSP(\mathbb{A}, \mathbb{B})$, asks for an algorithm that accepts input structures \mathbb{X} such that \mathbb{X} maps homomorphically to \mathbb{A} , and rejects whenever \mathbb{X} has no homomorphism to \mathbb{B} .

A substantial part of the algebraic approach to the CSP naturally carries over to the case of PCSP. The algebraic counterpart is the *polymorphism clonoid*: the set of all homomorphisms from finite powers of \mathbb{A} to \mathbb{B} . I will discuss some tractability and promise-NP-hardness results and a possible complexity classification of PCSPs.





On a combinatorial problem on directed graphs

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We consider for a directed graph $G = (V, A)$ equivalence relations on value functions $f, g : V \rightarrow \mathbb{N}$ on the vertices defined on terms of rearrangement functions $r : \{(x, y) | y \in S(x)\}$, satisfying the equalities $f(x) = \sum_{y \in S(x)} r(x, y)$ and $g(y) = \sum_{x \in P(y)} r(x, y)$. We determine the equivalence classes and count the elements of an equivalence class.

Finite group presentations, van Kampen diagrams and a combinatorial problem

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We associate with a group presentation $G = \langle g_1, \dots, g_k | r_1, \dots, r_l \rangle$ the directed graph of the van Kampen diagram and consider an equivalence relation on the set of value functions $f, g : G \rightarrow \mathbb{N}$, defined in terms of rearrangement functions. We determine the equivalence classes and count the elements of an equivalence class.

A semigroup-theoretical approach to the study of generalized inverses

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The purpose of this talk is to show that many fundamental concepts and results of the theory of generalized inverses of matrices can be expressed in purely semigroup-theoretical terms, as well as to present some new



results obtained in our recent research on generalized inverses in semigroups [2]. We provide various characterizations of outer and inner inverses of an element of a semigroup which belong to the prescribed Green's equivalence classes, what is a direct generalization of outer and inner inverses of matrices with the prescribed range and/or null space. In particular, we show that both Mary's inverse along an element [LAA 434 (2011) 1836–1844] and Drazin's (b,c)-inverse [LAA 436 (2012) 1909–1923] represent essentially the same concept, and both coincide with the outer inverse belonging to the prescribed Green's H -class. The only difference is in a way of representing the H -class. We also introduce the concept of a trace factorization of an element of a semigroup, that can be viewed as a semigroup-theoretical generalization of the full-rank factorization of matrices. Using trace factorizations we provide numerous representations of the outer inverse belonging to the prescribed Green's H -class.

We also discuss the same problems in the context of involutive semigroups, where we give new results concerning least-squares and minimum-norm outer inverses, Moore-Penrose inverses, and core and dual core inverses.

Extremely important results of our research are the existence criteria for outer and inner inverses belonging to the prescribed Green's equivalence classes, which are given in terms of solvability of certain systems of linear equations in a semigroup. Solving these systems of equations allows to effectively compute outer and inner inverses with the required properties. This has been done for matrices with entries in the field of complex numbers (in [4]) and fuzzy matrices with entries in residuated lattices (in [1]), as well as in the context of involutive residuated semigroups and involutive quantales (in [3]).

Joint research with PREDRAG STANIMIROVIĆ and JELENA IGNJATOVIĆ (Niš).

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A representation theorem for reduced Rickart rings

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A ring R is called a (left) Rickart ring iff for every element $a \in R$ there exists an idempotent $e \in R$ such that, for all $x \in R$, $ax = 0$ if and only if $ex = x$. By a Sussman ring (first described by I. Sussman and originally called an associate ring) we mean a subdirect product S of unitary rings without zero divisors R_i which has "enough" idempotents in the following sense: for every $(a_i)_{i \in I} \in S$, the ring S contains also the idempotent obtained by replacing every non-zero component of $(a_i)_{i \in I}$ by the unit of R_i .

We proved that a ring is a reduced Rickart ring if and only if it is isomorphic to a Sussman ring.

This talk is the result of joint work with JĀNIS CĪRULIS.

Dualizable algebras of arbitrary nilpotence class

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In the early 2000s, Szabó and Quackenbush showed that groups and rings with a nonabelian nilpotent congruence are not dualizable. More recently, Bentz and Mayr showed that algebras in a modular variety that have a nonabelian supernilpotent congruence are also nondualizable.



However, they did find an example of a dualizable algebra of nilpotence class 2 with infinite signature. We exhibit dualizable algebras of arbitrary nilpotence class with finite signature.

Solving weakly linear inequalities for matrices over max-plus semiring and applications to automata theory

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In this paper, we will consider matrix weakly linear inequalities $A \cdot X \leq X \cdot B$ and $X \cdot B \leq A \cdot X$ for (type matching) matrices over max-plus semiring \mathbb{R}_{max} . This semiring is additively idempotent and it is not complete (a complete idempotent semiring must have a maximal element), but it can be embedded in the complete semiring $\overline{\mathbb{R}_{max}}$. This enable us to define appropriate isotone functions in terms of residuation of underlying structure and to provide connection between their pre-fix points and the solutions to observed matrix inequalities. In the case that unknown matrix X is required to satisfy condition $X \leq Z$, for a given matrix Z , the iterative algorithms for testing existence and computing the greatest solutions (when they exist) are given.

Knowing that the behavior of a max-plus automaton is described using matrices over the semiring \mathbb{R}_{max} , we will apply previous methodology for testing behavioral equivalence of max-plus automata.

This is joint work with ZORANA JANČIĆ and IVANA MICIĆ (Niš).

Maltsev CSPs are definable in 2-sorted Datalog

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In this talk, we sketch a proof that CSPs on Maltsev templates can be solved by local consistency checks on the adjacency structure (i.e. the





Gaifman graph) of the template. In 2016, M. Kozik proved that Singleton Linear Arc Consistency (SLAC) characterizes the tractability of all CSPs over templates of bounded width. In fact, SLAC turns out to be a local consistency check which characterizes the solvability of any CSP over a finite Maltsev template. The universal algebraic tool used to prove this is the characterization of simple idempotent algebras by K. Kearnes.

Sandwich semigroups in locally small categories

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Let S be a semigroup and $a \in S$. A *variant* S^a of S is a new semigroup (defined on the same set S) by the operation

$$x * y = xay.$$

The concept of a *sandwich semigroup* is a generalisation of that of a variant, and it provides a method to turn any fixed hom-set of an arbitrary locally small category into a semigroup. In more detail, let S be a locally small category (a category in which the class of morphisms between any two fixed objects is a set), and let S_{ij} denote the set of all morphisms between objects i and j . Upon fixing an element $a \in S_{ji}$, we can define the *sandwich semigroup* S_{ij}^a of S (with respect to a) by

$$x *_a y = xay$$

for all $x, y \in S_{ij}$.

In this talk we present the results of our thorough algebraic and combinatorial analysis of sandwich semigroups in general locally small categories. In particular, we characterise Green's relations, regular and (left/right) stable elements. We show that under certain mild conditions, regular elements form a subsemigroup of the sandwich semigroup, and we study the structure of this regular subsemigroup. We also compute the ranks of sandwich semigroups and their regular subsemigroups, characterise their idempotent-generated subsemigroups, and compute the rank and idempotent rank of the latter.



Our general results are illustrated in categories **PT**, **T**, **I** of partial transformations, transformations, and partial injections, respectively. Investigations are underway for categories of order-preserving mappings and diagram categories (partition, Brauer, Temperley-Lieb, Motzkin, etc.).

This is joint work with IGOR DOLINKA (Novi Sad), JAMES EAST (Western Sydney), PREEYANUCH HONYAM, KRITSADA SANGKHANAN, JINTANA SANWONG, and WORACHEAD SOMMANEE (Chiang Mai).

The gossip monoid

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Given n people, each knowing a unique scandal and being in possession of a telephone, what is the minimum number of phone calls required until each person knows every scandal? This problem is known as the *gossip problem* and was studied in the 70s by Tijdeman and many others.

The *gossip monoid* arises from consideration of the gossip problem. The monoid is generated by particular idempotents known as *phone call matrices*. Elements of the monoid are $n \times n$ boolean matrices representing states of knowledge among the n gossiping parties, and right multiplication by a phone call matrix represents a phone call between two gossips.

In this talk I will introduce the gossip problem and the gossip monoid and present some results, including a polynomial time reduction which shows that the *gossip membership problem* (Given an arbitrary $n \times n$ boolean matrix, is it an element of the gossip monoid?) is NP-complete.

Coherency for monoids

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A monoid S may be represented via mappings of sets or, equivalently and more concretely, by S -acts. A (right) S -act is a set A together with a map $A \times S \rightarrow A$ where $(a, s) \mapsto as$, such that for all $a \in A$ and $s, t \in S$ we have $a1 = a$ and $(as)t = a(st)$.

A *finitary property for monoids* is a property defined for monoids that is guaranteed to be satisfied by any finite monoid, such as the maximal condition on the lattice of right ideals. Studying algebras via their finitary properties is a classic approach stretching back to Noether and Artin in the early part of the last century. The finitary property we are interested in here is coherency. We say that a monoid S is *right coherent* if every finitely generated S -subact of every finitely presented S -act is finitely presented. *Left coherency* is defined dually and S is *coherent* if it is both right and left coherent. These notions are analogous to those for a ring R (where, of course, S -acts are replaced by R -modules).

Coherency arises naturally from several directions, as this talk will explain. Of course one wants to know which monoids *are* (right) coherent. This question turns out to be hard to answer, in spite of us having a condition for coherency analogous to that of Chase for rings. Free commutative monoids and free monoids are coherent, as are free ample monoids. However, free inverse monoids are not.

This is joint work with MIKLÓS HARTMANN (Szeged) and NIK RUŠKUC (St Andrews).



The complexity of free combinations of temporal CSPs

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Let Γ_1 and Γ_2 be two structures with disjoint finite relational signatures τ_1, τ_2 . We study the computational complexity of the free combination of the constraint satisfaction problem for Γ_1 and Γ_2 , i.e. the computational problem to decide for a given primitive positive $(\tau_1 \cup \tau_2)$ -sentence φ whether $\{\varphi\} \cup T_1 \cup T_2$ is satisfiable where T_1, T_2 is the first-order theory of Γ_1, Γ_2 respectively. We show that if Γ_1 and Γ_2 are first-order expansions of $(\mathbb{Q}; <, \neq)$, then the combined CSP is in P if both Γ_1 and Γ_2 have tractable CSPs and binary injective polymorphisms, and is NP-hard otherwise.

Lattice representations with DCC posets

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The set of all preorders of a set naturally forms a lattice. Any lattice can be embedded in such a lattice in a way that all of the images are anti-symmetric, i.e. posets of the underlying set. However, the underlying set needs to be infinite even for most finite lattices. Sivak in 1978 characterised the finite lattices for which the underlying set can be finite, which coincided with the finite, lower (McKenzie-)bounded lattices.

Here, the finiteness of the underlying set could be exchanged to the condition that the posets in question do not contain infinite chains. In this talk, I relax this to the condition of posets not containing infinite descending chains. The class of lattices embeddable in such a way is strictly larger than the class of lower bounded lattices, but strictly smaller than the class of all finite lattices.





On limit closed classes of algebras

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Some metalogical properties of classes of algebras closed with respect to the formation of direct limits will be concerned. The paper *H. J. Keisler: Theory of Models with Generalized Atomic Formulas, The Journal of Symbolic Logic, Vol.25, No.1 (Mar.,1960), 1-26* will be used. It contains a characterisation of all first-order definable classes which are closed under the formation of direct limits built on natural numbers with linear order.

We will manipulate axiomatic classes defined by a single formula. Let Γ be the class of all sentences (in a given signature) equivalent to the universal closures of implications of the form

$$\alpha \rightarrow (\exists x)\beta, \quad (*)$$

where α is an open formula of finite length, β a positive formula (of an arbitrary length) and $\exists x$ a (possibly unbounded) string of existential quantifiers. The class of models of each sentence of Γ is closed under direct limits.

In the case of mono-unary algebras we allow α in (*) to be an arbitrary formula of infinite length.

The class of all connected mono-unary algebras is closed under the formation of direct limits but it is not first-order definable.

Lattice-valued functions

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The talk starts with the connection between cuts of lattice-valued functions and closure systems. This consideration leads to the canonical representations of lattice-valued functions, which turned out to be essential for studying invariance groups of lattice-valued functions. We discuss



some special cases of the invariance group problem. Finally I show our results about those lattice-valued Boolean functions that can be given by a linear combination.

Join work with BRANIMIR ŠEŠELJA and ANDREJA TEPAVČEVIĆ.

Systems of two-sided linear fuzzy relation equations and inequalities and their applications

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The study of systems of fuzzy relation equations and inequalities (FREIs) was initiated in 1974 by E. Sanchez. He used these systems in medical research, and later they found a wide field of application. Sanchez started the study of systems consisting of linear FREIs. Solvability and methods for computing the greatest solutions of systems of linear FREIs over various structures of truth values have been investigated in numerous papers.

The purpose of this talk is to present our recent results on new, more complex systems of FREIs which are called two-sided linear. They consist of equations/inequalities whose each side is a linear function of an unknown fuzzy relation or fuzzy set defined by means of the composition with the given fuzzy relation. We have proved that solutions of these systems form complete lattices, and we have provided methods for computing their greatest solutions. These methods are based on the concept of a residuated function and the Knaster-Tarski and Kleene Fixed Point Theorems for isotone functions on complete lattices. We also present applications of the obtained results in solving the fundamental problems of the theory of fuzzy automata (state reduction, simulation, bisimulation and equivalence), fuzzy social network analysis (position analysis and blockmodeling), and fuzzy formal concept analysis (simultaneous reduction of objects and attributes).

This is joint research with MIROSLAV ĆIRIĆ (Niš).



Subdirectly irreducible medial quandles

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A medial quandle is an algebra $(A, *, \setminus)$ satisfying

- $x * x = x$;
- $(x * y) * (z * u) = (x * z) * (y * u)$;
- $x \setminus (x * y) = y$;
- $x * (x \setminus y) = y$.

It turns out that all subdirectly irreducible medial quandles fall into exactly one of the following categories:

- divisible groupoids;
- reductive groupoids;
- groupoids with two minimal left ideals;

where reductive groupoids satisfy

$$\underbrace{((x y) y) \dots y}_{m\text{-times}} = y.$$

We give examples of all such types of subdirectly irreducible medial quandles.

Joint work with AGATA PILITOWSKA and ANNA ZAMOJSKA-DZIENIO.





On the characterization of orthogroups by disjunctions of identities

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Based on a general result by Clifford, Yamada proved that a semigroup is an orthogroup if and only if it is a semilattice of rectangular groups. In particular, any band is a semilattice of rectangular bands. Classes of semilattices of other particular rectangular groups can be characterized using the concept of *disjunctions of identities*. This concept, which is a generalization of the well-known identities, was introduced by Evseev. Using *disjunctions of identities*, we characterize orthogroups which are unions of groups with finite exponent as semilattices of certain classes of rectangular groups. In particular, we consider the case when the semilattice is a chain.



Identities in upper triangular tropical matrix semigroups and the bicyclic monoid

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Izhakian and Margolis have shown that the semigroup of all 2×2 tropical matrices satisfies a semigroup identity. A key step of their proof was establishing an identity for the upper triangular submonoid $UT_2(\mathbb{T})$, which is also of interest in its own right. Specifically, they showed that $UT_2(\mathbb{T})$ satisfies Adjan's celebrated identity

$$ABBA \cdot AB \cdot ABBA = ABBA \cdot BA \cdot ABBA$$

for the the bicyclic monoid \mathcal{B} .

Since \mathcal{B} embeds in $UT_2(\mathbb{T})$, every identity satisfied in the latter must also hold in the former. In view of this and their results, Izhakian and Margolis posed the natural question of whether the converse holds: do $UT_2(\mathbb{T})$ and \mathcal{B} satisfy exactly the same semigroup identities?





This talk concerns some joint work with LAURE DAVIAUD and MARK KAMBITES. We provide a necessary and sufficient condition for an identity to hold in the semigroup of $n \times n$ upper triangular tropical matrices, and use this result in the case $n = 2$ to give a positive answer the question posed above.

Representation of integral quantales by tolerances

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A *quantale* is an algebraic structure (Q, \vee, \cdot) such that (Q, \vee) is a complete semilattice, (Q, \cdot) is a semigroup and the multiplication distributes over all joins. A quantale is called *integral* if the greatest element 1 of the semilattice (Q, \vee) is the multiplicative identity element of (Q, \cdot) .

Note that the join operation of a quantale Q determines an order relation \leq on Q and the ordered set (Q, \leq) is actually a complete lattice. Thus, we may speak about underlying lattice of a quantale.

Let L be a complete lattice. Recently E. Bartl and M. Krupka showed that the set $\text{Tol}(L)$ of all tolerances of L forms an integral quantale with the intersection \cap in role of \vee and the multiplication \otimes defined as follows:

$$S \otimes T = (S \circ \geq \circ T) \cap (T \circ \leq \circ S)$$

where \circ denotes the usual relational product operation.

Our central result is a Cayley type theorem claiming that every integral quantale Q has a natural embedding into the integral quantale of complete tolerances on the underlying lattice of Q . The image of Q under this embedding is described as the set of all tolerances closed with respect to certain natural unary operations.

As an application, we show that the underlying lattice of any finite integral quantale is distributive in 1 and dually pseudocomplemented. Besides, we exhibit relationships between several earlier results. In particular, we give an alternative approach to so called ordered sets introduced





by S. Valentini in 1994 and show how the ordered sets are related to tolerances.

We also exhibit two situations when reflexive binary relations form an integral quantale with respect to set-theoretical intersection \cap and the relational product \circ . Moreover, we prove that any finite integral involutive quantale is obtained as the quantale of all compatible reflexive binary relations on a suitable finite majority algebra.

This is a joint work with SÁNDOR RADELECZKI (Miskolc).

Random walks on semigroups

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I will describe some recent joint research with ROBERT D. GRAY (Norwich) into random walks on finitely generated semigroups, and their connections to properties such as cogrowth and amenability.

Solving edge CSP with even delta-matroid constraints

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Edge CSP is a variant of the Constraint Satisfaction Problem (CSP) where each variable appears in exactly two constraints. One can draw an instance of edge CSP as a graph where edges are variables and vertices are constraints. We want to know the complexity of the edge CSP whose all constraints come from a fixed constraint language Γ .

We investigate edge CSP where each variable can take values 0 or 1. In order to shrink the gap between provably hard and polynomial time solvable cases, we present a polynomial time algorithm for the case where all constraints in Γ are even delta-matroids. This CSP generalizes the





problem of finding perfect matchings in graphs – in fact, our algorithm generalizes the classic Edmonds’ algorithm for perfect matchings.

This is a joint work with MICHAEL ROLÍNEK and VLADIMIR KOLMOGOROV (IST Austria).

On ordered hypersemigroups

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From every ordered semigroup an ordered hypersemigroup can be constructed in such a way that

- (1) A set A is a right (left) ideal, bi-ideal, quasi-ideal or interior ideal of the ordered semigroup if and only if it is a right (left) ideal, bi-ideal, quasi-ideal or interior ideal of the hypersemigroup.
- (2) The ordered semigroup is regular, intra-regular, left (right) regular, left (right) quasi-regular or semisimple if and only if the ordered hypersemigroup is, respectively, so.

So, from every example of a regular, intra-regular, left regular, left quasi-regular or semisimple ordered semigroup given by a table of multiplication and an order, a corresponding example of ordered hypersemigroup can be constructed having the same right (left) ideals, bi-ideals, quasi-ideals and interior ideals.

Spectral properties of partial automorphisms of a regular rooted tree

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Let \mathcal{J}_n be a semigroup of partial automorphisms of an n -level d -regular rooted tree T . To a randomly chosen $x \in \mathcal{J}_n$, we assign the matrix $A_x =$



$(\mathbf{1}_{\{x_b(i)=j\}})_{i,j=1}^{d^n}$ describing the action of x on the n th level of T . Let Ξ_n be a uniform probability measure of the eigenvalues of A_x , respecting the multiplicity. We show that this measure converges weakly in probability to the delta-measure concentrated at zero.

Linearization of certain non-trivial equations in oligomorphic clones

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A system of equations in a clone is called non-trivial if those equations are not satisfied by any set of projections. Equations are called linear if they do not involve compositions of operations.

Motivated by the dichotomy conjecture for constraint satisfaction problems (CSPs) of reducts of finitely bounded homogeneous structures, we are interested in systems of non-trivial equations in oligomorphic clones. In particular we would like to know under which conditions non-trivial equations are equivalent to linear ones.

A general answer to this question is not known. We are going to show that the answer is positive for certain strong non-trivial equations, in particular every operation that is totally symmetric modulo outer embeddings induces a system of non-trivial linear equations (under some additional structural assumptions).

This is part of a joint work with LIBOR BARTO, MIREK OLŠÁK, TRUNG VAN PHAM and MICHAEL PINSKER.



Homomorphic images of subdirectly irreducible rings

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An algebra is subdirectly irreducible (SI) if and only if the intersection of all its non-identical congruences is non-identical. By the well-known Birkhoff's theorem, every algebra can be embedded into a product of SI algebras. One can ask, on the other hand, about the characterization of all homomorphic images of SI algebras. Such a characterization was given by Stanovský '01 for the variety of groupoids. Analogical results were further obtained also for other varieties: all algebras of the given rich signature (Ježek, Kepka '02), semigroups (Bulman-Fleming, Hotzel, Wang '04), lattices (Freese, unpublished), groups (McKenzie, unpublished), loops and quasigroups (Stanovský '05) and unary algebras (Ježek, Marković, Stanovský '07). We give a similar characterization for the varieties of rings, rings with unity, radical rings and for their subvarieties of commutative rings. Recall that radical rings (also called quasi-regular rings) are precisely all Jacobson radicals of all rings. They can also be equivalently characterized as such rings that form a group with respect to the adjoint operation $a \circ b = a + b + ab$.



Units in quasigroups

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V. D. Belousov posed in 1967 the following question:

How to recognize identities which force quasigroups satisfying them to be loops?

Examples of such identities are the associativity and the four Moufang identities. In general, only partial answers are known.





We can generalize slightly Belousov's Problem by asking the question about identities forcing quasigroups to have left ($ex = x$), right ($xe = x$) and middle units ($xx = e$). An example is the left (right) Bol identity which implies existence of right (resp. left) unit.

We investigated several families of identities which give partial answers to the Belousov's problem and the proposed generalization. The results are from several papers under preparation by myself, or coauthored by V. A. SHCHERBAKOV and J. FEMPL-MAĐAREVIĆ.

Generating integer polynomials using function composition

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We describe those integer polynomials that can be obtained from x^2 using $+$, $-$ and function composition. We see that a polynomial $p = \sum_{i=1}^n c_i x^i \in \mathbb{Z}[x]$ lies in the subnear-ring of $(\mathbb{Z}[x], +, \circ)$ generated by $\{x^2\}$ if and only if for all $i \in \mathbb{N}$, $2^{s_2(i)-1}$ divides c_{2i} and $c_{2i+1} = 0$, where $s_2(i)$ denotes the digit sum of i to the basis 2. We state a similar description for x^3 . In the language of near-rings this means that we describe the subnear-rings of $(\mathbb{Z}[x], +, \circ)$ which are generated by $\{x^2\}$ and $\{x^3\}$. Furthermore, we exhibit descending and ascending chains of subnear-rings of $(\mathbb{Z}[x], +, \circ)$. The ascending chain will give us a subnear-ring which is not finitely generated.

This is a joint work with ERHARD AICHINGER (Linz).

Clones of small posets

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For five of the six classes of maximal clones, it has been shown that their members are finitely generated. The unsettled class is that of monotone



clones of bounded partial orders. Some results have already been obtained. The most famous one is due to Tardos who gave the first example of a non-finitely generated maximal clone. We investigate the clones of some small posets in terms of finite generability. In his proof Tardos described the obstacles of a particular eight element poset. In general, the obstacles of a poset are difficult to describe. We propose to replace obstacles by critical relations. It turns out that critical relations may be tractable even when obstacles are too unwieldy.

Generalized attributes in concept lattices

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To control the number of concepts generated from a binary context, many approaches have been proposed. One of this is to replace some attributes by their union. We call this new attribute a generalized attribute. This operation reduces the size of the attribute set, but not automatically the size of the concept lattice. In our talk we will present a worst case where the size increases exponentially and discuss a measure on attributes to check if the size will increase after generalization or not.

Fair semigroups

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Fair semigroups are non-additive analogues of xst-rings, introduced by Xu, Shum and Turner-Smith in 1993.

If S is a semigroup then a right S -act A_S is called *unitary* if $AS = A$. We say that a semigroup S is a *right fair semigroup* if every subact of a unitary right S -act is unitary. Dually one defines left fair semigroups. By a *fair semigroup* we mean a semigroup which is both left and right fair.



It turns out that a semigroup S is right fair if and only if for every sequence $(s_i)_{i \in \mathbb{N}}$ of elements of S there exist $n \in \mathbb{N}$ and $u \in S$ such that

$$s_n \dots s_2 s_1 u = s_n \dots s_2 s_1.$$

We will give a list of examples of fair semigroups and some basic facts about them. We also present several results about Morita theory of fair semigroups. Two semigroups are called Morita equivalent if certain categories of acts over them are equivalent. Strong Morita equivalence of semigroups is defined using Morita contexts. For example, we can say that each finite monogenic semigroup S is Morita equivalent to its group part G . If G is not equal to S then these semigroups cannot be strongly Morita equivalent.

This talk is based on joint research with LÁSZLÓ MÁRKI.

Non-associative MV-algebras



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I. Chajda and J. Kühr introduced so-called NMV-algebras. These are MV-algebras whose addition operation is not necessarily associative. We discuss interval NMV-algebras, idempotent elements of NMV-algebras and derivations on NMV-algebras. Since NMV-algebras are congruence regular, congruences on them are determined by their kernels. We present a characterization of these kernels.





Reflection-closed varieties of multisorted algebras and minor identities II

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We characterize the minor-equational theories of multisorted algebras, i.e., the closed sets of minor identities of the Galois connection Mod-mId .

We also discuss how reflection-closed varieties and usual varieties of multisorted algebras are related to each other. These notions can be quite different in general, but for varieties of multisorted algebras of a non-composable type, the only varieties that are not reflection-closed are in a certain sense trivial.

This is joint work with REINHARD PÖSCHEL and TAMÁS WALDHAUSER.



Identities in plactic and related monoids

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The plactic monoid is a fundamental algebraic object which captures a natural monoid structure carried by the set of combinatorial objects of semistandard Young tableaux. Other monoids arise in a similar way by considering different combinatorial objects: the hypoplactic monoid (the monoid of quasi-ribbon tableaux, connected with the theory of quasi-symmetric functions), the sylvester monoid (binary search trees), and the Baxter monoid (pairs of twin binary search trees, connected with the theory of Baxter permutations).

In this talk we discuss whether the plactic monoid and its plactic-like variants satisfy non-trivial identities. (An identity is a formal equality $u = v$, where u and v are words over some alphabet of variables, and is non trivial if u and v are not identical as words. It is satisfied by a monoid if every substitution of elements for variables yields equality in





the monoid.) We present new results showing that the plactic monoid does not satisfy a non-trivial identity, whereas the above mentioned related monoids do satisfy non-trivial identities.

The results were obtained in collaboration with A. CAIN, G. KLEIN, Ł. KUBAT, and J. OKNIŃSKI.

This work was partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID/MAT/00297/2013 (Centro de Matemática e Aplicações) and the project PTDC/MHC-FIL/2583/2014.

Morita equivalence of semigroups revisited: Firm semigroups

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We define firm semigroups and firm acts as non-additive analogues of firm rings and firm modules. Using the categories of firm acts we develop Morita theory for firm semigroups. We show that equivalence functors between categories of firm acts over two firm semigroups have to be tensor multiplication functors. Our main result states that the categories of firm right acts over two firm semigroups are equivalent if and only if these semigroups are strongly Morita equivalent, which means that they are contained in a unitary Morita context with bijective mappings. This is a far-reaching generalization of Lawson's result which states this for semigroups with local units.

We also investigate other categories of acts which have been used earlier to develop Morita equivalence. The main tool in our work is adjoint functors. We prove that over firm semigroups all the considered categories are equivalent to the category of firm acts.

All this suggests that firm semigroups and firm acts are the natural environment to study Morita equivalence of semigroups.

This is joint work with VALDIS LAAN and ÜLO REIMAA (Tartu).



Decent Mal'cev conditions which hold in all locally finite congruence meet-semidistributive varieties I

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A strong Mal'cev condition is a finite set of identities in some language. A strong Mal'cev condition is realized in a variety of algebras if there is an interpretation of its operation symbols as terms so that all identities of the strong Mal'cev condition become true identities of that variety. We define a strong Mal'cev condition to be *decent* if it is linear (there is no composition of operations), idempotent, has one operation symbol and its equations use only the variables x and y . The requirement of one operation symbol can be achieved at no cost, given idempotence, but the others are true restrictions.

We characterize all decent Mal'cev conditions which are realized in all locally finite congruence meet-semidistributive varieties in several ways: They are realized iff they are realized in one particular variety generated by a 4-element algebra, iff they fit a syntactical description we give, iff a tractable algorithm we devised verifies they are realized.

Our work follows from, and generalizes, earlier work by M. Kozik, A. Krokhin, M. Valeriote and R. Willard, and also by J. Jovanović, R. McKenzie, M. Moore and the speaker, but Z. Brady managed to independently prove much the same results in a completely different way, a little before us. We will compare our result with his. This talk is presenting joint work with N. DRAGANIĆ, V. ULJAREVIĆ and S. ZAHIROVIĆ. The talk will be more focused on the obtained results and their significance, while Samir Zahirović, in a related talk, will speak on our methods in more detail.



A Ramsey theorem for relational structures consisting of several partial orders

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In the parlance of relational structures, the Finite Ramsey Theorem states that the class of all finite chains has the Ramsey property. Another classical result claims that the class of all finite posets with a linear extension has the Ramsey property. In 2012 M. Sokić proved that the class of all finite structures consisting of several linear orders has the Ramsey property. This was followed by a 2017 result of S. Solecki and M. Zhao that the class of all finite posets with several linear extensions has the Ramsey property. Using the categorical reinterpretation of the Ramsey property in this talk we prove a common generalization of all these results. We consider multiposets to be structures consisting of several partial orders and several linear orders. We allow partial orders to extend each other in an arbitrary but fixed way, and require that every partial order is extended by at least one of the linear orders. We then show that the class of all finite multiposets conforming to a fixed template has the Ramsey property.

This is joint work with NEMANJA DRAGANIĆ.

The complexity of quantified constraint satisfaction on monoids

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The quantified constraint satisfaction problem QCSP for a fixed finite relational structure has as input a first order sentence over this structure built from atoms, conjunction, and both existential and universal quantifiers. The problem is then to decide whether the sentence is true.



This generalizes quantified satisfiability of Boolean formulas and is well-known to be always in PSPACE. We aim to classify structures by the computational complexity of their QCSP.

Using an established algebraic viewpoint we can replace a relational structure by its polymorphism algebra and ask about the complexity of the QCSP for this algebra instead. We show that the QCSP of any finite monoid is either in P or NP-complete.

An algorithmic ordering condition for groups

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Ordering conditions for groups provide useful tools for the study of various relationships between group theory, universal algebra, and topology. In this work, we establish a new “algorithmic” ordering condition for extending partial orders on groups to total orders. We then use this condition to show that the problem of extending a finite subset of a free group to a total order corresponds to the problem of checking validity of a certain inequation in the variety of representable lattice-ordered groups (or, equivalently, the class of totally ordered groups). As a direct consequence, we obtain a new proof that free groups are orderable.

Constructive semigroups with apartness – a new algebraic theory

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We will present basic background and tools for the main aspects of the theory of constructive semigroups with apartness. Constructive mathematics is, roughly speaking, mathematics with intuitionistic logic. Here, the focus is on E. Bishop’s approach to constructive mathematics (**BISH**), [1].



By an *apartness* on S we mean a binary relation $\not\approx$ on S which satisfies the axioms of irreflexivity, symmetry and cotransitivity: $\neg(x \not\approx x)$, $x \not\approx y \Rightarrow y \not\approx x$, $x \not\approx z \Rightarrow \forall y (x \not\approx y \vee y \not\approx z)$. We then say that $(S, \simeq, \not\approx)$ is a *set with apartness*. A tuple $(S, \simeq, \not\approx, \cdot)$ is a *semigroup with apartness* with $(S, \simeq, \not\approx)$ as a set with apartness, \cdot a binary operation on S which is associative, i.e. $\forall a, b, c \in S [(a \cdot b) \cdot c \simeq a \cdot (b \cdot c)]$, and strongly extensional, i.e. $\forall a, b, x, y \in S (a \cdot x \not\approx b \cdot y \Rightarrow (a \not\approx b \vee x \not\approx y))$. Although this talk will be based on material given in [2,3], it is by no means an attempt to give a complete overview of our existing results. An example of application(s) of these ideas can be found in [4]. We will report a few open issues too. Once again we want to emphasize that

semigroups with apartness are a *new approach*,
and not a new class of semigroups.

This is joint work with SINIŠA CRVENKOVIĆ (Novi Sad) and DANIEL ABRAHAM ROMANO (Banja Luka).

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The lattice of all clones definable by binary relations on three-element set

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Using a computer computation, we have constructed the lattice of all clones definable by binary relations on three-element set. It turns out that there are 2,079,040 such clones.





Moreover, we have proved that if a binary relation p is expressible using a primitive positive formula over some set M of binary relations (where $\{false, true, eq\} \subseteq M$), then p can be obtained from M by applying the following operations:

$$\begin{aligned} perm(p)(x_1, x_2) &\equiv p(x_2, x_1), \\ conj(p_1, p_2)(x_1, x_2) &\equiv p_1(x_1, x_2) \wedge p_2(x_1, x_2), \\ comp(p_1, p_2)(x_1, x_2) &\equiv \exists y (p_1(x_1, y) \wedge p_2(y, x_2)). \end{aligned}$$

Finally, if we consider the lattice of all clones definable by relations of arity r on k -element set, it is known that the number of all clones is greater than 2^{50} if $r = 2$ and $k \geq 4$ or if $r \geq 3$ and $k \geq 3$ (see D. Zhuk, S. Moiseev "On the Clones Containing a Near-Unanimity Function", ISMVL 2013: 129-134). Thus, we consider it to be practically infeasible to construct the lattice of all clones for these cases.

Higher Delta and supernilpotence for congruence modular varieties

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The binary commutator for a congruence modular variety is defined equivalently by

- (1) a term condition, or
- (2) examining a congruence (usually called Delta) that collapses the diagonal.

Bulatov's formulation of the higher commutator is defined with a generalization of the term condition. In this talk we will discuss a ternary commutator that is defined with a generalization of Delta. For congruence modular varieties, this commutator is the same as the term condition ternary commutator. Some key properties of the higher commutator are easily proved with this formulation. Furthermore, we will show that every congruence modular variety has a 7-ary term operation that satisfies a weakened set of cube identities. This term operation, which we





call a three dimensional ‘wobbly’ cube term, is the three-dimensional analogue of a difference term.

Cross-connections and variants of the full transformation semigroup

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Cross-connection theory propounded by K. S. S. Nambooripad describes the ideal structure of a regular semigroup using the categories of principal left (right) ideals. A variant of the full transformation semigroup (T_X, \cdot) for an arbitrary $\theta \in T_X$ is the semigroup $T_X^\theta = (T_X, *)$ with the binary operation

$$\alpha * \beta = \alpha \cdot \theta \cdot \beta$$

where $\alpha, \beta \in T_X$. In this talk, we discuss the ideal structure of the regular part $\text{Reg}(T_X^\theta)$ of the variant of the full transformation semigroup using cross-connections. We describe the constituent categories of $\text{Reg}(T_X^\theta)$ and illustrate how they are cross-connected by a functor induced by the sandwich transformation θ . This leads us to a structure theorem for the semigroup and gives the representation of $\text{Reg}(T_X^\theta)$ as a cross-connection semigroup.

Transferring Davey’s theorem on annihilators and m -completeness to modular congruence lattices

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Congruence lattices of semiprime algebras from semi-degenerate congruence-modular varieties fulfill the equivalences from [1, Theorem 1]. I have proven this by transferring Davey’s Theorem from bounded distributive lattices to such congruence lattices through a certain lattice morphism. I have also shown that, if the congruence lattice of such an



algebra satisfies the equivalent conditions from this theorem, then so does the reticulation of that algebra, as well as the fact that the properties from this theorem are preserved by finite direct products of algebras of this kind.

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The extensions of Green's relations on upper triangular tropical matrices

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We study the semigroup $U_n(\mathbb{FT})$ of upper triangular tropical matrices. Taylor has shown that $U_n(\mathbb{FT})$ is not regular for $n \geq 3$. We therefore consider the extensions $\tilde{\mathcal{R}}$ and $\tilde{\mathcal{L}}$, and \mathcal{R}^* and \mathcal{L}^* , of Green's relations \mathcal{R} and \mathcal{L} on $U_n(\mathbb{FT})$. We show that for every element A of $U_n(\mathbb{FT})$ there are idempotents $E, F \in U_n(\mathbb{FT})$ such that $E \tilde{\mathcal{R}} A \tilde{\mathcal{L}} F$, that is, $U_n(\mathbb{FT})$ is *weakly abundant* or a *Fountain semigroup*. However, it does not have the congruence condition. In other words, $\tilde{\mathcal{R}}$ and $\tilde{\mathcal{L}}$ are not, respectively, left and right congruences. As for Green's $*$ -relations, we show that $\mathcal{R}^* = \mathcal{R}$ and $\mathcal{L}^* = \mathcal{L}$ so that consequently, $U_n(\mathbb{FT})$ is not abundant.

This is a joint work with VICTORIA GOULD.



The weakest nontrivial idempotent equations

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An equational condition (virtually identical to a better known name “strong Maltsev condition”) is a set of equations in an algebraic language, and an algebraic structure satisfies such a condition if it possesses terms that meet the required equations. Equational conditions serve as the main organizing principle in universal algebra and provide a successful approach to several problems in computational complexity. We find a single nontrivial equational condition which is implied by any nontrivial idempotent equational condition.

Infinite algebras with few subpowers

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Finite algebras with few subpowers play a significant role for classifying complexity of CSPs with finite domain. Namely, they mark the widest scope where an algorithm based on computing generating sets of solutions can be used. Motivated by CSPs on infinite domains, we will talk about infinite algebras having *few subalgebras of powers*, i.e., algebras \mathbf{A} for which there exists a polynomial p such that the number of subalgebras of the n -th power of \mathbf{A} is finite and at most $2^{p(n)}$. We will describe how the standard classification theorems of finite algebras with few subpowers generalize to countably infinite algebras, and in particular those that can be obtained by taking polymorphisms of a countable relational structure.





Reconstructing the topology of the elementary self-embedding monoids of countable saturated structures

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Every transformation monoid comes equipped with a canonical topology – the topology of pointwise convergence. For some structures, the topology of the endomorphism monoid can be reconstructed from its underlying abstract monoid. This phenomenon is called *automatic homeomorphicity*.

In this talk we are going to present our newest findings about the question when the self-embedding monoid of a countable homogeneous structure has automatic homeomorphicity.

As an application we will strengthen a classical result by Lascar that states that whenever \mathbf{A} is a countable \aleph_0 -categorical G -finite structure and whenever \mathbf{B} is any other countable structure, then the restriction of every monoid-isomorphism between the monoids of elementary self-embeddings to the respective automorphism groups is a homeomorphism.

This is joint work with MAJA PECH.

On the structure of finite commutative totally ordered monoids

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We follow up our previous research in which we gave a description of the structure of finite negative totally ordered monoids (f. n. tomonoids for short). Given a f. n. tomonoid L , we have explained all those f. n. tomonoids that are by one element larger than L and from which L arises by means of a Rees congruence.





Here, negativity means that the top element is also the monoidal identity. Dropping this requirement, we are working towards a description of general finite tomonoids. In the approach that we will present we will focus on the commutative case. We adapt the concept of closure conditions known from the branch of differential geometry called web geometry.

This is a joint work with THOMAS VETTERLEIN.

Action of endomorphism semigroups on definable sets

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The aim of the talk is to construct and describe the Galois-type correspondence between subsemigroups of the endomorphism semigroup $\text{End}(A)$ of an algebra A and sets of formulas. Such Galois-type correspondence forms a natural frame for studying algebras by means of actions of different subsemigroups of $\text{End}(A)$ on definable sets over A . We treat some applications of this Galois correspondence.

Joint work with G. MASHEVITZKY (Beer-Sheva University).

Syntax versus semantics in knowledge bases

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The goal of this talk is to show that in the constructed knowledge base model syntax and semantics are tightly related via some structures of the categorical nature.

Joint work with E. ALADOVA.





The classification of symmetric conservative clones with a finite carrier and its applications in Computational Social Choice

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In the paper the classical problem of study of the lattice of clones with finite carrier A is regarded in context of applications to Computational Social Choice (CSC). Our work is inspired by the paper [1] and uses some of its ideas. We consider the lattice of clones $\mathcal{F} \subseteq \mathcal{O}(A)$ with the class of Galois connections $(\text{inv}_Q, \text{pol})$ (where Q is a finite set) generated by preservation relation between a function $f \in \mathcal{O}(A)$ and a set $\mathcal{C} \subseteq A^Q$. A clone $\mathcal{F} \subseteq \mathcal{O}(A)$ is called *conservative* if it consists only of conservative functions, i.e. of functions satisfying

$$(\forall x_0, \dots, x_{n-1} \in A) \bigvee_{i < n} (f(x_0, \dots, x_{n-1}) = x_i).$$

A clone $\mathcal{F} \subseteq \mathcal{O}(A)$ is called *symmetric* if $(\forall f \in \mathcal{O}(A)) f \in \mathcal{F} \rightarrow f_\sigma \in \mathcal{F}$ for any permutation σ of A where $(\forall x_0, \dots, x_{n-1} \in A) f_\sigma(x_0, \dots, x_{n-1}) = \sigma^{-1}(f(\sigma(x_0), \dots, \sigma(x_{n-1})))$. We give an explicit classification of all symmetric conservative clones with finite carrier A and their Q -invariant sets (for any finite set Q). In application of CSC, we obtain the complete description of symmetric classes of restricted choice functions without *Arrow's propriety* (see [1]) and *possibility theorems* for a non-Arrowian aggregation consistent with rational preferences.

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Reflection-closed varieties of multisorted algebras and minor identities I

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Reflections (as introduced by L. Barto, J. Opršal, M. Pinsker) generalize the classical operators of taking subalgebras and homomorphic images. We generalize this notion to multisorted algebras.

In this talk we ask for a characterization of reflection-closed varieties (RP-varieties) and consider the Galois connection $\text{Mod} - \text{mld}$ between multisorted algebras and minor identities. Analogously to the classical Birkhoff theorem, it turns out that the Galois closures of this Galois connection are just the reflection-closed varieties of multisorted algebras, i.e. $\text{Mod mld } \mathcal{K} = \text{RP } \mathcal{K}$.

This is a joint work with ERKKO LEHTONEN and TAMÁS WALDHAUSER.



\aleph_0 -categorical semigroups

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A countable first order structure M (relational or algebraic) is \aleph_0 -categorical if it can be characterized, up to isomorphism, by its first-order properties. This condition has an alternative formulation that does not directly involve model theoretic notions, due to Engeler, Ryll-Nardzewski and Svenonius, namely that $\text{Aut}(M)$ has only finitely many orbits in its action on M^n , for each $n \geq 1$. As such, there has been a great deal of interest in \aleph_0 -categoricity for a range of structures, both from a model theoretic and algebraic viewpoint.

A number of algebraic structures have been considered from the point of view \aleph_0 -categoricity, such as groups and rings. Until recently, little was known in the context of semigroups. Here we consider \aleph_0 -categoricity for a semigroup S , taking the approach of relating the \aleph_0 -categoricity of



S to that of its natural constituents. Our most complete results are for classes of inverse and orthodox semigroups.

Weak congruences on categories

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Starting with the notion of a congruence on a category, we define weak congruences on a category in an analogous way. We show that this definition is a natural generalisation of that of weak congruences on algebras. We prove that the collection of weak congruences on a category is an algebraic lattice and that equivalent categories have isomorphic weak congruence lattices. We analyze the structure of this lattice. In particular, we compare this approach with the algebra case where the diagonal plays an important role. Here we have also a thin diagonal, which we define and show that it has the same importance as the standard diagonal. We look also at the behavior and structure of weak congruences on special categories, like groupoids.



On homotopies of universal algebras

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The notions of homotopy and isotopy, which can be considered as generalizations of the notions of homomorphism and isomorphism, respectively, were well-known in quasigroup theory from Albert's works, and defined for arbitrary algebraic systems in [1]. Here we use appropriately defined notions of pseudo-algebra and pseudo-congruence to establish isotopy theorems generalizing the standard isomorphism theorems of abstract algebra.

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On partial matroids, geometric posets and semimodular posets

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Recall that a matroid is defined as a set A together with a closure operator $\bar{\cdot} : \mathcal{P}(A) \rightarrow \mathcal{P}(A)$ on A such that for all $X \subseteq A$ and for all $x, y \in A$ we have

- M_1 : $x \notin \bar{X}$ and $x \in \overline{X \cup \{y\}}$ imply $y \in \overline{X \cup \{x\}}$;
 M_2 : there exists a finite Y such that $Y \subseteq X$ and $\bar{Y} = \bar{X}$.

Also recall [1] that a partial closure operator C on a set S is a partial mapping $C : \mathcal{P}(S) \rightarrow \mathcal{P}(S)$ that satisfies:

- P_{C_1} : if $C(X)$ is defined, then $X \subseteq C(X)$;
 P_{C_2} : if $C(X)$ and $C(Y)$ are defined, then $X \subseteq Y$ implies $C(X) \subseteq C(Y)$;
 P_{C_3} : if $C(X)$ is defined, then $C(C(X))$ is also defined and $C(C(X)) = C(X)$;





Pc_4 : $C(\{x\})$ is defined for every $x \in S$.

We introduce *partial matroid*, which is a structure analogous to matroid but with closure operator replaced by partial closure operator, in fact, a particular kind of it that we call *sharp* partial closure operator [2]. Motivated by a well-known correspondence between matroids and geometric lattices, we define a natural generalization of the notion of geometric lattices to posets, and then show that the analogous correspondence holds between partial matroids and geometric posets.

Finally, since (finite) geometric lattices are in fact precisely semimodular and atomistic lattices, at the end we show how to generalize the notion of semimodularity to posets in such a way that this characterization of geometric lattices is transferred to posets.

This is a joint work with B. ŠEŠELJA and A. TEPAVČEVIĆ.

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Divisibility of ultrafilters

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The Stone-Čech compactification of a topological space S is the space of ultrafilters on S . A semigroup operation on a discrete space S can be extended to its Stone-Čech compactification βS so that a right-topological semigroup is obtained. Using the extension of the multiplication on the set N of natural numbers to βN we find several ways to extend the divisibility relation.

One of these relations turns out to have a number of properties resembling the properties of divisibility on N . For example, there are minimal





elements, which we call prime ultrafilters (more precisely, there are 2^c of them). There are prime ultrafilters below any other, and we can classify all ultrafilters according to the set of primes they are divisible by. In this way a hierarchy is defined: there are countably many levels of ultrafilters divisible by finitely many primes (and the subdivision of levels according to the prime factors), but there are also those divisible by infinitely many. There are even maximal ultrafilters, divisible by all others.

The product of ultrafilters works in accordance with this hierarchy: $p \cdot q$ is divisible exactly by those primes that p and q are divisible by. The most interesting problem seems to be determining the number of ultrafilters divisible by given primes. Being a Ramsey ultrafilter, a P-point etc. has a certain effect on this.

The global dimension of the algebra of the monoid of all partial functions on an n -set

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Given a finite monoid M , we can consider the monoid algebra $\mathbb{C}M$ (where \mathbb{C} is the field of complex numbers). It is of interest to study fundamental invariants of the algebra $\mathbb{C}M$ such as its quiver presentation, Cartan matrix and global dimension. The global dimension is the supremum over the (minimal) lengths of all projective resolutions of modules over the algebra.

Recently, Steinberg proved that the global dimension of the algebra of the full transformation monoid T_n is $n - 1$. Using a different approach, we will prove that the global dimension of the algebra of the monoid of partial transformations PT_n is also $n - 1$.

Results of Nico and Putcha imply that $n - 1$ is an upper bound, but proving equality requires finding a projective resolution of length $n - 1$. Due to the special structure of PT_n we are able to reduce the question to the representation theory of its maximal subgroups – symmetric groups. We



then use combinatorial methods involving Young diagrams and standard homological techniques to find a module whose projective dimension is $n - 1$.

Some properties of monotonous fuzzy set operator applied to fuzzy set equations and inequations

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We consider some general properties of the monotonous fuzzy set operators and apply them to existential and constructive problems connected with fuzzy set equations and inequations. Every monotonous operator gives rise to a closure operator that maps each fuzzy set to the greatest fuzzy set that may be reached by applying the initial closure operators and the set union infinitely many times. Dually we get an openness operator, and applying these two operators we get the smallest or the greatest solutions of fuzzy set equations and inequations, which in some cases may be constructed in at most countably many steps.

On products of inverse semigroups

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Wreath product and semidirect product are fundamental constructions in group theory. A widely known theorem due to Kaloujnine and Krasner says that each extension of a group K by a group G is embeddable in the wreath product of K by G .

In the theory of inverse semigroups, the best known constructions which play similar role in the study of extensions are λ -wreath product and λ -semidirect product, see [1] and [4]. However, as it has been recently formulated in [3], these constructions do not really fit in the context of

inverse semigroups since the actions involved in them are by endomorphisms rather than by partial automorphisms.

In the talk we introduce a wreath product of inverse semigroups of partial bijections which generalizes the wreath product of permutation groups and also Houghton's wreath product for inverse semigroups [2]. Moreover, we define a product of semidirect type for inverse semigroups where the action involved is by partial automorphisms. Finally, we discuss how these and the former constructions relate to each other from the point of view of which extensions of inverse semigroups are embeddable in them.

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Algebras of incidence structures: representing regular double p-algebras

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It is widely known that the lattice of subsets of a set naturally forms a boolean algebra. Conversely, every completely distributive and atomic boolean algebra is isomorphic to a lattice of subsets. More generally, given some class of objects, one may wish to characterise the structure of the subobject lattices defined by that class. Reyes and Zolfaghari tackle this problem by studying toposes, and characterise the toposes whose subobject lattices are double-Heyting algebras.

One particular consequence they observe is that the lattice of subgraphs of a graph is a double-Heyting algebra. It turns out that the lattice of



subgraphs actually forms a (congruence-)regular double p-algebra. Varlet proved that the class of regular double p-algebras is an equational class, Katriňák further proved that the class is term equivalent to a subvariety of double-Heyting algebras, so this generalises the observation of Reyes and Zolfaghari.

Naturally, we ask ourselves which regular double p-algebras are isomorphic to a lattice of subgraphs. It is easy to find one that isn't. Instead, we adjust our focus towards incidence structures. An incidence structure is a standard geometric object consisting of a set of points, a set of lines and an incidence relation specifying which points lie on which lines. This concept generalises, for example, graphs, hypergraphs and projective planes.

In this talk we prove a result analagous to the case for boolean algebras. Every completely distributive and doubly atomic regular double p-algebra is isomorphic to a substructure lattice, and every regular double p-algebra embeds into one.



Reconstructing the topology on monoids and polymorphism clones of reducts of the rationals



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Transformation Monoids and Clones on a set A carry a natural topology, induced by the topology of point-wise convergence. The endomorphism monoids $\text{End}(\mathcal{A})$ and polymorphism clones $\text{Pol}(\mathcal{A})$ of a relational structure \mathcal{A} are viewed abstractly as topological monoids and topological clones, respectively. Their topology is the natural one. In this talk we show how to reconstruct the topology on the monoid of endomorphisms and the polymorphism clone of reducts of the rationals, which are the betweenness relation, circular order, and separation relation on \mathbb{Q} .





Cancellable elements of lattices of semigroup and epigroup varieties

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An element x of a lattice $\langle L; \vee, \wedge \rangle$ is called *modular* if

$$(\forall y, z \in L) \quad y \leq z \longrightarrow (x \vee y) \wedge z = (x \wedge z) \vee y$$

and *cancellable* if

$$(\forall y, z \in L) \quad x \vee y = x \vee z \ \& \ x \wedge y = x \wedge z \longrightarrow y = z.$$

It is easy to see that any cancellable element is a modular one. In [1], commutative semigroup varieties that are modular elements of the lattice **SEM** of all semigroup varieties were completely classified. Now we prove that, for commutative varieties, the properties of being modular and cancellable elements are equivalent. So, we completely determine all commutative varieties that are cancellable elements of **SEM**.

An *epigroup* is a semigroup S such that some power of any elements in S lies in some subgroup of S . It is well known that epigroups may be considered as *unary semigroups*, i.e., semigroups equipped with certain additional unary operation called *pseudoinversion* (see [2], for instance). This allows us to consider varieties of epigroups. Commutative epigroup varieties that are modular elements of the lattice **EPI** of all epigroup varieties are completely determined in [3]. We verify that, for epigroup varieties, the analog of the mentioned semigroup result is true. Namely, for commutative epigroup varieties, the properties of being modular and cancellable elements in the lattice **EPI** are equivalent. So, we classify all commutative epigroup varieties that are cancellable elements of **EPI**.

This is the joint work with S. V. GUSEV and D. V. SKOKOV.

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Cut-continuous pomonoids

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We investigate a generalisation of residuated posets. The latter's defining condition is that the monoidal product is residuated in each argument; we define cut-continuous pomonoids by requiring that the monoidal product is in each argument only cut-continuous. A mapping between posets is cut-continuous if the inverse image of each principal ideal is a cut, that is, it coincides with the set of lower bounds of its upper bounds. If the poset underlying a cut-continuous pomonoid is a complete lattice, we deal with unital quantales. If the poset is a chain, the condition of cut-continuity means that the multiplication distributes over existing suprema.

We choose the morphisms between cut-continuous pomonoids in a way consistent with the case of residuated lattices. Any congruence, defined accordingly, is then induced by a filter, in the same way as known for residuated lattices.

Our particular interest concerns coextensions: given cut-continuous pomonoids K and C , we wonder how we can determine the cut-continuous pomonoids L such that C is a filter of L and the quotient of L induced by C is isomorphic to K . The practical construction of coextensions of K by C requires determining all possible C -modules as well as the bimorphisms between them. The latter need has motivated us to study tensor products of modules over cut-continuous pomonoids. Whereas in the context of residuated lattices the known results on this topic are largely negative, the present context is more convenient. Using results of M. Ern e and D. Picado on closure spaces, we show a tensor product to exist. An application is the construction of residuated structures related to fuzzy logics, in particular left-continuous t-norms.

This is a joint work with DAVID KRUML and JAN PASEKA.



Rearrangement problem of two-dimensional arrays by prefix reversals

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We consider a rearrangement problem of two dimensional arrays in which upper or lower case letters are written by prefix reversals. A similar problem for linear strings is known as a burnt pancake sorting problem and has been studied extensively. An unburnt version of the rearrangement problem of two dimensional arrays is clarified in “Rearranging two dimensional arrays by prefix reversals, A. Yamamura, RP 2015, Lecture Notes in Computer Science, Springer-Verlag, Vol. 9328, pp 153-165, (2015)” using group theory. To formalize a similar problem for two dimensional arrays with upper or lower case letters, we employ groupoids, that is, a category in which every morphism is invertible (or inverse semigroups). We shall prove rearrangement is always possible for an array in which number of either row or column is odd, otherwise impossible.



Free idempotent generated semigroups: subsemigroups, retracts and maximal subgroups

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Let S be a subsemigroup of a semigroup T and let $IG(E)$ and $IG(F)$ be the free idempotent generated semigroups over the biordered sets of idempotents of E of S and F of T , respectively. We examine the relationship between $IG(E)$ and $IG(F)$, including the case where S is a retract of T . We give sufficient conditions satisfied by T and S such that for any $e \in E$, the maximal subgroup of $IG(E)$ with identity e is isomorphic to the corresponding maximal subgroup of $IG(F)$. We then apply this result to some special cases and, in particular, to that of the partial endomorphism monoid $P\text{End } A$ and the endomorphism monoid $\text{End } A$ of an independence algebra A of finite rank. As a corollary, we obtain the





result of Dolinka for the partial transformation monoid PT_n and the full transformation monoid T_n over a finite n -element set.

This is joint work with VICTORIA GOULD and THOMAS QUINN-GREGSON.

Decent Mal'cev conditions which hold in all locally finite congruence meet-semidistributive varieties II

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We outline the proof of the characterization of the Mal'cev conditions in the title. Our proof changes languages several times.

Initially we represent decent Mal'cev conditions as equivalence relations on the power set of the set of positions in the operation. This is used to describe the manipulations of the Mal'cev conditions, making them more restrictive at each stage, but keeping them realized in certain fixed four-element algebra.

The final equivalence relation we arrive at, after all manipulations, has four classes, two are singletons and trivial, and one of the other classes encodes the whole relation. This class is then viewed as a partially ordered set with disjointness, order being the inclusion, and disjointness a symmetric binary relation subject to some abstract axioms (which are all satisfied by actual disjointness of subsets). We prove a Ramsey theoretic lemma about finding arbitrary finite monochromatic posets with disjointness in any n -colored power set of a large enough set.

This sets the stage for the final shift in our language, where we speak about a Constraint Satisfaction Problem instance which encodes the realization of the decent Mal'cev condition. To prove the instance has a solution we use Libor Barto's bounded width theorem about (2,3)-minimal instances. To prove our desired Mal'cev condition is realized entails finding the set of variables which encode the related poset with disjointness and are all mapped by the solution to the same element. But this is exactly what our Ramsey lemma gives us, completing the proof.



This is joint work with NEMANJA DRAGANIĆ, PETAR MARKOVIĆ and VLADO ULJAREVIĆ, and Petar Marković will introduce the topic in the related talk.

Restriction semigroups and λ -Zappa-Szép products

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The semidirect product and Zappa-Szép product of two inverse semigroups need not be inverse in general. Billhardt showed how to get round this difficulty by modifying the definition of semidirect product of two inverse semigroups to obtain what he termed a λ -semidirect product. Gilbert and Wazzan generalised Billhardt's concept of λ -semidirect product to what they named as λ -Zappa-Szép products.

We unify these concepts by considering what we name the *scaffold* of a Zappa-Szép product $S \bowtie T$ where S and T are restriction. Under certain conditions this scaffold becomes a category. If one action is trivial, or if S is a semilattice and T a monoid, the scaffold may be ordered so that it becomes an inductive category. A standard technique, developed by Lawson and based on the Ehresmann-Schein-Nambooripad result for inverse semigroups, allows us to define a product on our category. We thus obtain restriction semigroups that are λ -semidirect products and λ -Zappa-Szép products, extending the work of Billhardt and of Gilbert and Wazzan.





PROGRAMME

Wednesday, 14 June

16:00 – 21:00 Registration (Room 16, DMI – Mathematics & Informatics building, ground floor)

Thursday, 15 June – Morning

7:45 – 8:30 Registration (Rectorate – The central building of UNS, ground floor)

	Amphitheatre (ground floor)	Room (floor I)	Room (floor II)
9:00 – 9:15	<i>Opening ceremony</i>		
	SESSION A1 (Chair: K. Kearnes)		
9:15 – 10:10	Brian Davey: <i>The homomorphism lattice induced by a finite algebra</i>		
10:10 – 10:45		<i>Coffee break</i>	
		SESSION B1 (Chair: M. Maróti)	SESSION C1 (Chair: M. V. Volkov)
10:45 – 11:10	Reinhard Pöschel: <i>Reflection-closed varieties of multisorted algebras and minor identities I</i>	Emília Halušková: <i>On limit closed classes of algebras</i>	Karl Auinger: <i>On the variety of strict pseudosemitlattices</i>
11:15 – 11:40	Erkko Lehtonen: <i>Reflection-closed varieties of multisorted algebras and minor identities II</i>	Andrew Moorhead: <i>Higher Delta and supernilpotence for congruence modular varieties</i>	Boris M. Vernikov: <i>Cancellable elements of the lattice of semigroup varieties</i>
11:45 – 12:10	Kalle Kaarli: <i>Representation of integral quantales by tolerances</i>	Eran Crockett: <i>Dualizable algebras of arbitrary nilpotence class</i>	Alexander Jende: <i>On the characterization of orthogroups by disjunctions of identities</i>

Thursday, 15 June – Afternoon

	Amphitheatre (ground floor)	Room (floor D)	Room (floor II)
14:30 – 15:25	SESSION A2 (Chair: M. Ploščica) Gábor Czédli: <i>From weak congruence lattices to circles and an online game</i>		
15:30 – 15:55	Eszter K. Horváth: <i>Lattice-valued functions</i>	SESSION B2 (Chair: P. Marković) Dejan Delić: <i>Maltsev CSPs are definable in 2-sorted Datalog</i>	SESSION C2 (Chair: M. Kamrites) Valdis Laan: <i>Fair semigroups</i>
16:00 – 16:25	Léonard Kwuida: <i>Generalized attributes in concept lattices</i>	Jakub Bulín: <i>Clonoids and Promise CSP</i>	Eugenia Kochubinska: <i>Spectral properties of partial automorphisms of a regular rooted tree</i>
16:25 – 17:00		<i>Coffee break</i>	
	SESSION A3 (Chair: G. Czédli)		
17:00 – 17:25	Dušan Radičanin: <i>Weak congruences on categories</i>	Alexandr Kazda: <i>Solving edge CSP with even delta-marroid constraints</i>	Eugenia A. Bondar: <i>\mathcal{B}-cross-sections of the semigroup of order-preserving transformations of a finite chain</i>
17:30 – 17:55	Anna Slivková: <i>On partial matroids, geometric posets and semimodular posets</i>	Jakub Opršal: <i>Infinite algebras with few subpowers</i>	Azeef Muhammed P.A.: <i>Cross-connections and variants of the full transformation semigroup</i>
18:00 – 18:25	Vanja Stepanović: <i>Some properties of monotonous fuzzy set operator applied to fuzzy set equations and inequations</i>	Johannes Greiner: <i>The complexity of free combinations of temporal CSPs</i>	Rida-e Zenab: <i>Restriction semigroups and λ-Zappa-Szép products</i>
18:30 – 19:25	Miroslav Ploščica: <i>Critical points for congruence lattices</i>		

The official programme of the Special Session "Lattices and Ordered Structures" (dedicated to the retirement of Prof. Branimir Šešelja) is shaded in violet.

Friday, 16 June – Morning

	Amphitheatre (ground floor)	Room (floor I)	Room (floor II)
	SESSION A4 (Chair: I. Dolinka)		
9:00 – 9:55	Nik Ruškuc: <i>Congruence lattices of diagram monoids</i>		
9:55 – 10:30		<i>Coffee break</i>	
		SESSION B3 (Chair: D. N. Zhuk)	SESSION C3 (Chair: B. Davey)
10:30 – 10:55	Victoria Gould: <i>Coherency for monoids</i>	Stanislav Moiseev: <i>The lattice of all clones definable by binary relations on a three-element set</i>	Claudia Mureşan: <i>Transferring Davey's theorem on annihilators and m-completeness to modular congruence lattices</i>
11:00 – 11:25	Mark Kambites: <i>Random walks on semigroups</i>	Ádám Kunos: <i>Clones of small posets</i>	Christopher Taylor: <i>Algebras of incidence structures: representing regular double p-algebras</i>
11:30 – 11:55	Mária B. Szendrei: <i>On products of inverse semigroups</i>	Miroslav Oľšák: <i>The weakest nontrivial idempotent equations</i>	Gergő Gyenizse: <i>Lattice representations with DCC posets</i>

Friday, 16 June – Afternoon

	Amphitheatre (ground floor)	Room (floor D)	Room (floor II)
14:00 – 14:55	SESSION A5 (Chair: N. Ruškuc) Mikhail V. Volkov: <i>Local finiteness for Green's relations in semigroup varieties</i>	SESSION B4 (Chair: R. Pöschel)	SESSION C4 (Chair: D. Delić)
15:00 – 15:25	Melanija Mitrović: <i>Constructive semigroups with apartness – a new algebraic theory</i>	Denis I. Saveliev: <i>On homotopies of universal algebras</i>	Petar Marković: <i>Decent Mal'cev conditions which hold in all locally finite congruence meet-semidistributive varieties I</i>
15:30 – 15:55	László Márki: <i>Morita equivalence of semigroups revisited: Firm semigroups</i>	Dmitry Bredikhin: <i>On groupoids of relations with primitive-positive operations</i>	Samir Zahirović: <i>Decent Mal'cev conditions which hold in all locally finite congruence meet-semidistributive varieties II</i>
15:55 – 16:30		Coffee break	
16:30 – 16:55	António Malheiro: <i>Identities in plactic and related monoids</i>	Eugene Plotkin: <i>Action of endomorphism semigroups on definable sets</i>	Paolo Agliano: <i>A general algebraic motivation for loops</i>
17:00 – 17:55	Miodrag Sokić: <i>Diameter 3</i>		

19:30 – ... *Conference dinner*

The official programme of the Special Session "Semigroups" (dedicated to the retirement of Prof. Siniša Crvenković) is shaded in turquoise.

Saturday, 17 June – Morning

	Room (floor I)	Room (floor II)	Room (ground floor)
	Amphitheatre (ground floor)		
	SESSION A6 (Chair: M. Pech)		
9:00 – 9:55	Manuel Bodirsky: <i>A dichotomy for first-order reducts of unary structures</i>		
9:55 – 10:30	Coffee break		
	SESSION B5 (Chair: A. Trepavčević)		
10:30 – 10:55	Thomas Vetterlein: <i>Cut-continuous pomonoids</i>	Peter Fenner: <i>The gossip monoid</i>	SESSION D1 (Chair: M. Chiş) Codruţa Chiş: <i>On a combinatorial problem on directed graphs</i>
11:00 – 11:25	Milan Petrik: <i>On the structure of finite commutative totally ordered monoids</i>	Dandan Yang: <i>Free idempotent generated semigroups: subsemigroups, retracts and maximal subgroups</i>	Bojan Bašić: <i>A problem on squares and its application to the square packing problem</i>
11:30 – 11:55	Helmut Länger: <i>Non-associative MV-algebras</i>	Akihiro Yamamura: <i>Rearrangement problem of two-dimensional arrays by prefix reversals</i>	George Metcalfe: <i>An algorithmic ordering condition for groups</i>
	Mike Behrisch: <i>Centralizing monoids with majority witnesses on four-element domains</i>		
	Edith Vargas-García: <i>Reconstructing the topology on monoids and polymorphism clones of reducts of the rationals</i>		
	Christian Pech: <i>Reconstructing the topology of the elementary self-embedding monoids of countable saturated structures</i>		

Saturday, 17 June – Afternoon

	Room (floor I)	Room (floor II)	Room (ground floor)
	Amphitheatre (ground floor)		
	SESSION A7 (Chair: M. Bodirsky)		
14:30 – 15:25			
	SESSION B6 (Chair: L. Márkai)		
	SESSION C6 (Chair: M. B. Szendrői)		
15:30 – 15:55			
16:00 – 16:25			
16:25 – 17:00	<i>Coffee break</i>		
17:00 – 17:25			
17:30 – 17:55			
18:00 – 18:55			

19:30 – ... Round table devoted to the recently proposed algebraic solutions of the CSP dichotomy conjecture (Department of Mathematics and Informatics, Amphitheatre I) – moderated by R. Willard

The official programme of the Special Session "Clones and Relation Algebras" (dedicated to the 70th birthday of Prof. Reinhard Pöschel) is shaded in light green.

Sunday, 18 June – Morning

	Room (floor I)	Room (floor II)	Room (ground floor)
	Amphitheatre (ground floor)		
	SESSION A8 (Chair: E. Aichinger)		
8:45 – 9:40	David Stanovský: <i>Quandles and universal algebra</i>	SESSION C7 (Chair: M. Johnson)	SESSION D3 (Chair: P. Agliano)
9:45 – 10:10	Libor Barto: <i>Cores of oligomorphic clones revisited</i>	Itamar Stein: <i>The global dimension of the algebra of the monoid of all partial functions on an n-set</i>	Aleksandar Krapež: <i>Units in quasigroups</i>
10:10 – 10:45	<i>Coffee break</i>		
10:45 – 11:10	Zarathustra Brady: <i>Spirals</i>	Nada Damljanović: <i>Solving weakly linear inequalities for matrices over max-plus semiring and applications to automata theory</i>	Marco Bonatto: <i>Taylor and Mal'tsev quandles</i>
11:15 – 11:40	Peter Mayr: <i>The complexity of quantified constraint satisfaction on monoids</i>	Tanya Plotkin: <i>Syntax versus semantics in knowledge bases</i>	Přemysl Jedlička: <i>Subdirectly irreducible medial quandles</i>
11:45 – 12:40	Ágnes Szendrei: <i>Cube terms and the Subpower Membership Problem</i>		
12:40 – 12:50	Closing		