Functions, Dynamic Geometry and CAS: offering possibilities for learners and teachers

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Functions, DG and CAS

- The CaSyOPEE research group
- What can we learn from math education research about functions?
- How can we use it to implement situations of use of CAS and DG?

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The CaSyOPEE research group

- 1995-2000 DERIVE, the TI-92.
  Instrumental Approach
- 2000-2006Transposing CAS
  Building a CAS tool for classroom
- 2006-2010The ReMath project
  Focus on multi-representation
- 2010-… Dissemination to teachers.
  - usable tool
  - conceptual framework about functions and algebra

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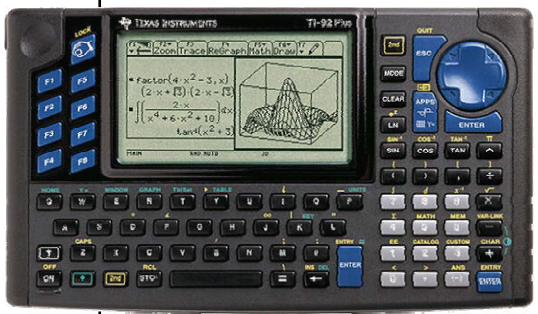
The Casyopee research group

1995-2000 DERIVE, the TI-92. Instrumental Approach	*Complex calculators in the classroom: theoretical and practical reflections on teaching pre-calculus* INTERNATIONAL JOURNAL OF COMPUTERS FOR MATHEMATICAL LEARNING Volume 4, Number 1 (1999)	•Tasks and techniques to help students to develop an appropriate instrumental genesis for algebra and functions •Potential of the calculator for connecting enactive representations and theoretical calculus



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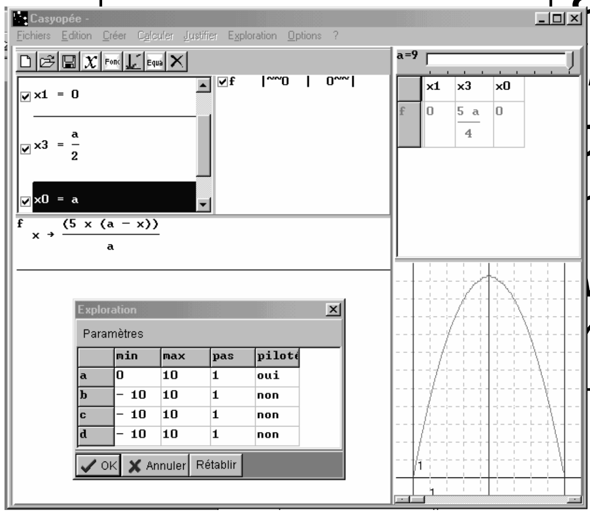
2000-2006 Transposing CAS Building a tool for the classroom	Curriculum, classroom practices, and tool design in the learning of functions through technology-aided experimental approaches *International journal of computers for mathematical learning Volume 10, Number 2 (2005)*	• while symbolic calculation is a basic tool for mathematicians, curricula and teachers are very cautious • design and experiment of a computer environment as means to contribute to an evolution of curricula and classroom practices



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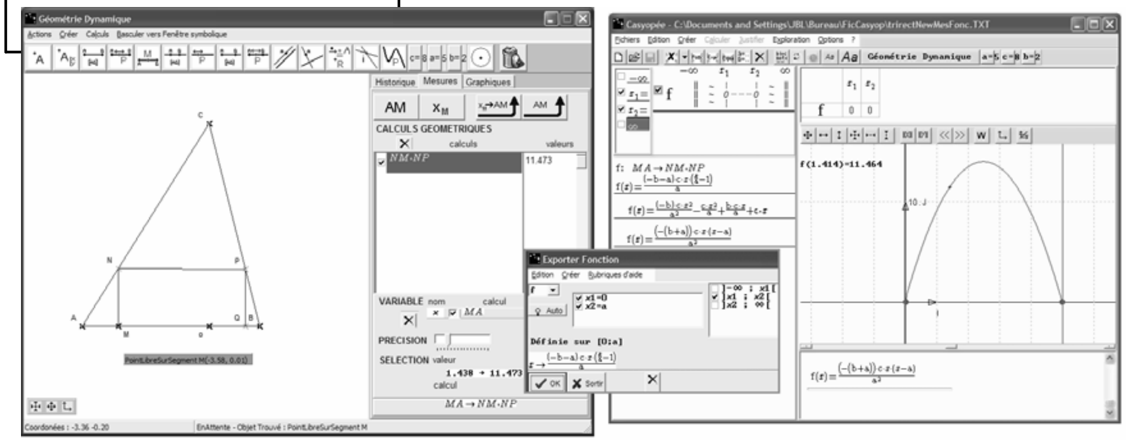
2006-2010 The ReMath project Focus on multi-representa-tion	Teaching and learning about functions at upper secondary level: designing and experimenting the software environment Casyopée. *International Journal of Mathematical Education in Science and Technology* Volume 41, Issue 2, 2010	An experimental teaching unit carried out in the ReMath European project focusing on the approach to functions via multiple representations for the 11th grade.



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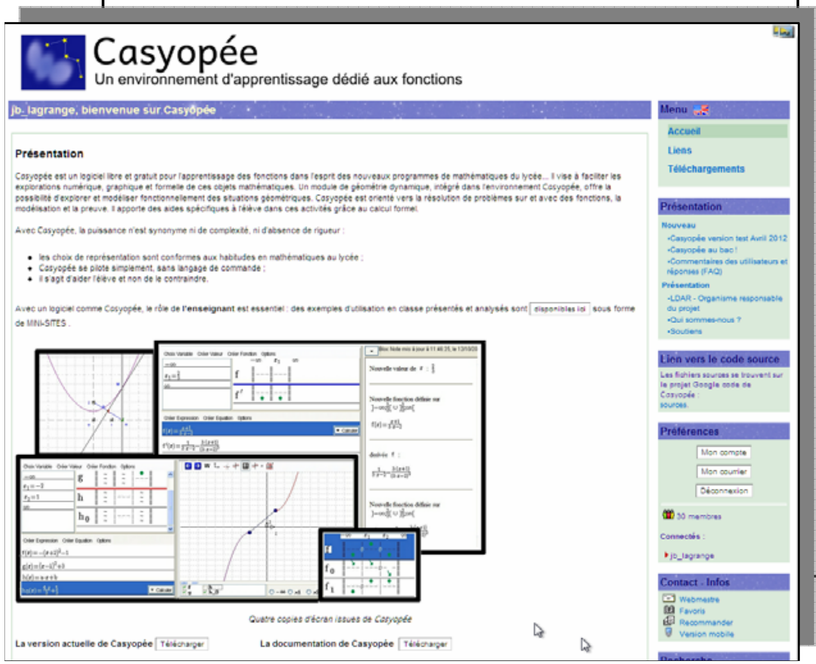
The Casyopee research group

2010-… Dissemination to teachers. -usable tool -conceptual framework about functions and algebra	Students' activities about functions at upper secondary level *33rd PME Conf. July 2009* Working with teachers: Innovative software at the boundary between research and classroom. *35th PME Conf. July 2001*	•A grid that organises and connects various students' activities about functions at upper secondary level. •Diffusion of research outcomes through communal work involving researchers and teachers .



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- What can we learn from math education research about functions?
  - A functional perspective on the teaching of algebra (Kieran)
  - Some key ideas (Lagrange Psycharis)

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A functional perspective ***on the teaching of algebra (Kieran)***

- Quite widespread in the world
- Attempt to include elements of both traditional (rational expressions and equations) and functional orientations to school algebra.
- The orientation toward the solving of realistic problems, with the aid of technological tools, allows for an algebraic content that is less manipulation oriented.
- Such orientations also emphasize multirepresentational activity with a shift away from the traditional skills of algebra

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A *functional perspective* on the teaching of algebra (Kieran)

Objections

1. Expressions can be used without describing functions, and functions can be expressed without using algebra
2. Students become confused regarding distinctions between equations and functions, not being able to sort out, for example, how equivalence of equations is different from equivalence of functions.
3. There is a strong presumption that symbolic forms are to be interpreted graphically, rather than dealt with, technology being used to insist on screen (graphical) interpretations of functions

➔ no meaning given to symbolic forms.

Teaching/learning about functions : **some key ideas**

- From process-object to co-variation aspect of functions
- The role of symbolism
- Understanding co-variation
- Understanding independent variable

From process-object …

- early nineties : distinction between
  - **process view** characterised by students' focus on the **performance of computational actions** following a sequence of operations (i.e. computing values)
  - **object view** based on the **generalization of the dependency relationships** between input-output pairs of two quantities/magnitudes

… to co-variation

- **Object-oriented** views of function emphasize the co-variation aspect of function
- The **co-variation** view of functions
  - understanding the manner in which dependent and independent variables change as well as the coordination between these changes.
- A shift in understanding an expression $f(x) = 2^x$
  - from a single input-output view,
  - to a more dynamic way : ‘running through’ a continuum of numbers’
- Not obvious for the students

The role of symbolism

- Critical role of symbolism in the development of the function concept.
  - "confronted in very different forms such as graphs and equations"
- Need for students' investigation of algebraic and functional ideas in different contexts such as the geometric one.

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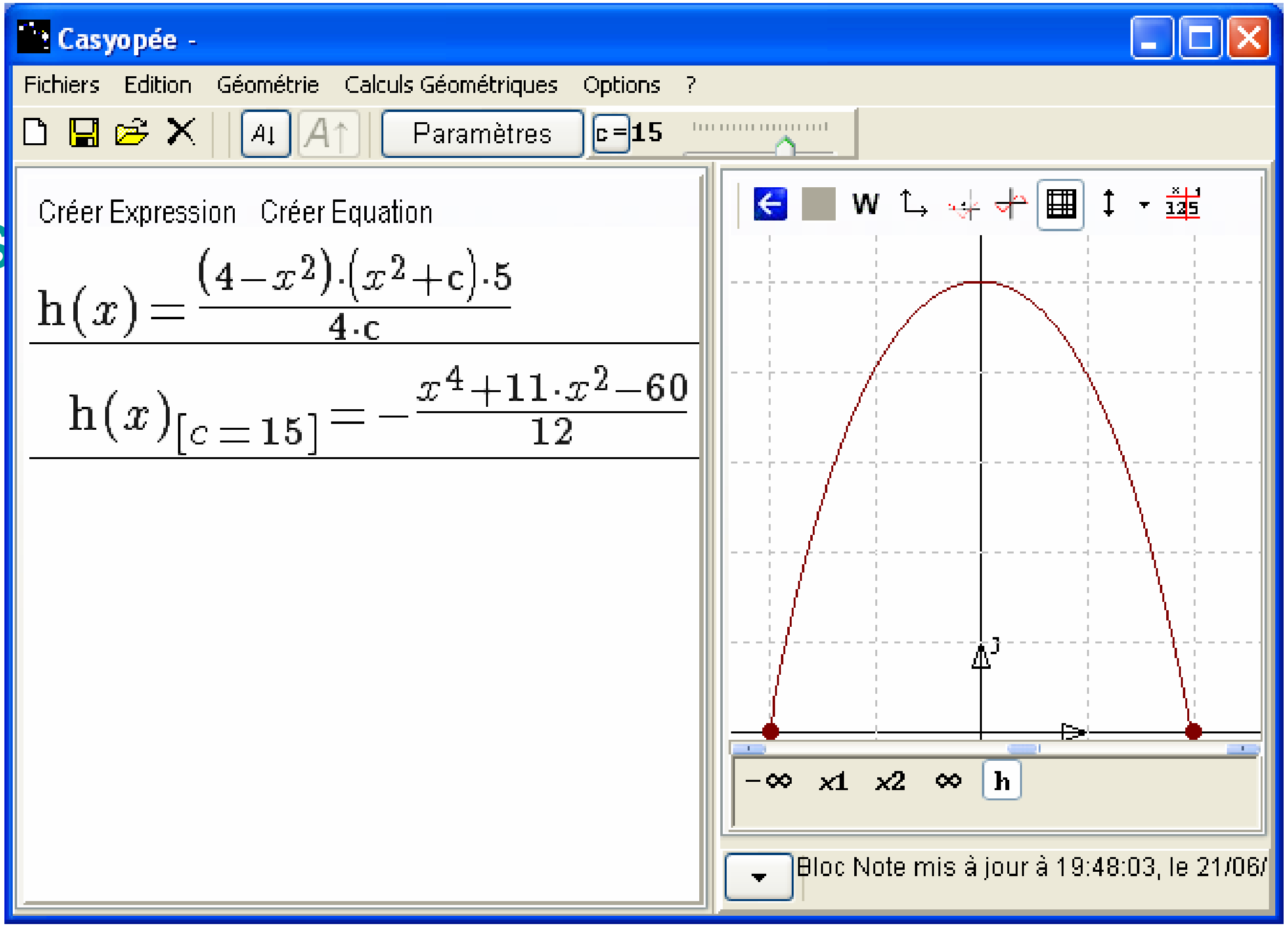
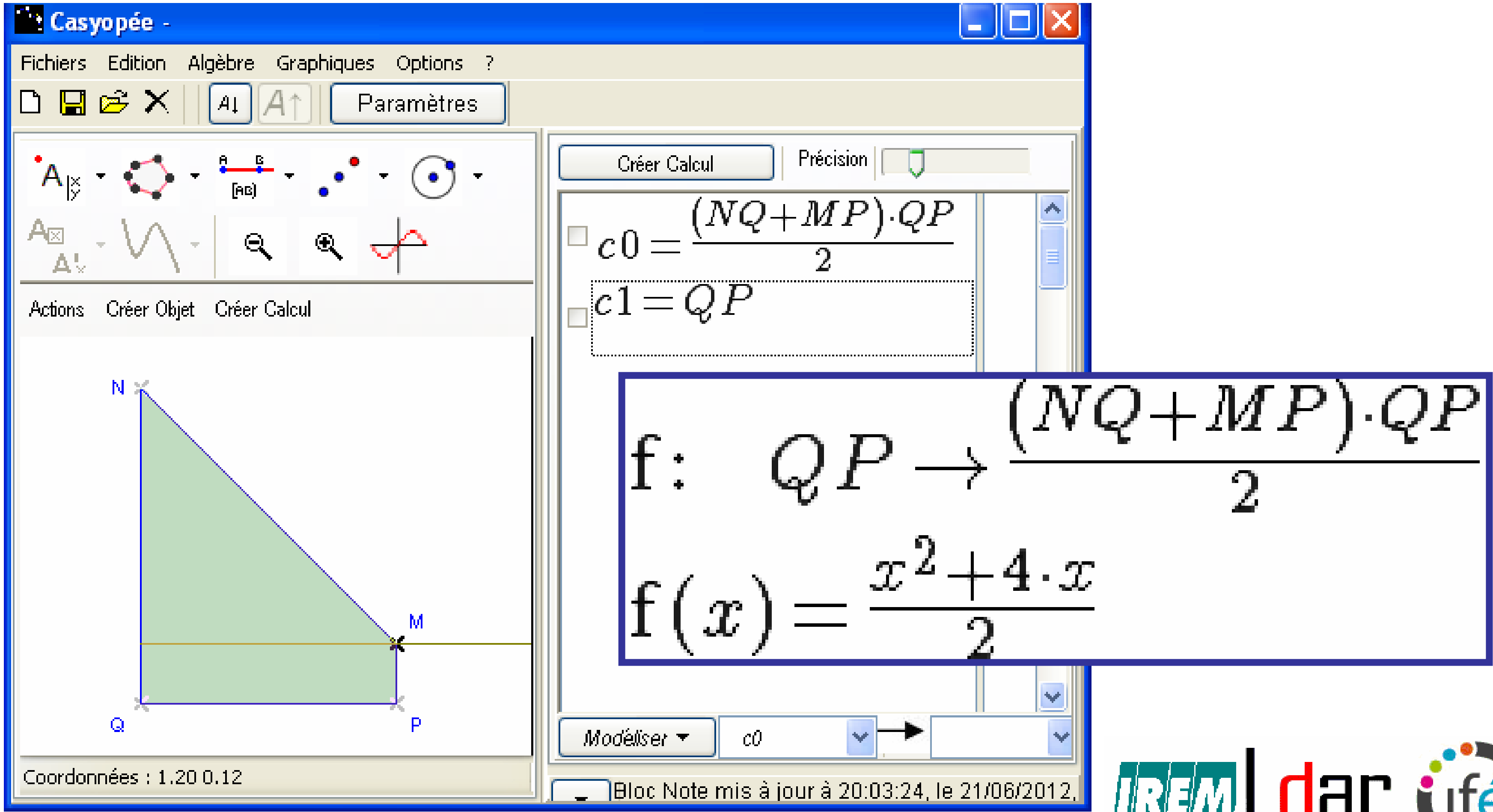


- Need for students' investigation of algebraic and functional ideas in different contexts such as the geometric one.



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Understanding **co-variation**

- Built upon understanding of correspondence in a **very long term process**.
- Situations based on modeling dynamic phenomena
  - connect functions with a **sensual experience of dependency**
  - have a potential to **help students reach awareness of co-variation**.
  - can help students to gradually **understand the properties of functions by connecting in a meaningful way its different representations**

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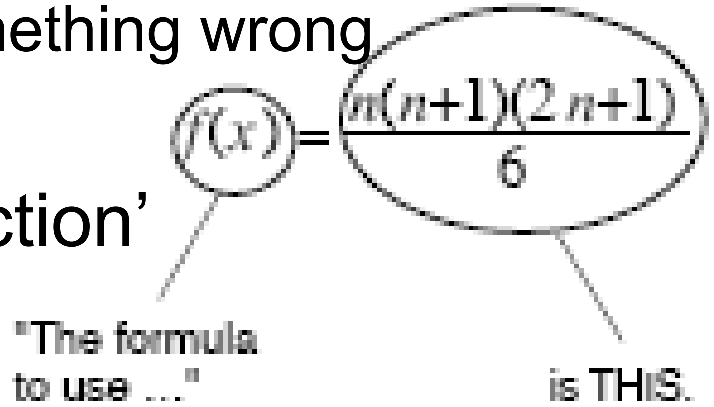
Understanding **independent variable**

- Students' persistent **'mal-formed concept images** showing up in the strangest places".
- An example: the formula for the sum of first n squares
  - A student wrote $f(x) = n(n+1)(2n+1)/6$
  - none of the students found something wrong
- Predominant image evoked
   in students' by the word 'function'
  two disconnected expressions linked by the equal sign.

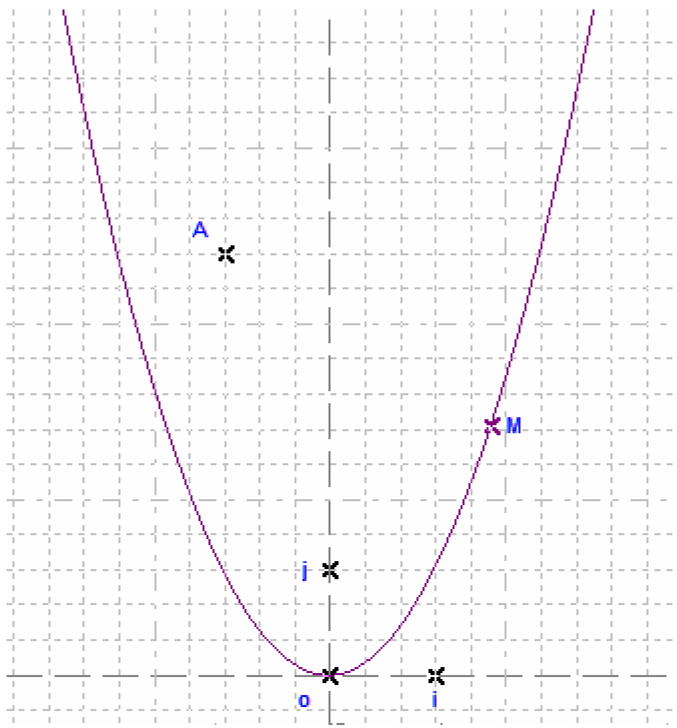


How can we use these ideas to implement situations of use of CAS and DG?

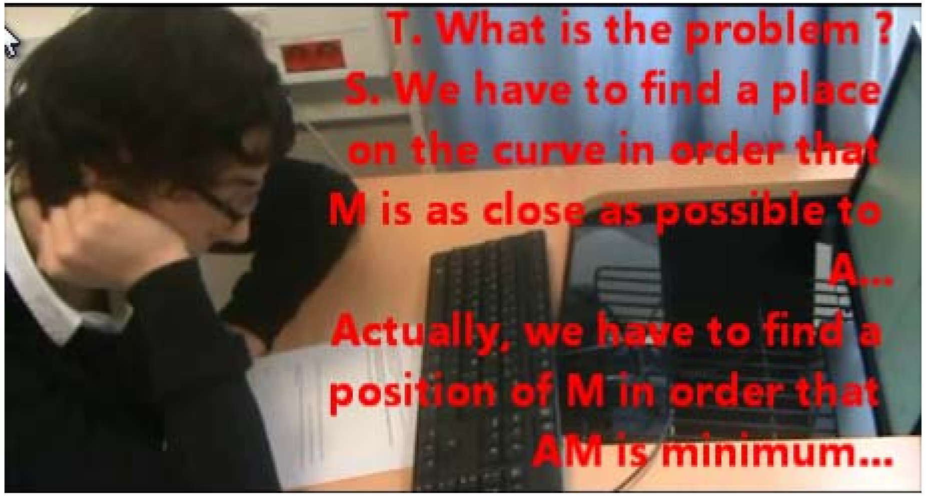
Two examples

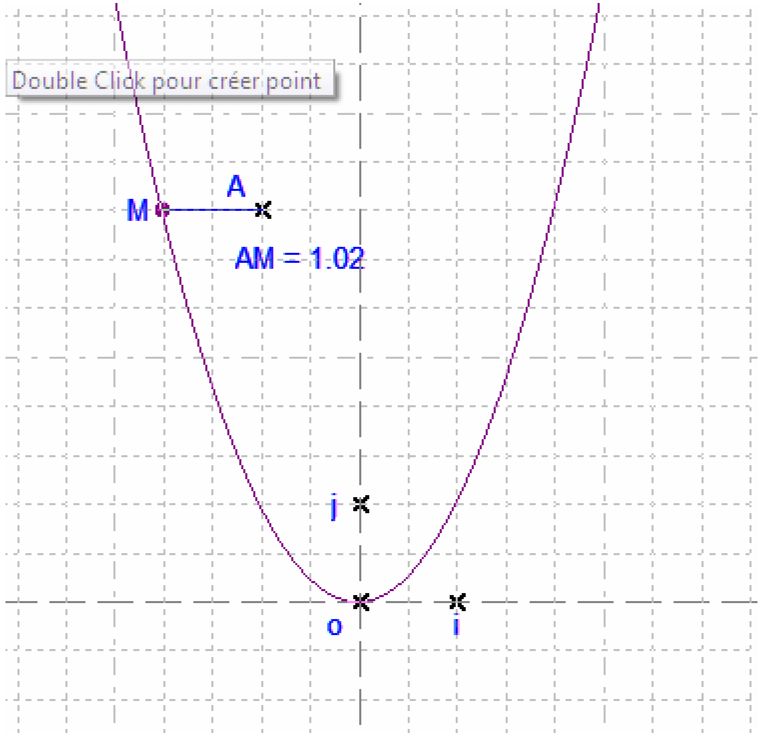
- A problem of minimum at 10th grade
- Real life experience and differentiability at 12th grade

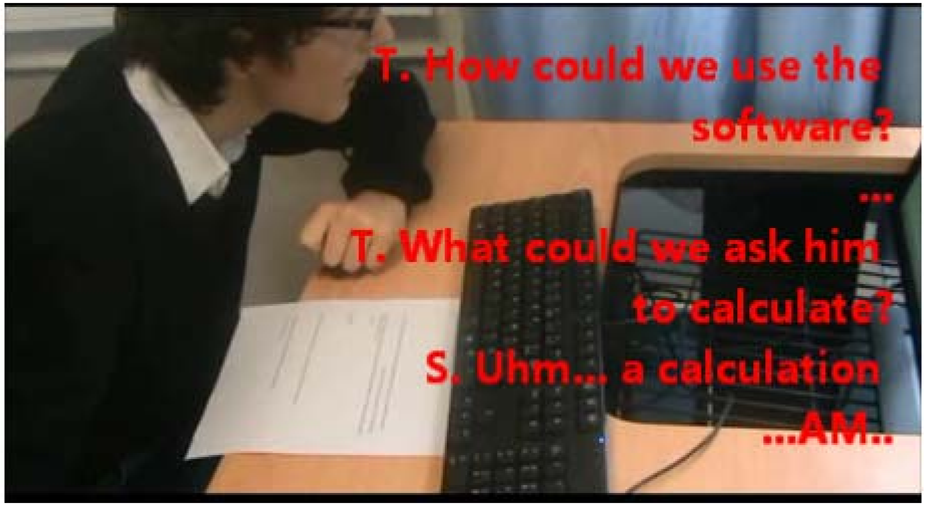
A problem of minimum at 10th grade

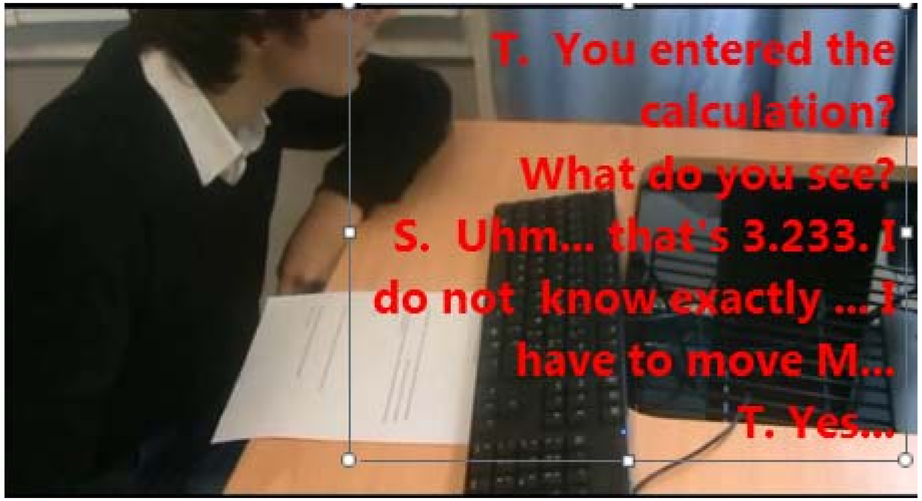


- M is a point on the parabola representing $x \rightarrow x^2$
  The goal is to find position(s) of M as close as possible to A.
- Make a dynamic geometry figure and explore.
- Use the software to propose a function modeling the problem
- Use this function to approach a solution

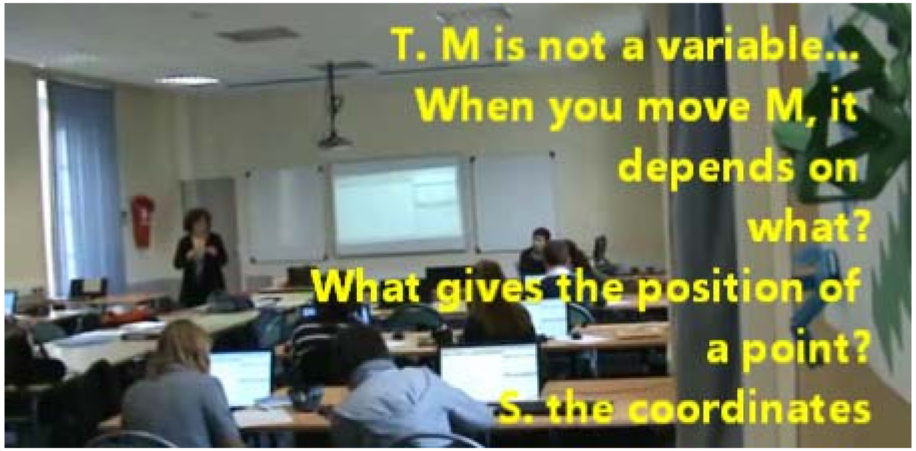

T. What is the problem ?
S. We have to find a place on the curve in order that M is as close as possible to A...
Actually, we have to find a position of M in order that AM is minimum...

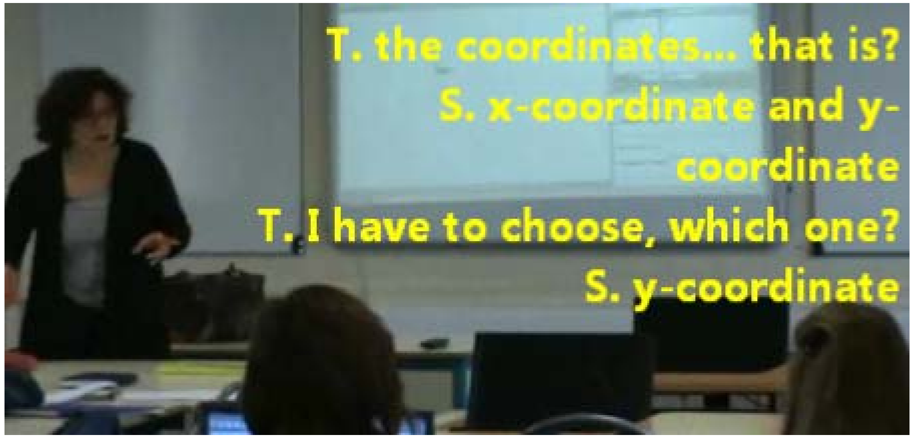

Double Click pour créer point
A
M
AM = 1.02
j
o
i

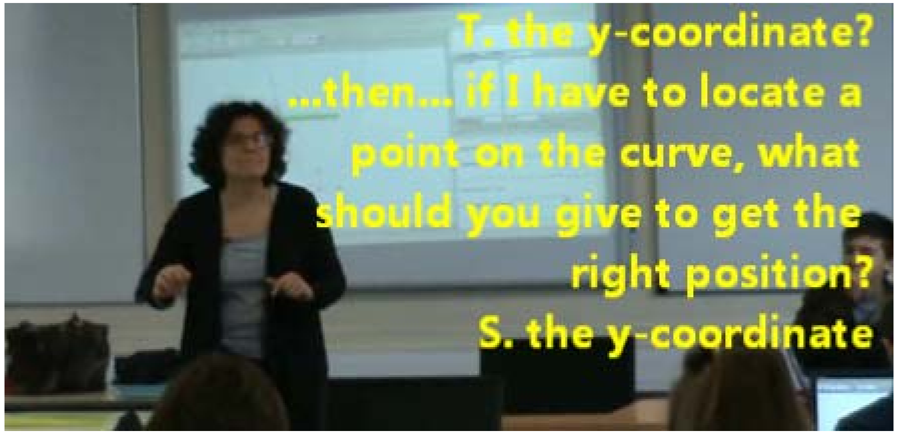

T. How could we use the software?
...
T. What could we ask him to calculate?
S. Uhm... a calculation ...AM...


T. You entered the calculation?
What do you see?
S. Uhm... that's 3.233. I do not know exactly ... I have to move M...
T. Yes...

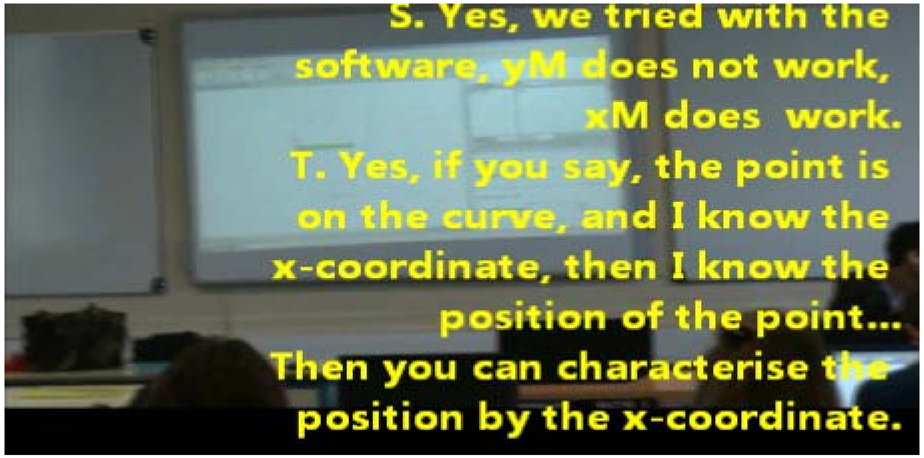

T. ...in order to get a better approximation, we need to define a function whose value is AM... but depending on what...
S. on M..

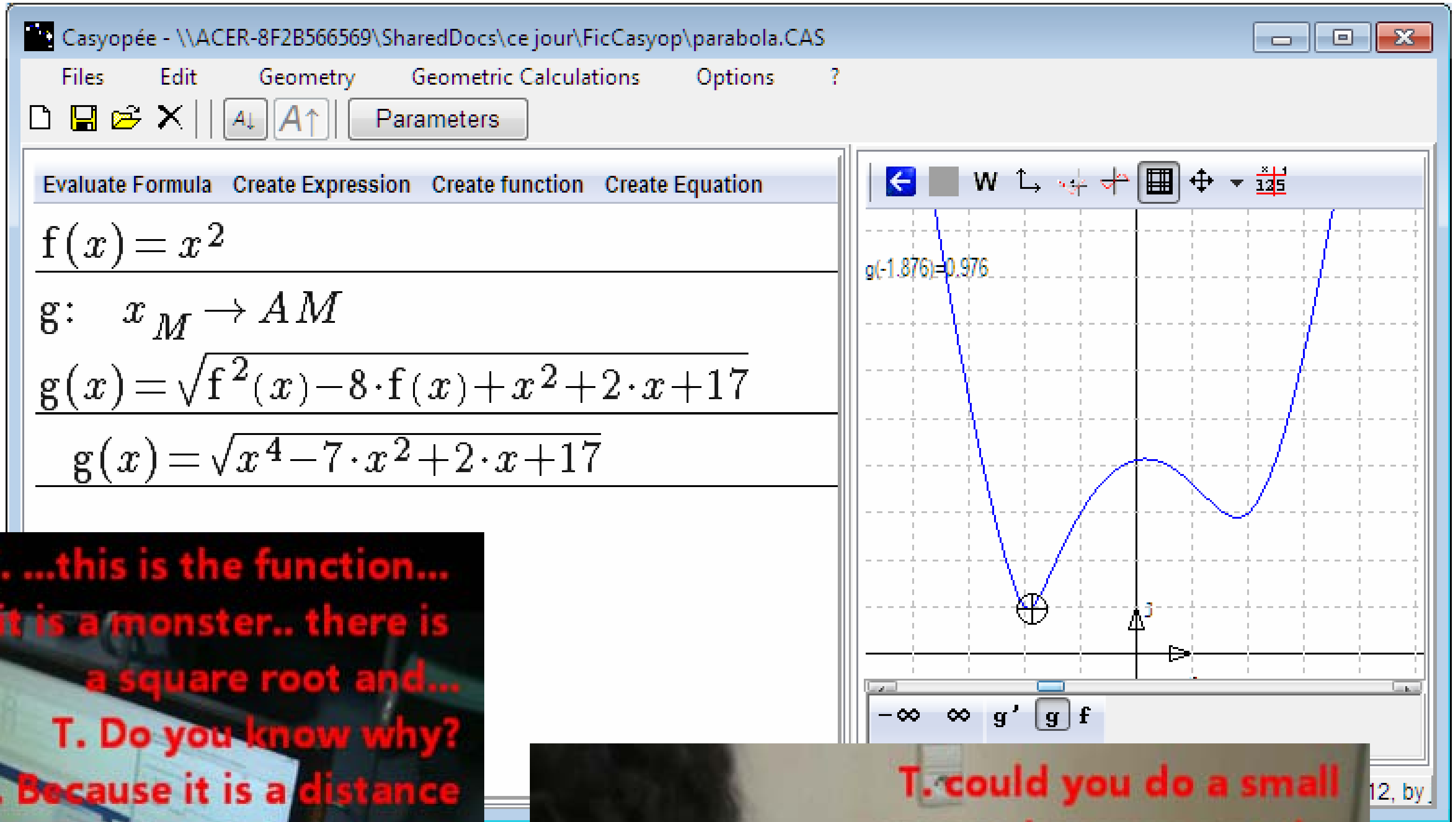

T. M is not a variable... When you move M, it depends on what?
What gives the position of a point?
S. the coordinates

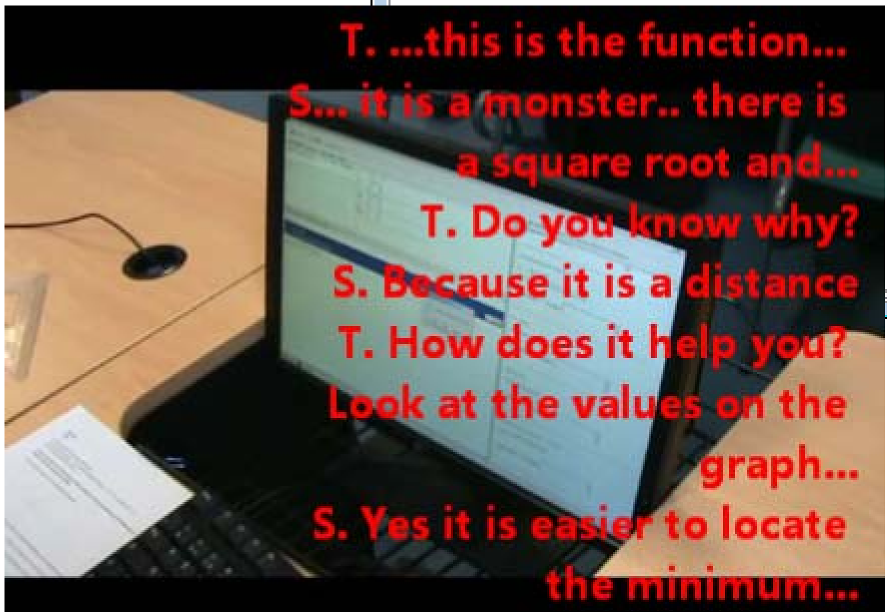

T. the coordinates... that is?
S. x-coordinate and y-coordinate
T. I have to choose, which one?
S. y-coordinate

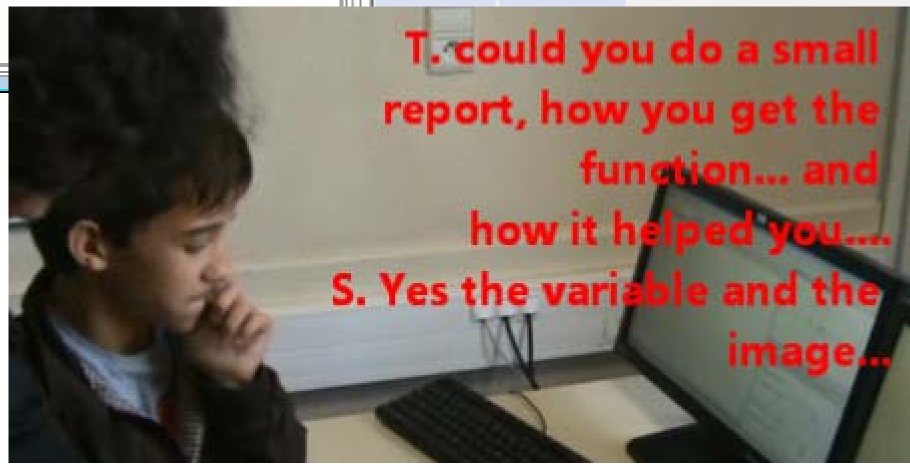

T. the y-coordinate?
...then... if I have to locate a point on the curve, what should you give to get the right position?
S. the y-coordinate


T. if you ask me for the y-coordinate 4...
S. there are two points... we need to give the x-coordinate
T. with the x-coordinate,
is it correct?


S. Yes, we tried with the software, yM does not work, xM does work.
T. Yes, if you say, the point is on the curve, and I know the x-coordinate, then I know the position of the point...
Then you can characterise the position by the x-coordinate.







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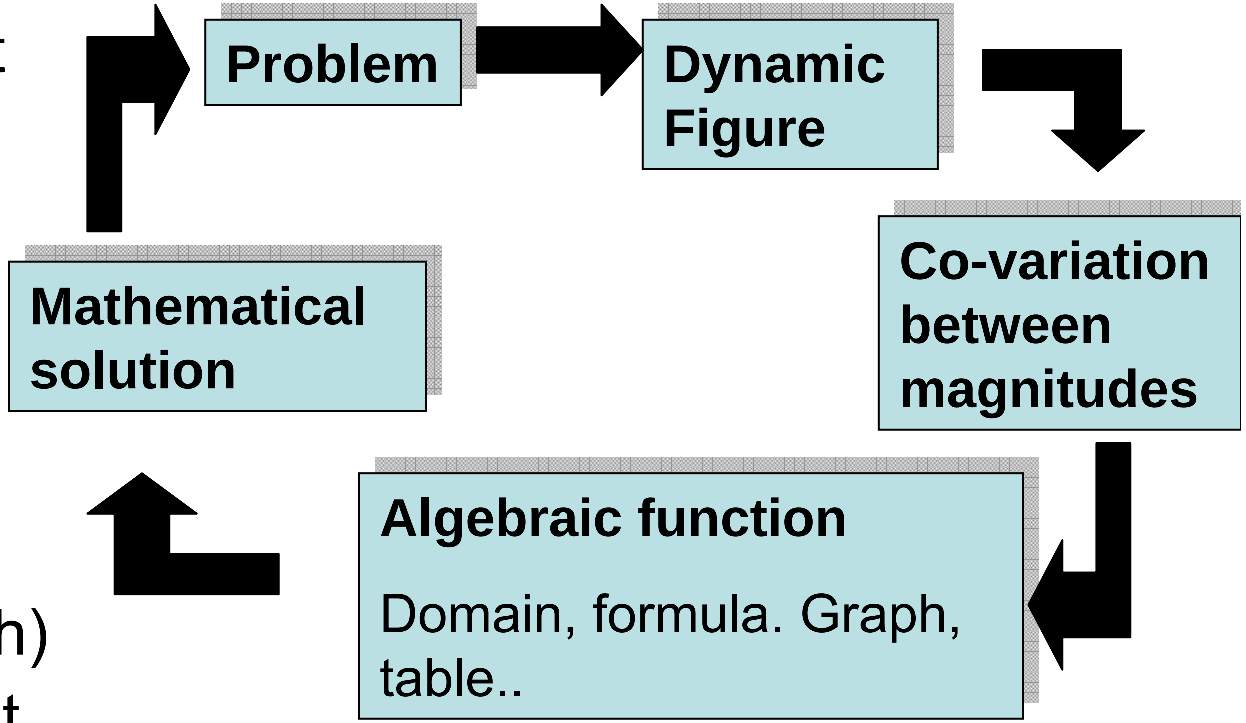
Understanding co-variation

- Contribution of the situation
- Contribution of the software

Contribution of the situation: a cycle of modelling

1. From a problem to a dynamic figure
2. Identifying relevant magnitudes
3. Understanding co-variation as a functional relationship
4. Using an algebraic representation (reading on a graph)
5. Connecting a result to the problem



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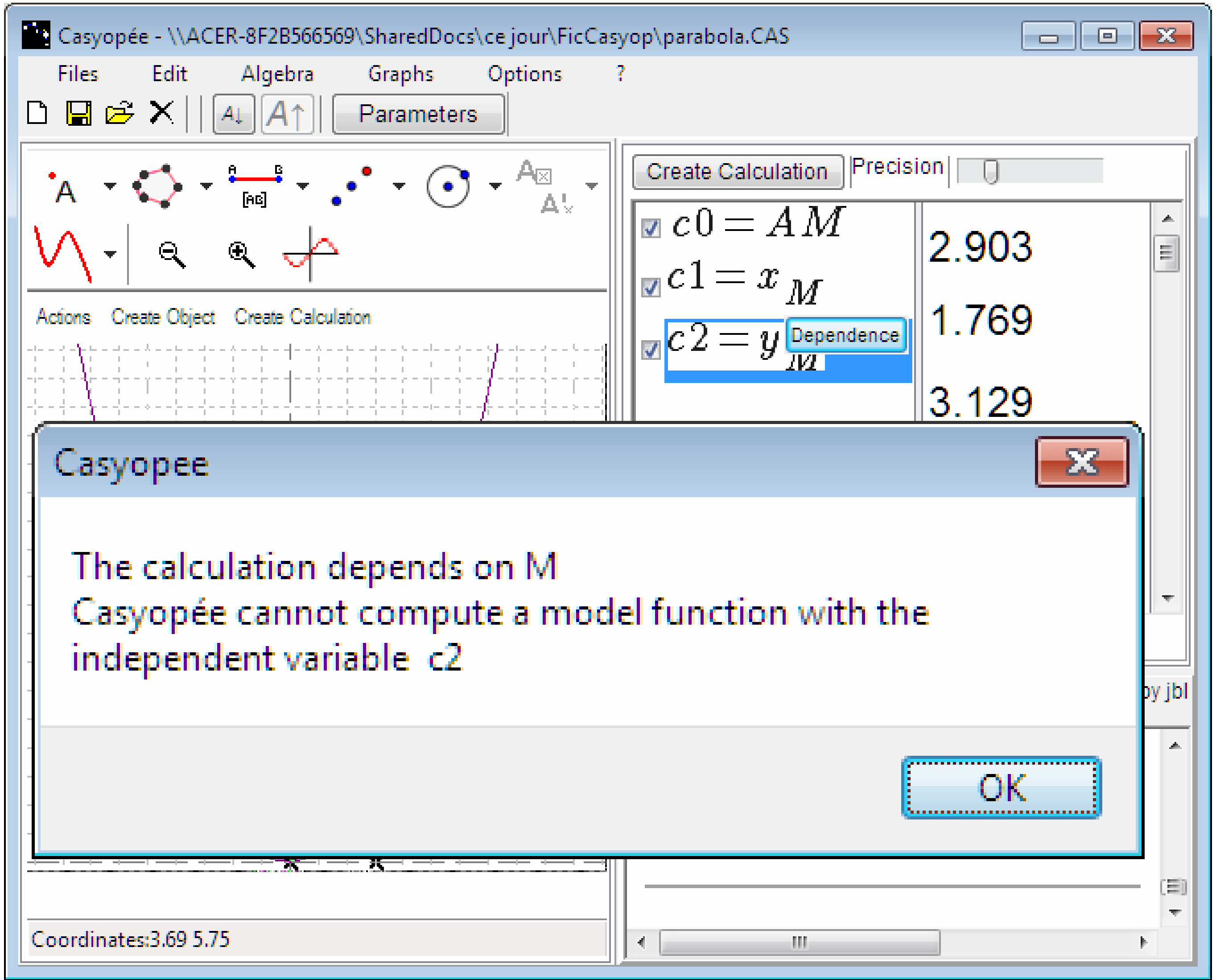
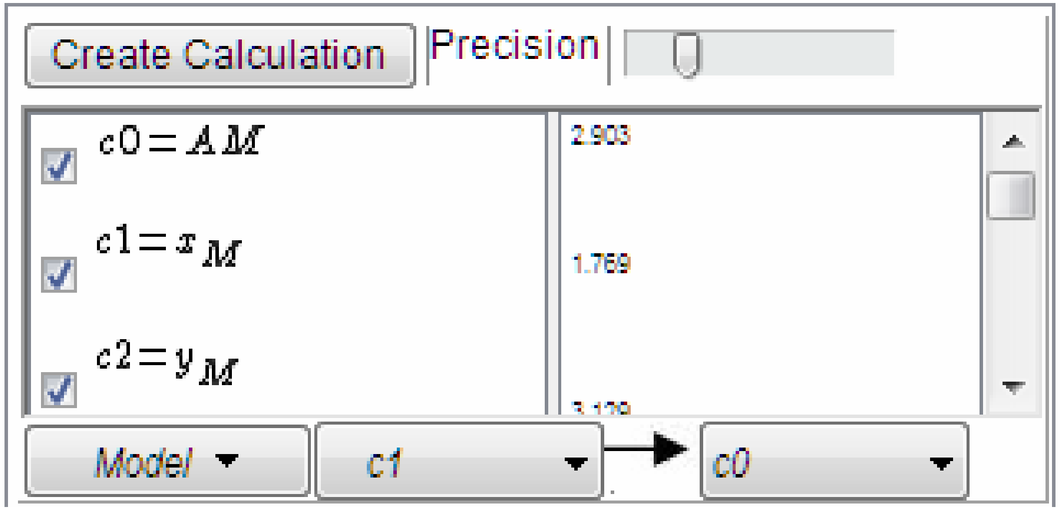
Contribution of the software

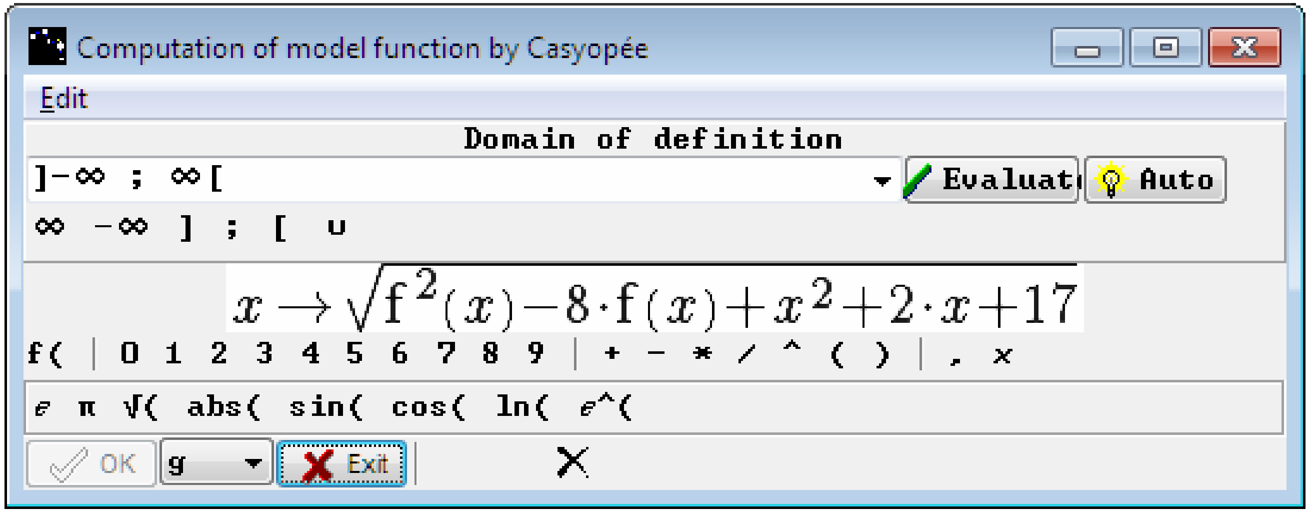
- Focus on independent and dependent variables (feedback)



Contribution of the software

- Focus on the algebraic function
  - CAS feature as an help to compute functions





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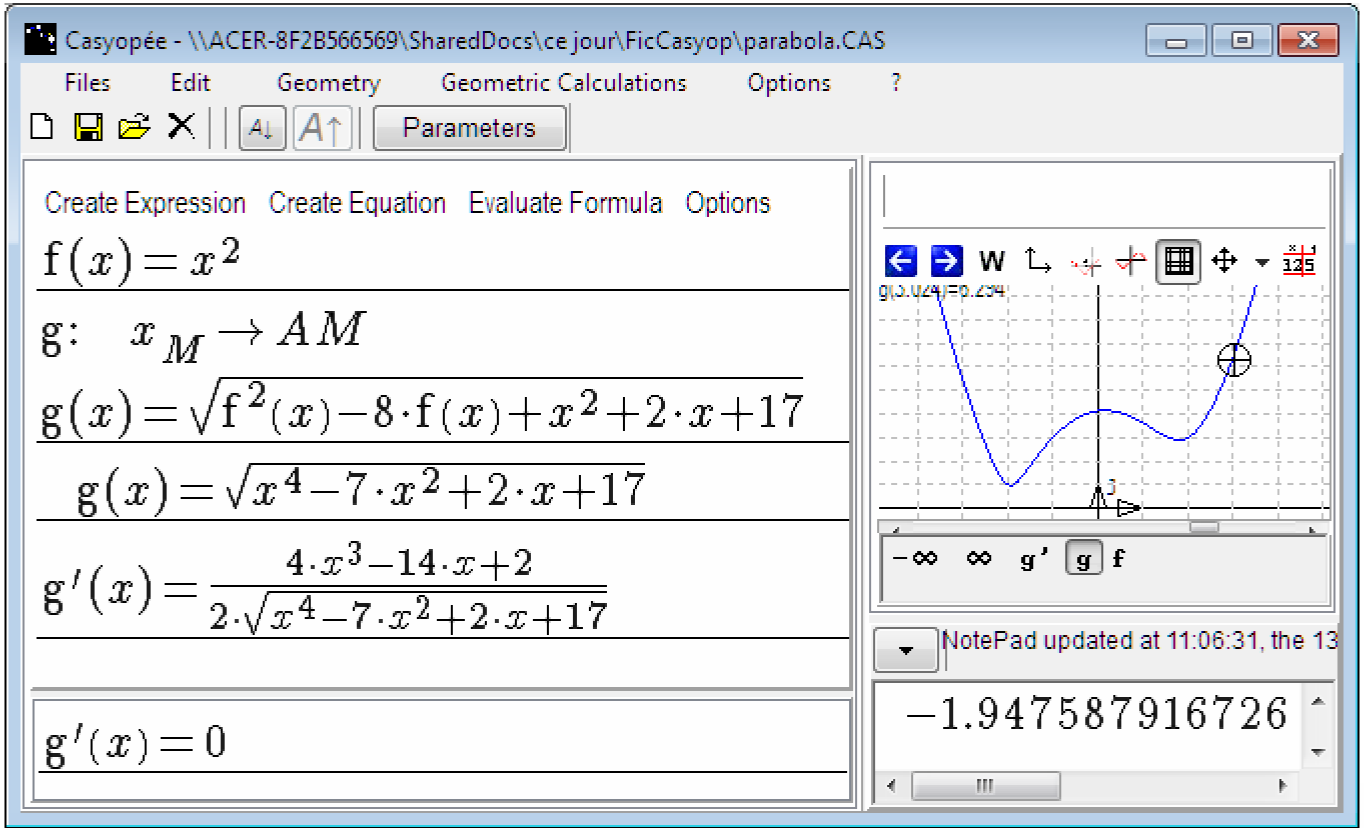
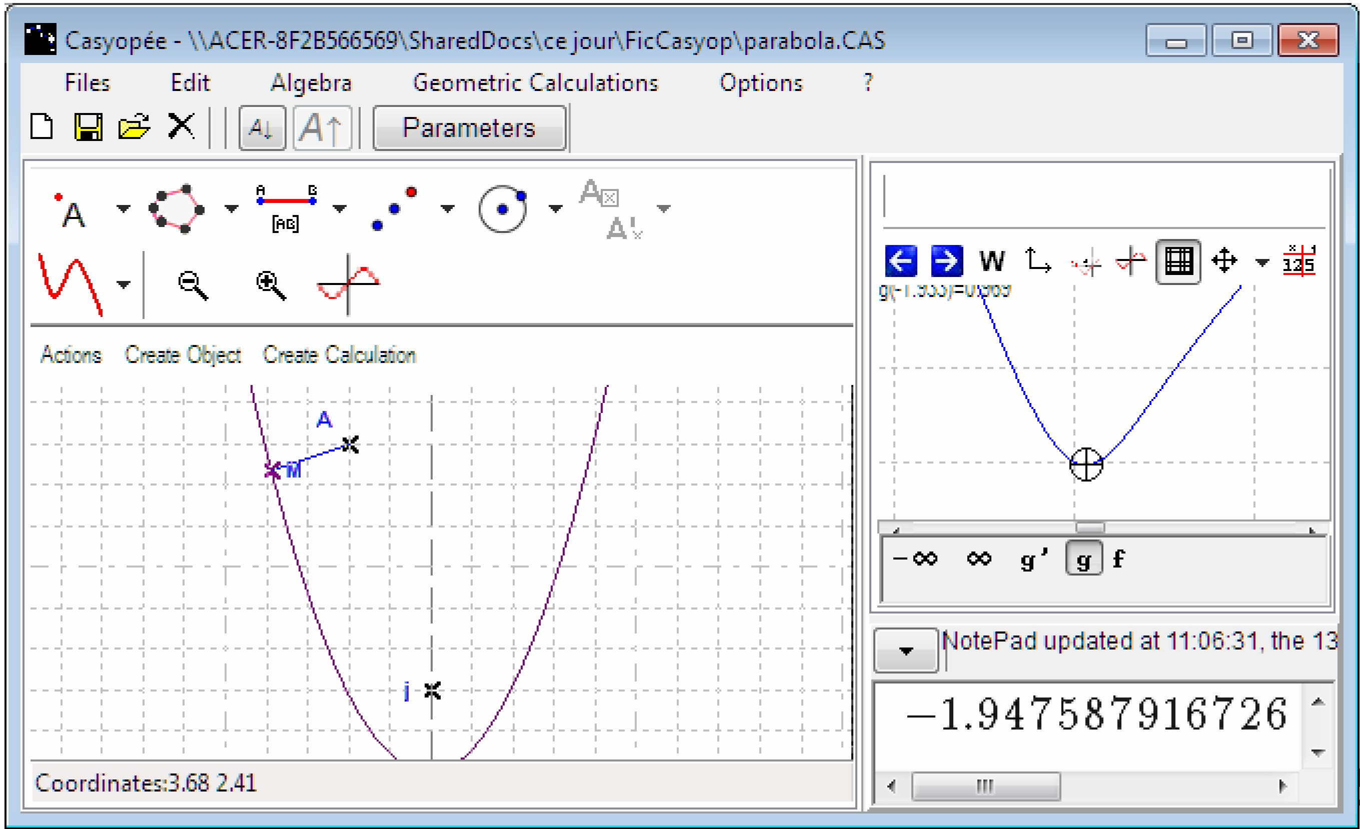
Contribution of the software

- Focus on the algebraic function
  - A full algebraic environment   Without a command language



Contribution of the software

- Focus on the algebraic function
  - in connection with the problem



2nd example: A challenge

- Considering “Irregular” functions
  - transition to university level
- Connecting
  - sensual experience of movements
  - with analytic properties of model functions

➔ The amusement park ride:

functional modeling and differentiability

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The amusement park ride: functional modeling and differentiability

- A wheel rotates with uniform motion around its horizontal axis. A rope is attached at a point on the circumference and passes through a guide. A car is hanging at the other end.
- Motion chosen in order that a person placed in the car feel differently the transition at high and low point.

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The amusement park ride: objectives

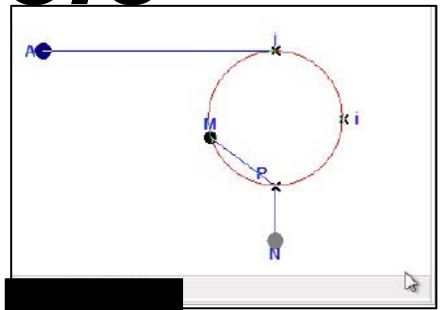
It is expected that students will

- identify the difference
- associate this with different properties of the function (non-differentiability and differentiability)
- after modelling the movement.

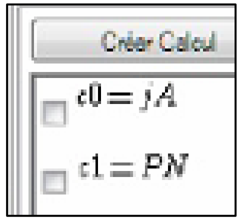
The modelling cycle

Problem

Dynamic Figure

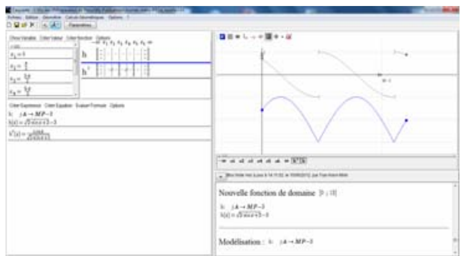


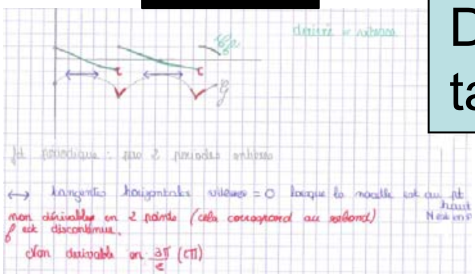
Co-variation between magnitudes



Algebraic function

Domain, formula. Graph, table..

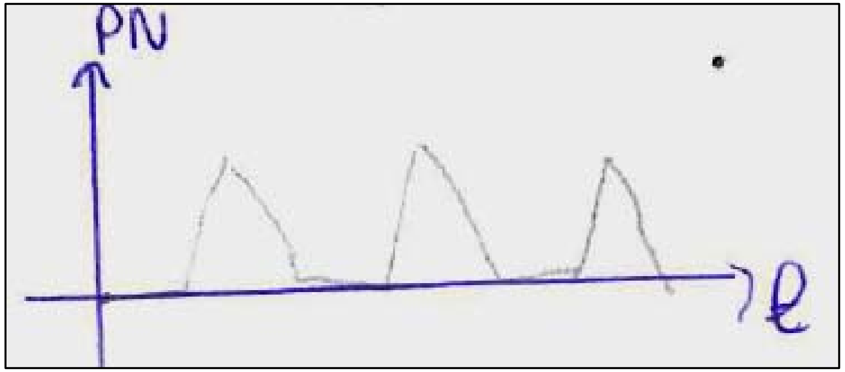


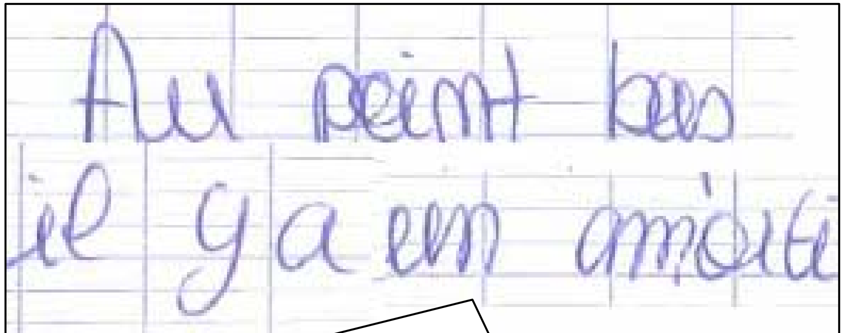


Mathematical solution

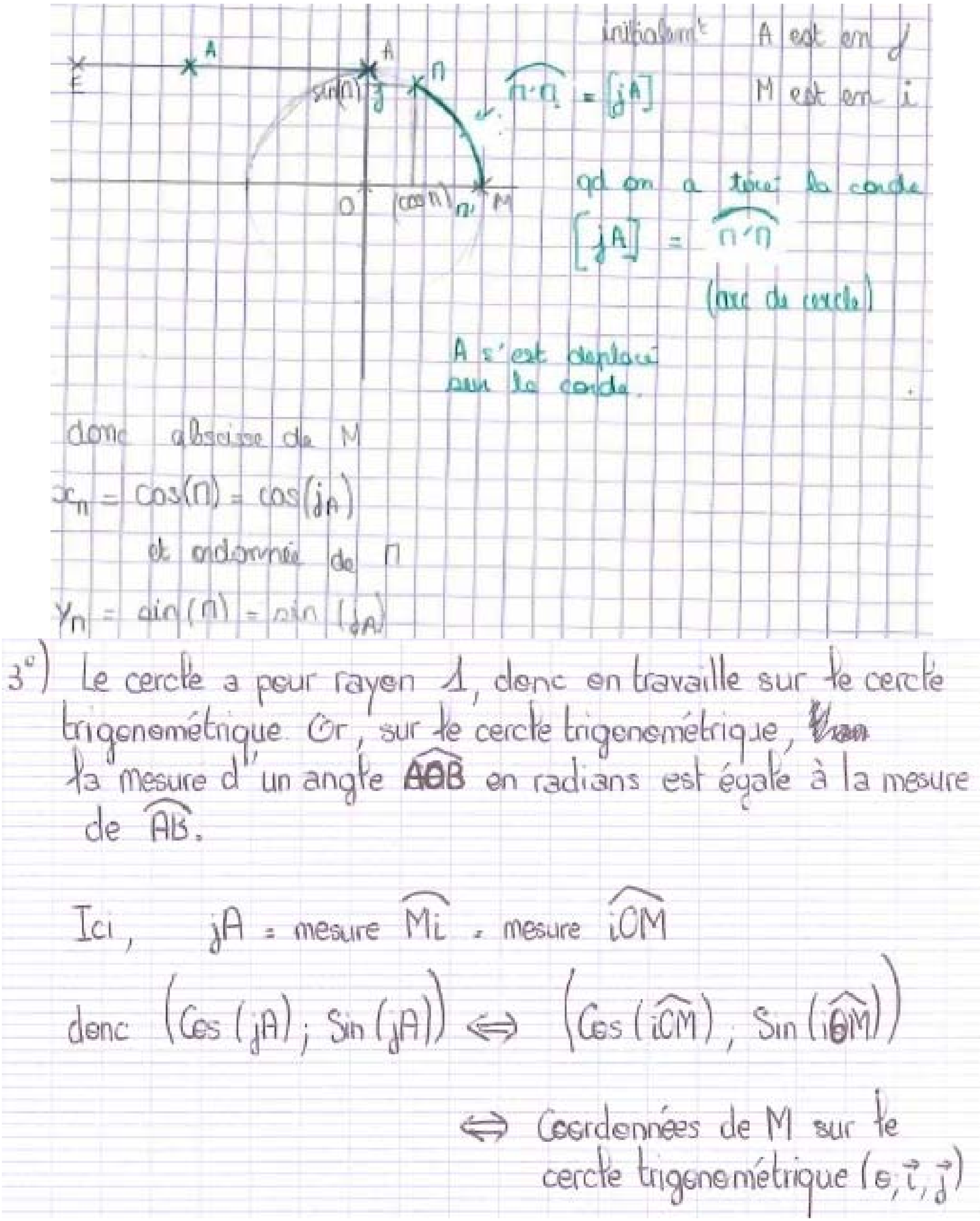
Classroom Observation

- Physical situation and spontaneous model
  - Students stick to piecewise uniform movements
  - Students are more or less aware of differences between high and low points





At the lower point, there is a drop shot



Building a geometric model

- Students need
  - Knowing about the artefact (dynamic geometry)
  - Associated Mathematical knowledge

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Students more or less aware of the choice of dependant and independent variables

On choisit la distance AJ en variable,

AJ en fonction des coordonnées de N

We choose distance AJ as the (independent) variable

AJ is a function of the coordinates of N

y = PN → position de la [illegible]
x = AJ → longueur de la corde tirée

y=PN → the car's position

x=AJ → the length of the rope drawn

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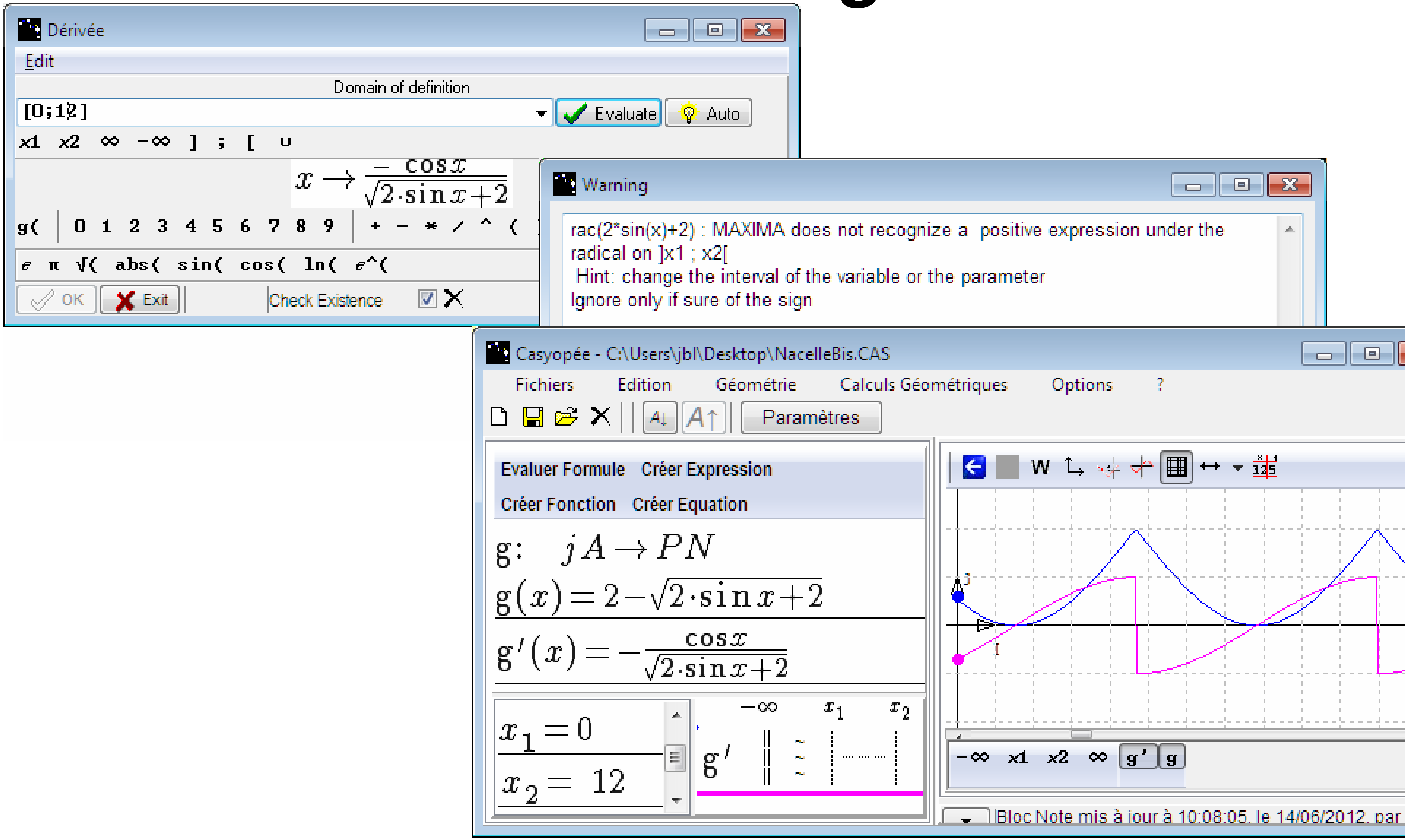
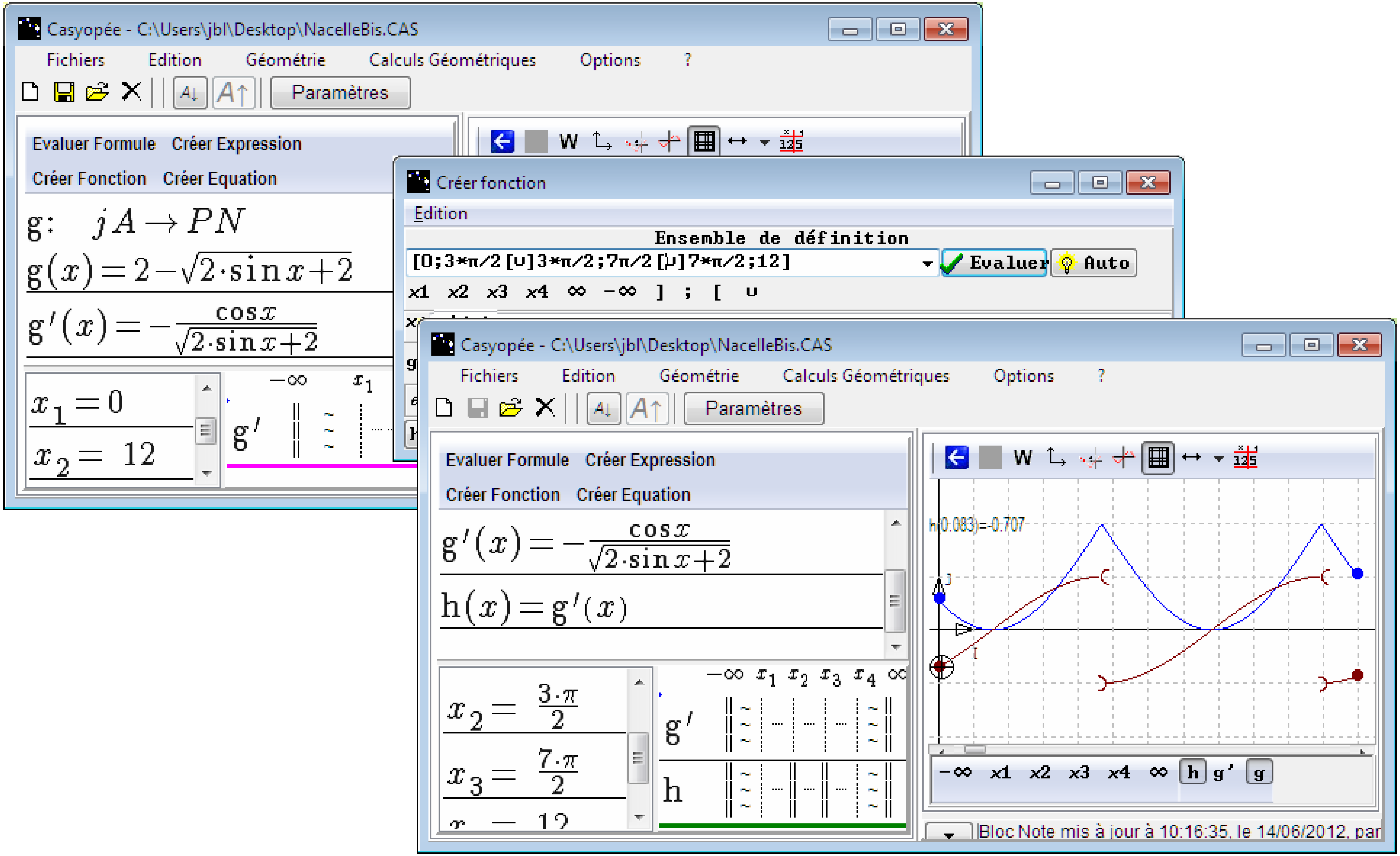
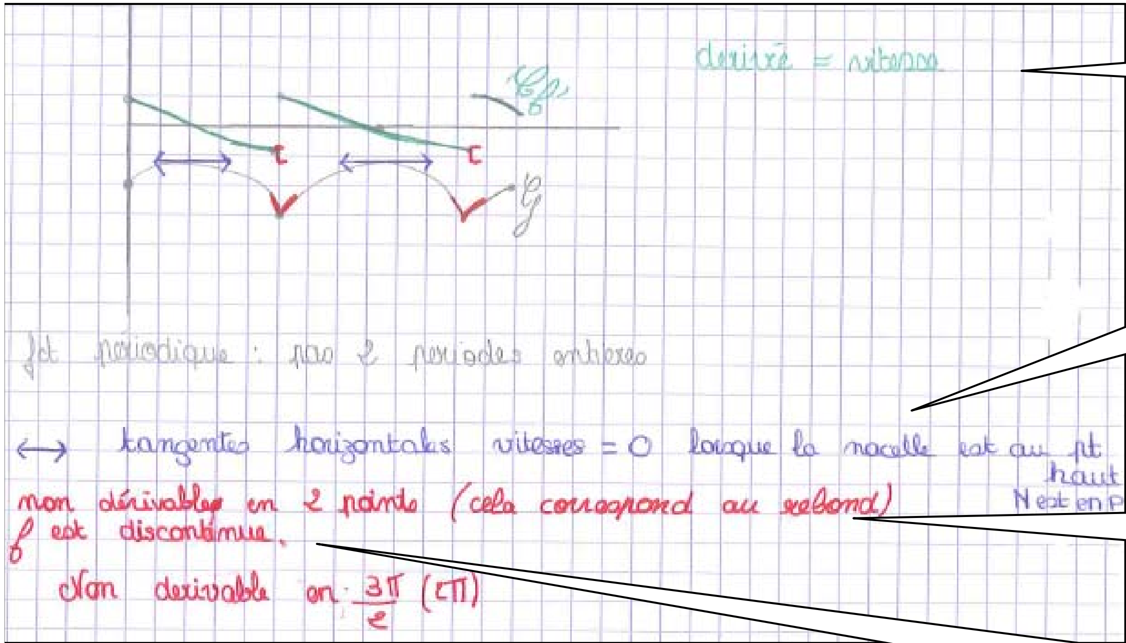
Students ignore Casyopée's warning



A mediation by the teacher



Math Solution ➔ Problem

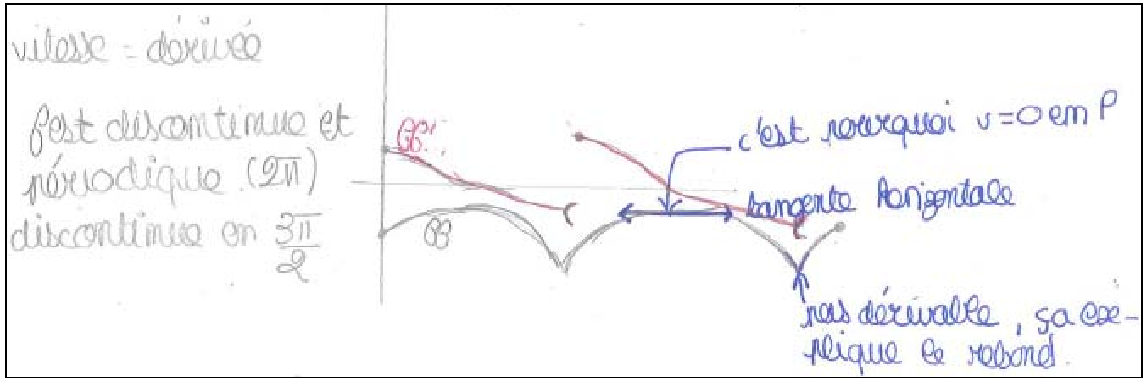


Derivative = speed

Horizontal tangent
Speed=0 High point

Not differentiable at
low point: *rebound*

The function is not
continuous



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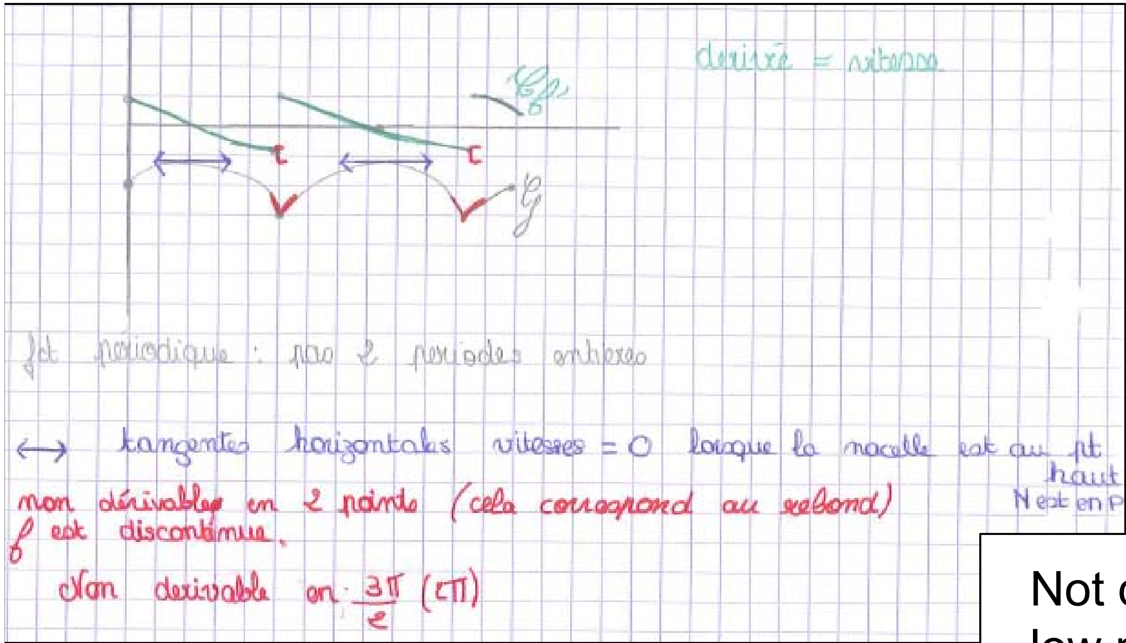
Math Solution ➔ Problem



Not differentiable at low point: *rebound*

Derivative = speed

Horizontal tangent Speed=0 High point

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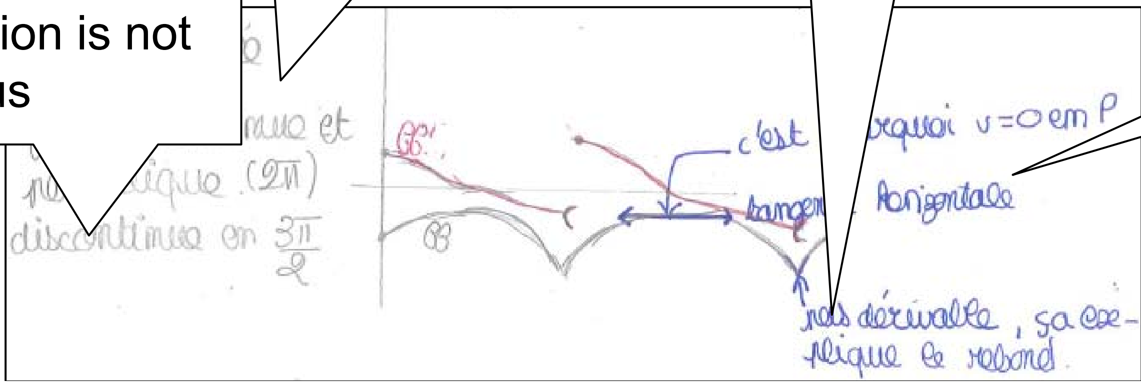
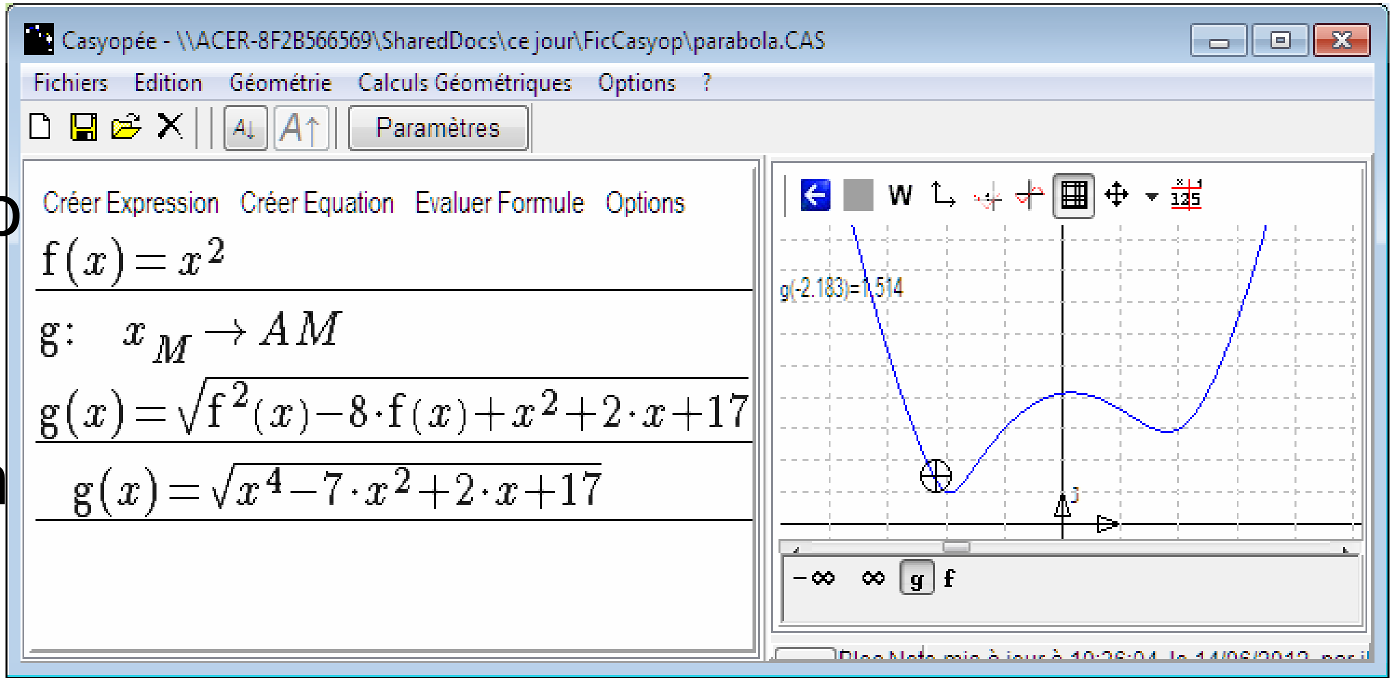


Kieran's objections

Strong presumption that symbolic forms are to be interpreted graphically, rather than dealt with directly.

Technology being used to insist on graphical interpretation



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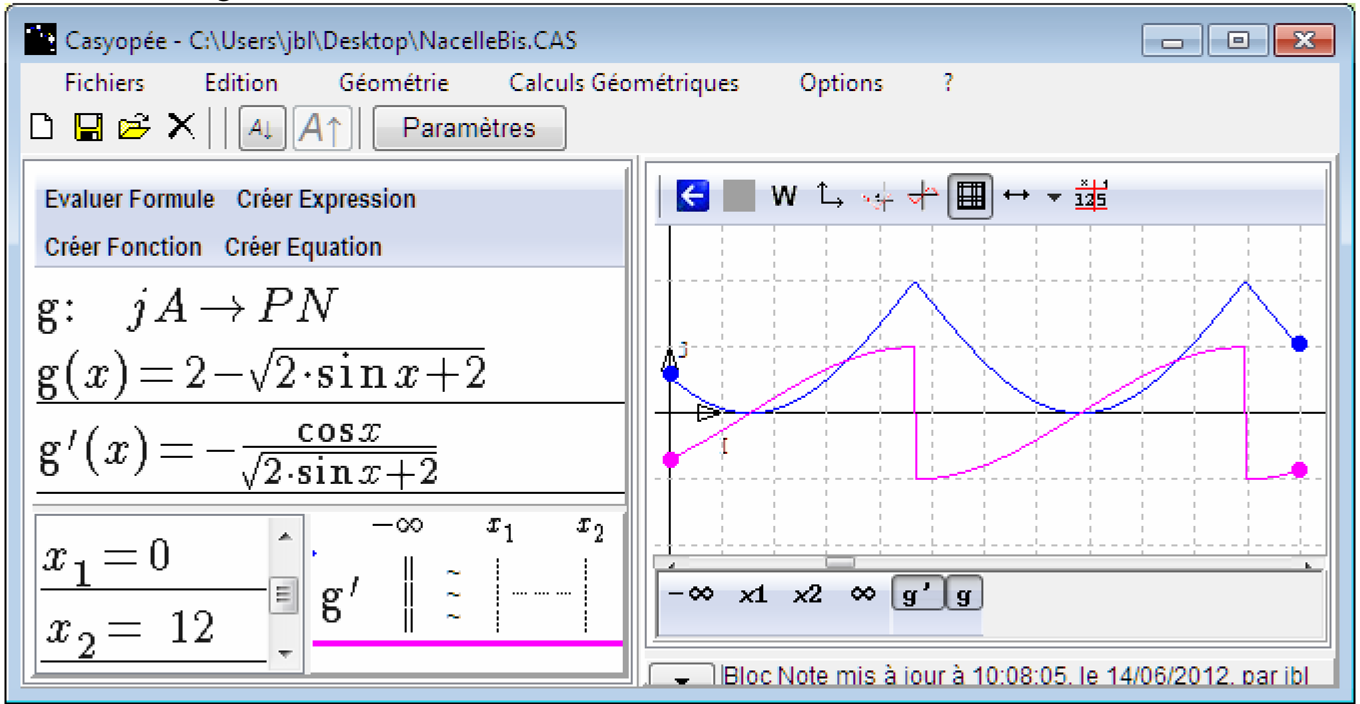

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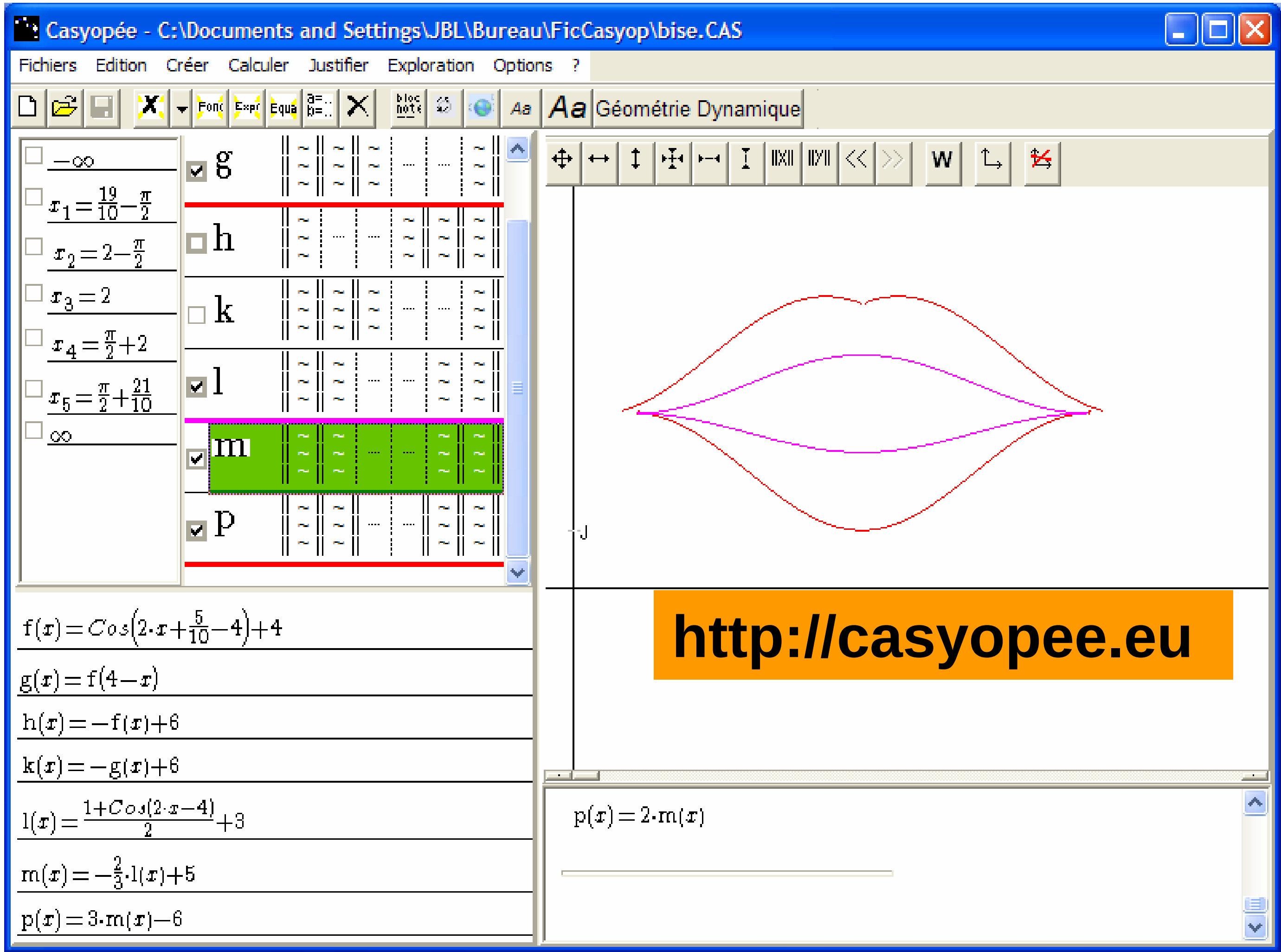
Thank you !



Casyopée - C:\Documents and Settings\JBL\Bureau\FicCasyop\bise.CAS
Fichiers Edition Créer Calculer Justifier Exploration Options ?
Géométrie Dynamique
$-\infty$
$x_1=\frac{19}{10}-\frac{\pi}{2}$
$x_2=2-\frac{\pi}{2}$
$x_3=2$
$x_4=\frac{\pi}{2}+2$
$x_5=\frac{\pi}{2}+\frac{21}{10}$
∞
g
h
k
l
m
p
$f(x)=Cos\left(2\cdot x+\frac{5}{10}-4\right)+4$
$g(x)=f(4-x)$
$h(x)=-f(x)+6$
$k(x)=-g(x)+6$
$l(x)=\frac{1+Cos(2\cdot x-4)}{2}+3$
$m(x)=-\frac{2}{3}\cdot l(x)+5$
$p(x)=3\cdot m(x)-6$
$p(x)=2\cdot m(x)$
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