The Necessary Components of Materials and Textbooks Used in Math Classes

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And

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Kaneko

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KETpic project

Our Introduction

(1) We are mathematics teachers in Japan.

(2) We give lessons in collegiate math classes.

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(1) We are mathematics teachers in Japan.

(2) We give lessons in collegiate math classes.

(3) We use printed materials together with textbooks.

(4) We have been editing and publishing a series of textbooks.

Our Introduction

(5) We use LaTeX and KETpic on a daily basis to make materials and textbooks.

(6) We developed KETpic to make it easy into insert figures to LaTeX documents.

(7) We have brought up KETpic to be more useful and comprehensive LaTeX helper.

What is KETpic

(1) It is a CAS macro package for inserting graphs easily into LaTeX documents

Maple, Mathematica, Maxima,

Scilab, Matlab (in part), R

(2) It is free downloadable from

`www.ketpic.com`

(3) It has some TeX macros generated by KETpic to help us make materials.

Components of Materials

Using TEX

(1) We can make documents with mathematical expressions easily and finely.

(2) Materials and textbooks require various components from an educational perspective.

(3) TEX itself cannot handle these components easily.

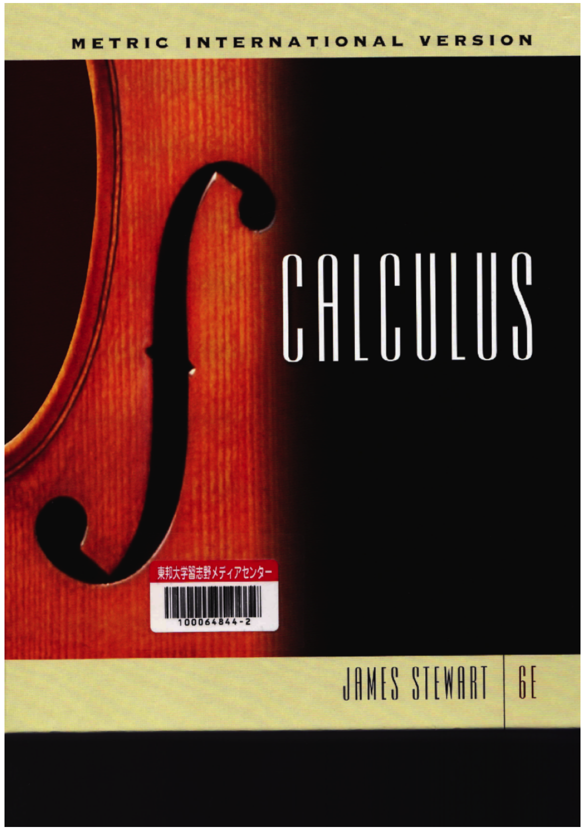
(4) KETpic shores up this weekness of TEX.

Necessary Components

- Sentences
- Mathematical Formulae
- Figures
- Symbols
- Tables
- Pointers, Highlighting
- Pagination (Layout)

Necessary Components

Stewart's *Culculus*, Brook/Cole



Very heavy

Weight : 2500 g

Thickness : 4.5cm

Number of pages : 1300

With deep cosideration from an educational point of view

Plenty of Figures

It can be shown that the lower approximating sums also approach $\frac{1}{3}$, that is,

$$\lim_{n\to\infty} L_n = \tfrac{1}{3}$$

From Figures 8 and 9 it appears that, as n increases, both L_n and R_n become better and be ter approximations to the area of S. Therefore, we *define* the area A to be the limit of t sums of the areas of the approximating rectangles, that is,

TEC In Visual 5.1 you can create pictures like those in Figures 8 and 9 for other values of n.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n = \tfrac{1}{3}$$

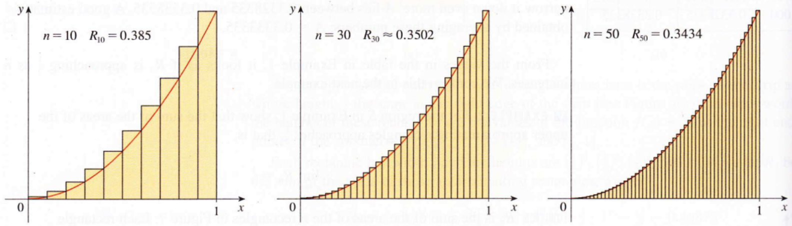


FIGURE 8

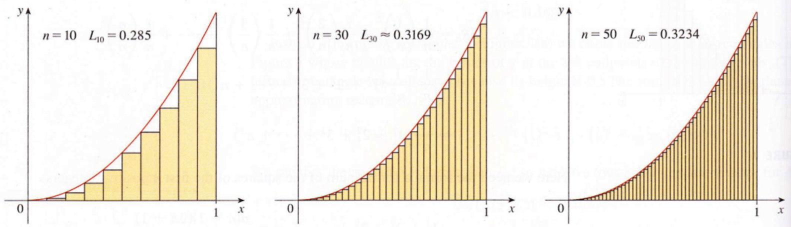


FIGURE 9

The area is the number that is smaller than all upper sums and larger than all lower sums

Let's apply the idea of Examples 1 and 2 to the more general region S of Figure 1. W start by subdividing S into n strips $S_1, S_2, \ldots, S_n$ of equal width as in Figure 10.

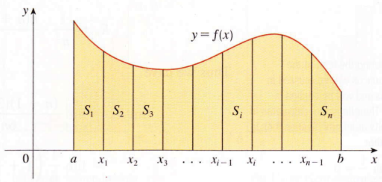
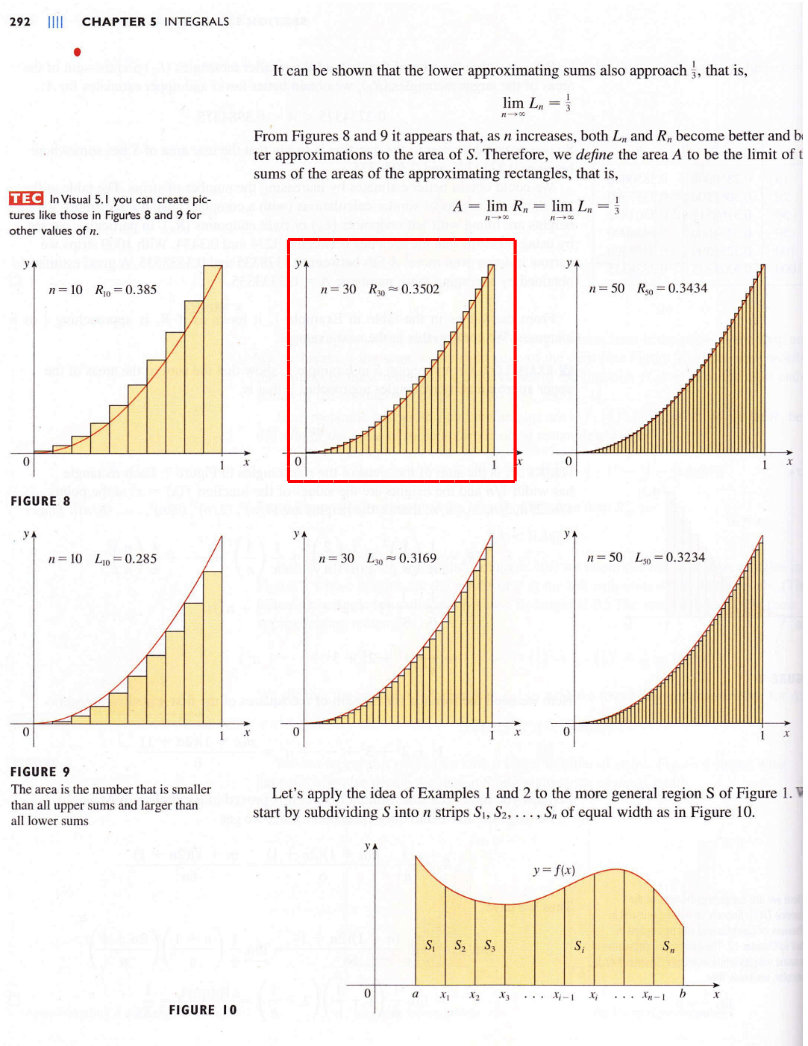


FIGURE 10

Plenty of Figures



292 |||| CHAPTER 5 INTEGRALS

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$n = 10$ $R_{10} = 0.385$ | $n = 30$ $R_{30} \approx 0.3502$ | $n = 50$ $R_{50} = 0.3434$

FIGURE 8

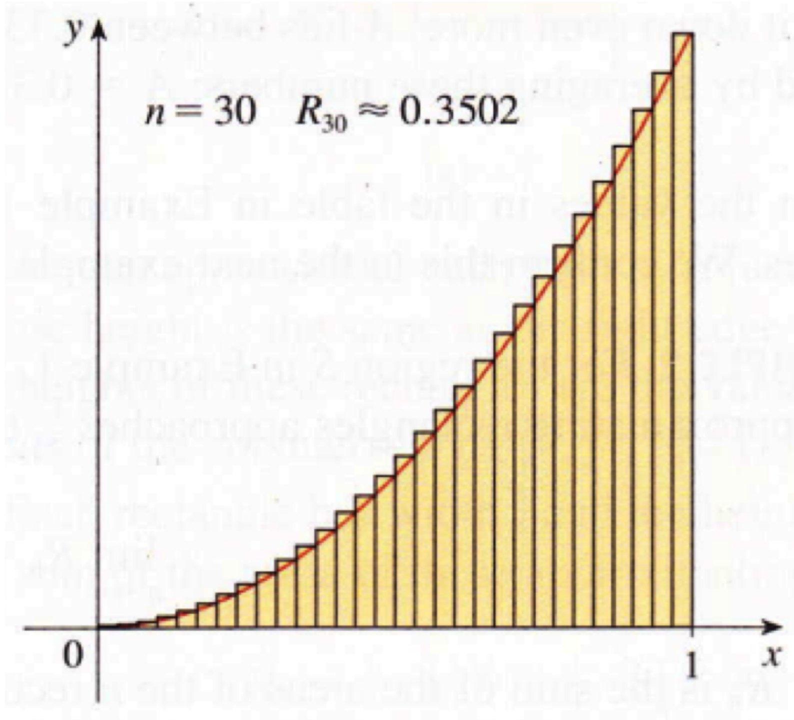
$n = 10$ $L_{10} = 0.285$ | $n = 30$ $L_{30} \approx 0.3169$ | $n = 50$ $L_{50} = 0.3234$

FIGURE 9
The area is the number that is smaller than all upper sums and larger than all lower sums

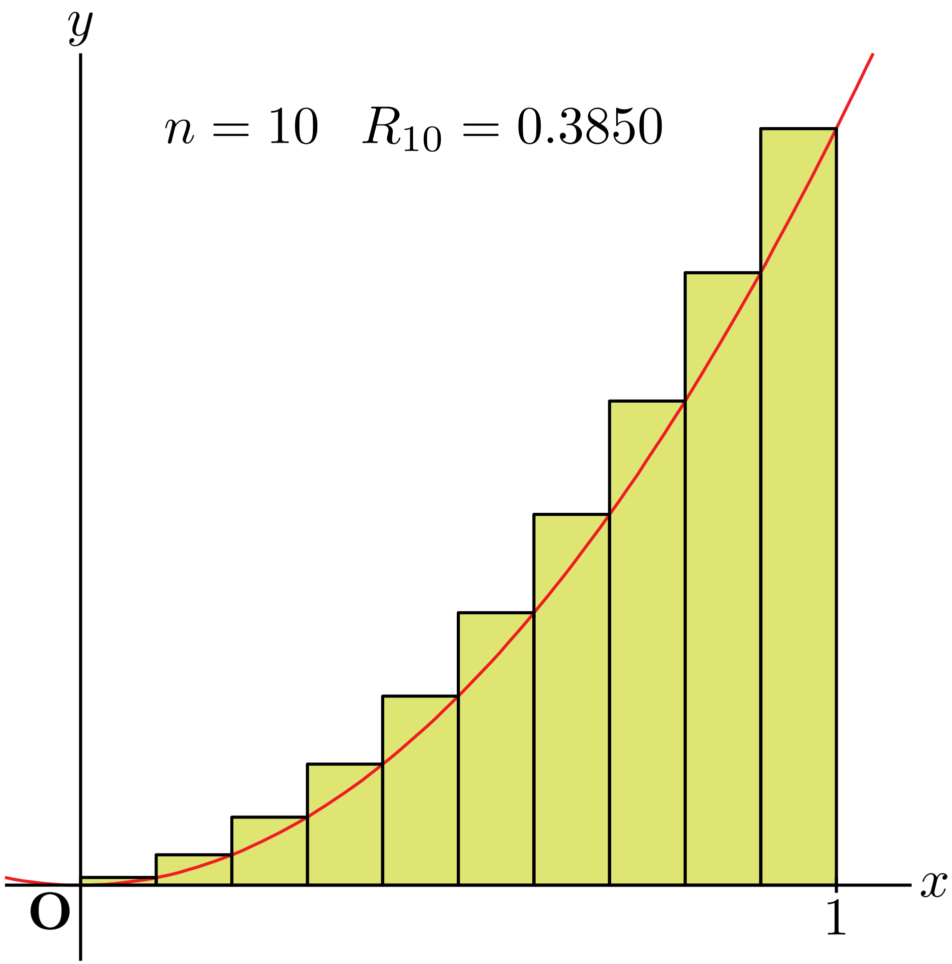
Let's apply the idea of Examples 1 and 2 to the more general region S of Figure 1. W start by subdividing S into n strips $S_1, S_2, \ldots, S_n$ of equal width as in Figure 10.

$y = f(x)$

FIGURE 10

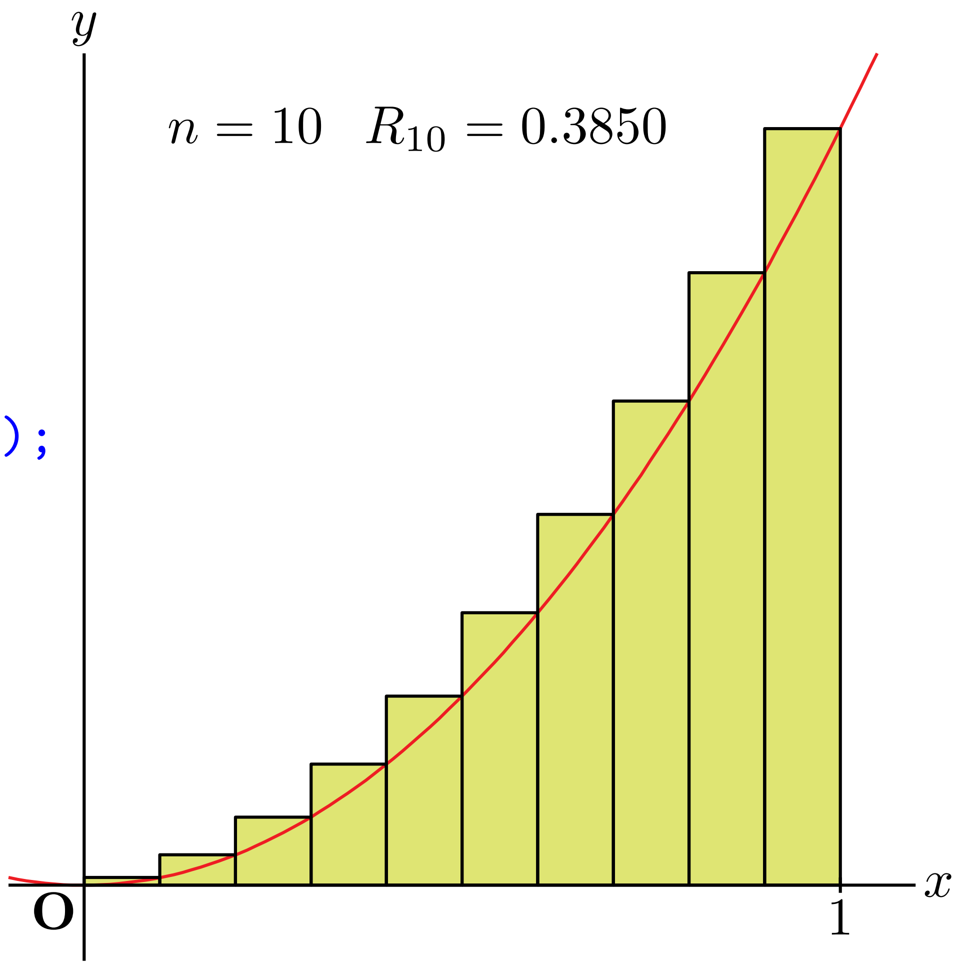


KETpic's Way



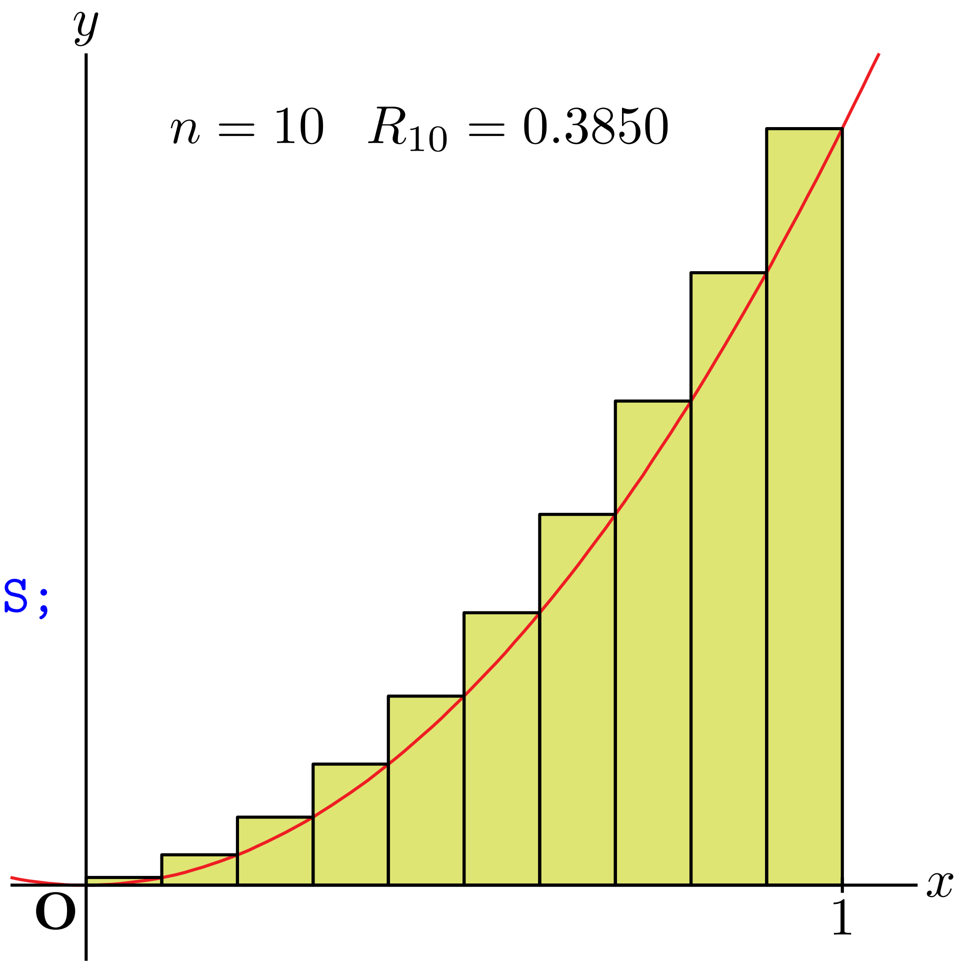
KETpic's Way

```
G1=Plotdata("x^2","x");
Dx=1/N;
G2=list(); Sum=0;
for I=1:N
  X1=Dx*(I-1);X2=Dx*I;Y2=X2^2;
  G2($+1)=Framedata([X1,X2],[0,Y2]);
  Sum=Sum+Dx*Y2;
end;
Openfile(Fname);
  Beginpicture("5cm");
    Texcom("\color{GreenYellow}");
    Shade(G2);
```



KETpic's Way

```
    Texcom("\color{Red}");
    Drwline(G1);
    Texcom("\color{black}");
    Drwline(G2);
    Fontsize("n");
    StN=string(N);
    StS=msprintf("%5.4f",Sum);
    Str="n="+StN+"R_{"+StN+"}="+StS;
    Expr([0,1],"e4",Str);
    Htickmark(1,"1");
  Endpicture(1);
Closefile();
```



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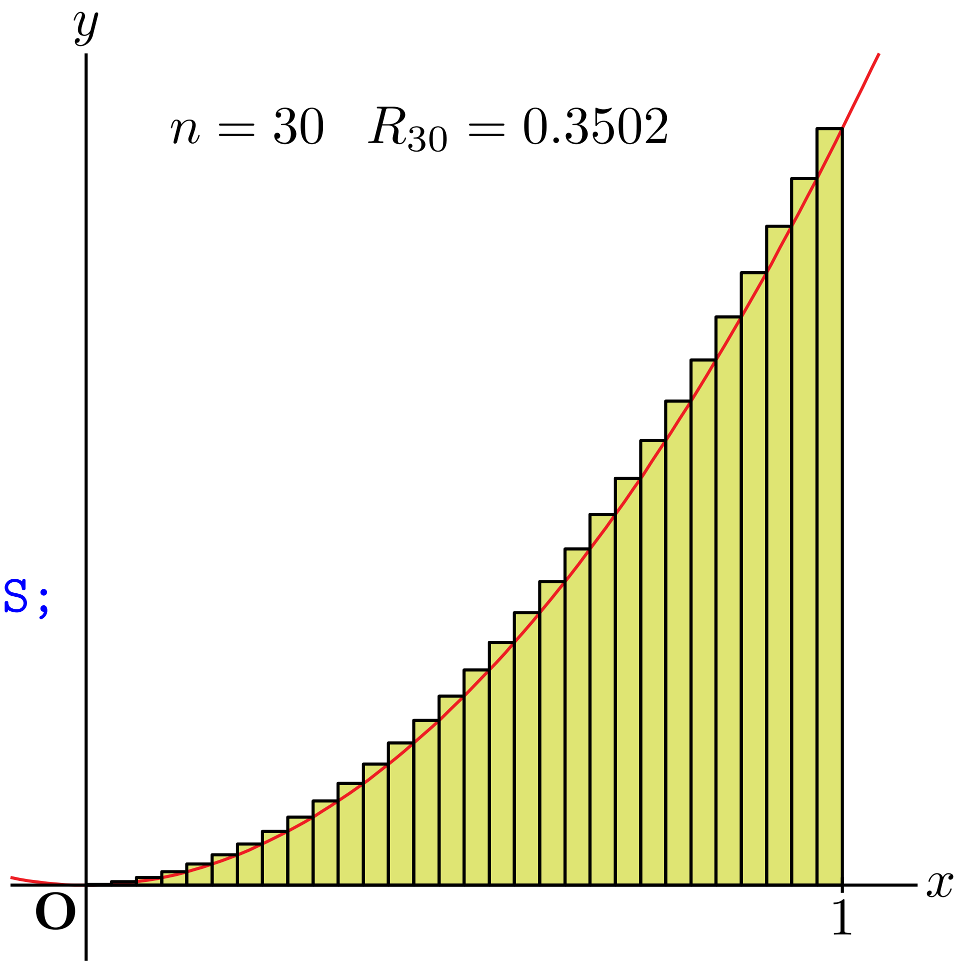

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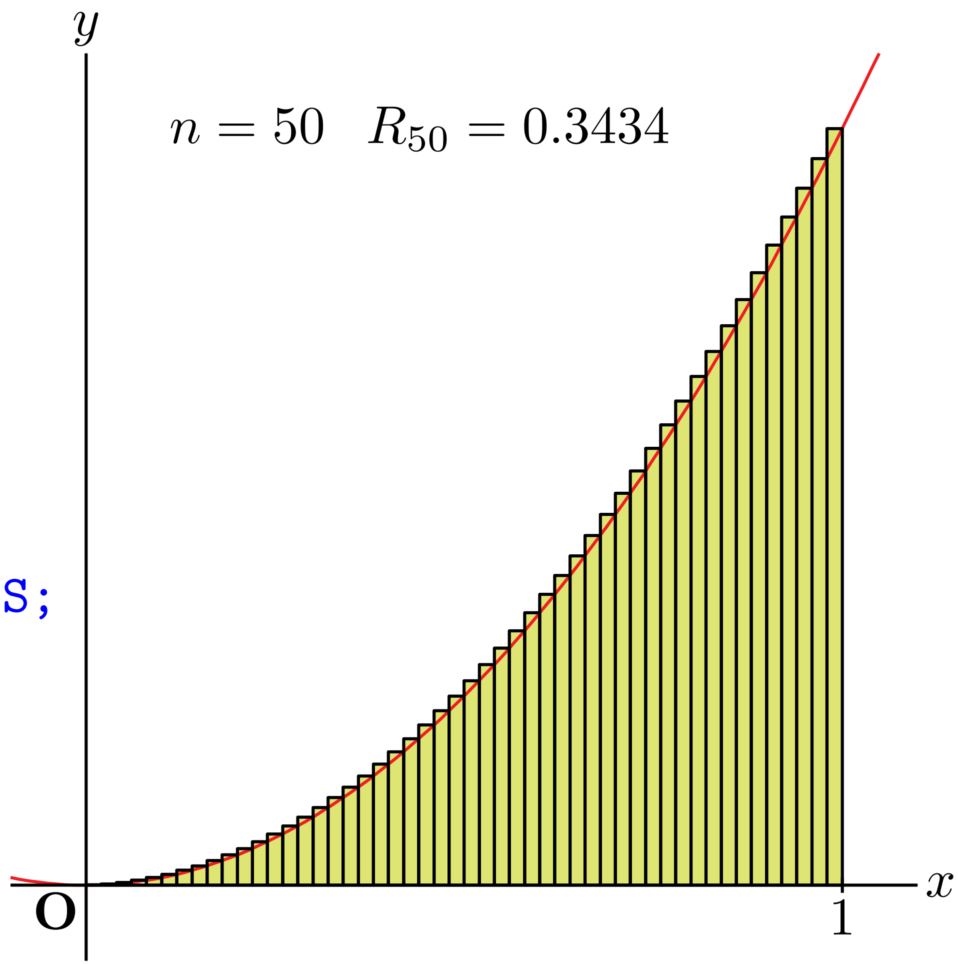
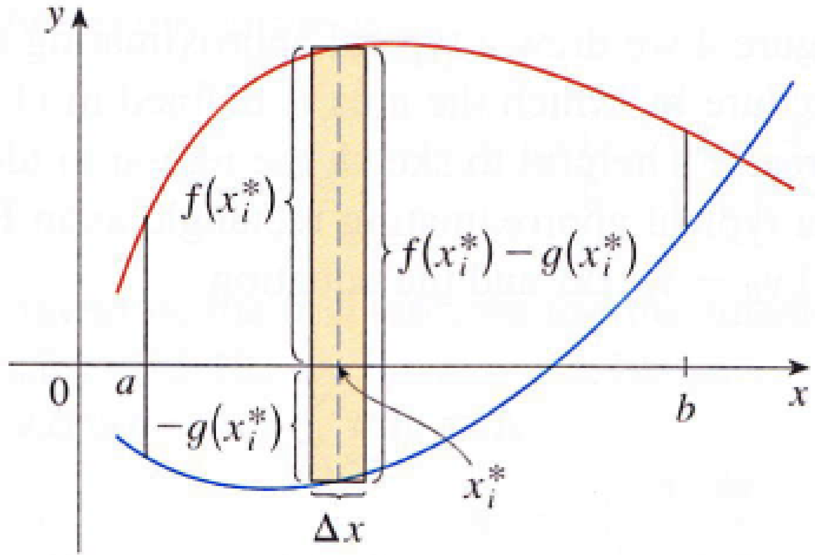


New Symbols and Pointers

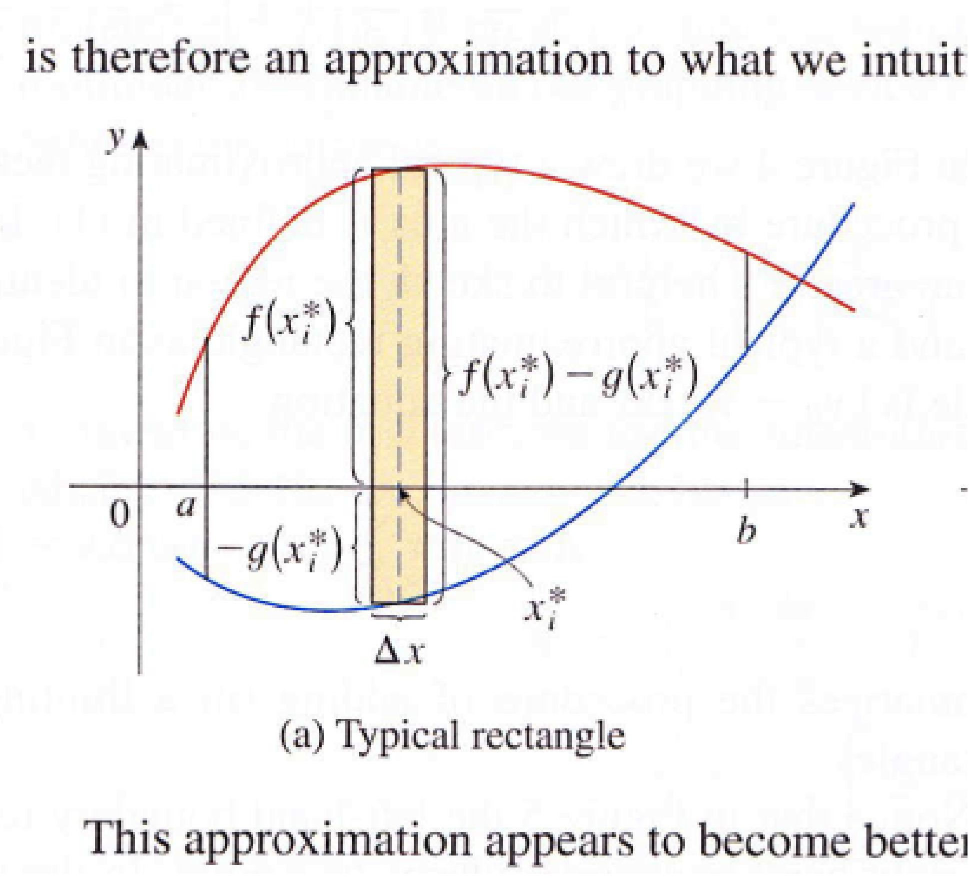
is therefore an approximation to what we intuit



(a) Typical rectangle

This approximation appears to become bettei

New Symbols and Pointers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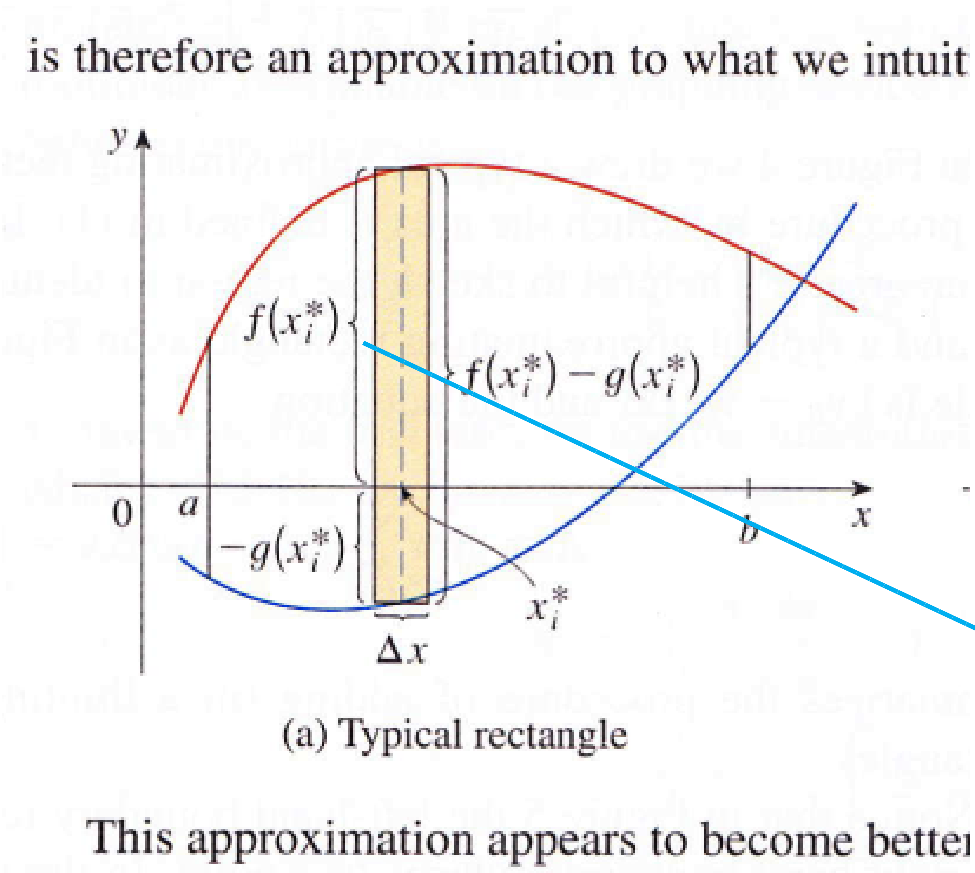
(a) Typical rectangle

This approximation appears to become bette

$$\begin{cases} x + y + z = 2 \\ 2x - 3y - z = 5 \\ x + 3y = 2 \end{cases}$$

New Symbols and Pointers



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Tables

SOLUTION $$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x-2)(x+1)$$

To use the I/D Test we have to know where $f'(x) > 0$ and where $f'(x) < 0$. This depends on the signs of the three factors of $f'(x)$, namely, $12x$, $x - 2$, and $x + 1$. We divide the real line into intervals whose endpoints are the critical numbers -1, 0, and 2 and arrange our work in a chart. A plus sign indicates that the given expression is positive, and a minus sign indicates that it is negative. The last column of the chart gives the conclusion based on the I/D Test. For instance, $f'(x) < 0$ for $0 < x < 2$, so f is decreasing on $(0, 2)$. (It would also be true to say that f is decreasing on the closed interval $[0, 2]$.)

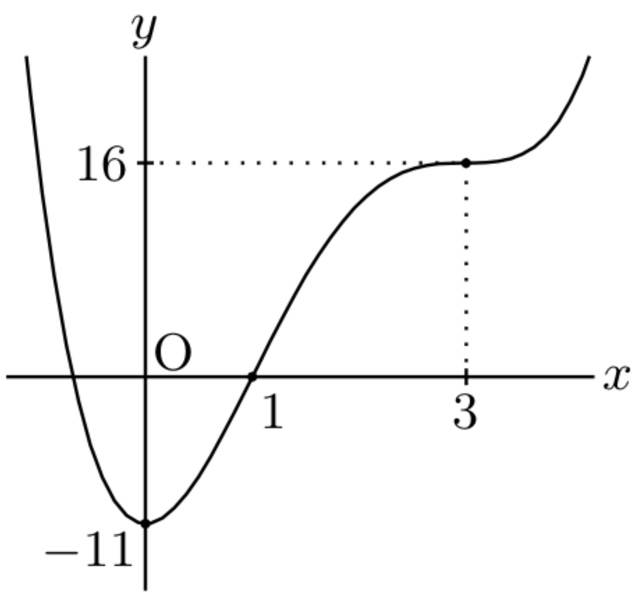
Interval	$12x$	$x-2$	$x+1$	$f'(x)$	f
$x < -1$	$-$	$-$	$-$	$-$	decreasing on $(-\infty, -1)$
$-1 < x < 0$	$-$	$-$	$+$	$+$	increasing on $(-1, 0)$
$0 < x < 2$	$+$	$-$	$+$	$-$	decreasing on $(0, 2)$
$x > 2$	$+$	$+$	$+$	$+$	increasing on $(2, \infty)$

The graph of f shown in Figure 2 confirms the information in the chart. □

Tables

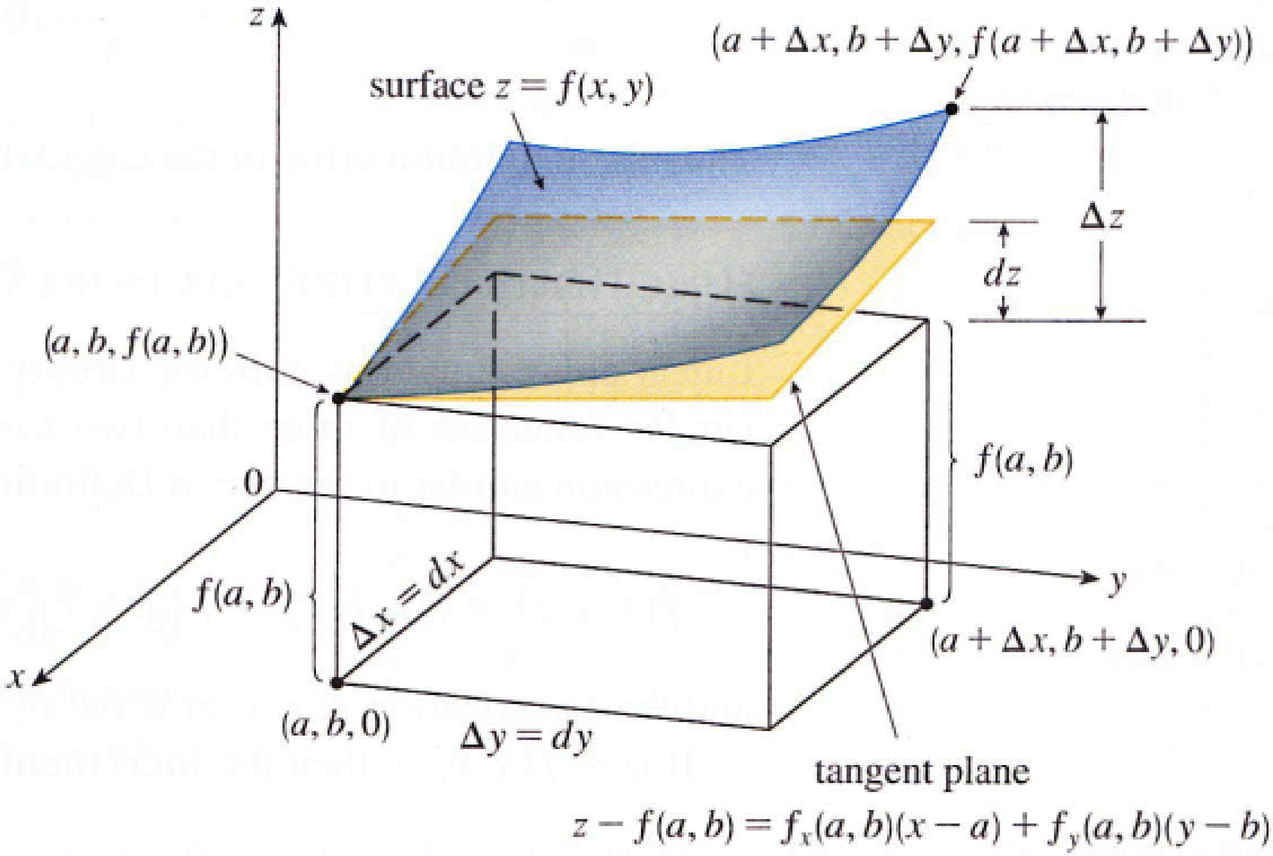
$$y = x^4 - 2x^3$$
$$y' = 4x^3 - 6x^2$$
$$y'' = 12x^2 - 12x = 12x(x-1)$$

x	$\cdots$	0	$\cdots$	1	$\cdots$	3	$\cdots$
y'	$-$	0	$+$	$+$	$+$	0	$+$
y''	$+$	$+$	$+$	0	$-$	0	$+$
y	↘	-11	↗	0	↗	16	↗

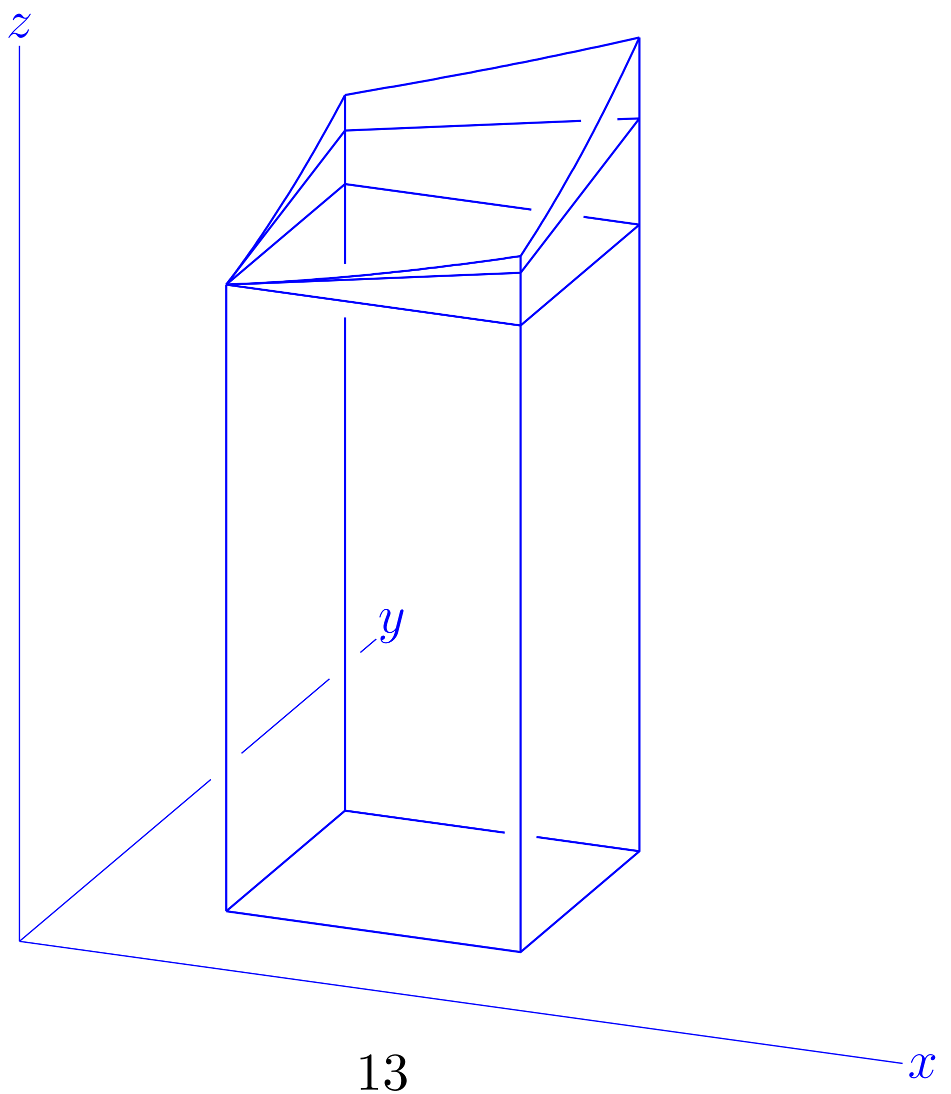


3D Figures(1)

the tangent plane, whereas Δz represents the change in height of the surface $z = f(x, y)$ when (x, y) changes from (a, b) to $(a + \Delta x, b + \Delta y)$.



3D Figures(1)



3D Figures(2)

EXAMPLE 7 Figure 12 shows a solid with a circular base of radiu
sections perpendicular to the base are equilateral triangles. Find

SOLUTION Let's take the circle to be $x^2 + y^2 = 1$. The solid, its ba
section at a distance x from the origin are shown in Figure 13.

TEC Visual 6.2C shows how the solid in Figure 12 is generated.

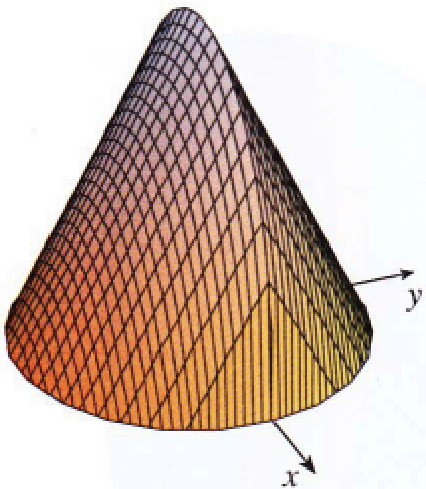
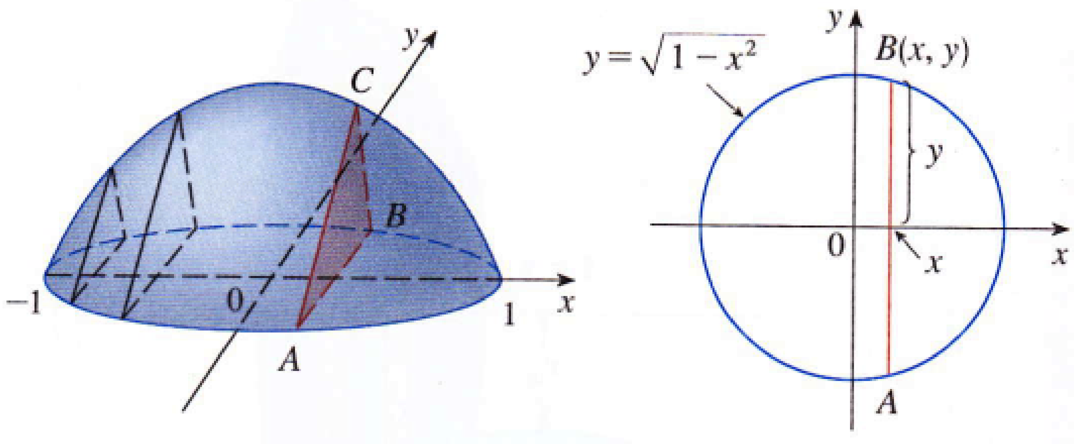


FIGURE 12
Computer-generated picture of the solid in Example 7

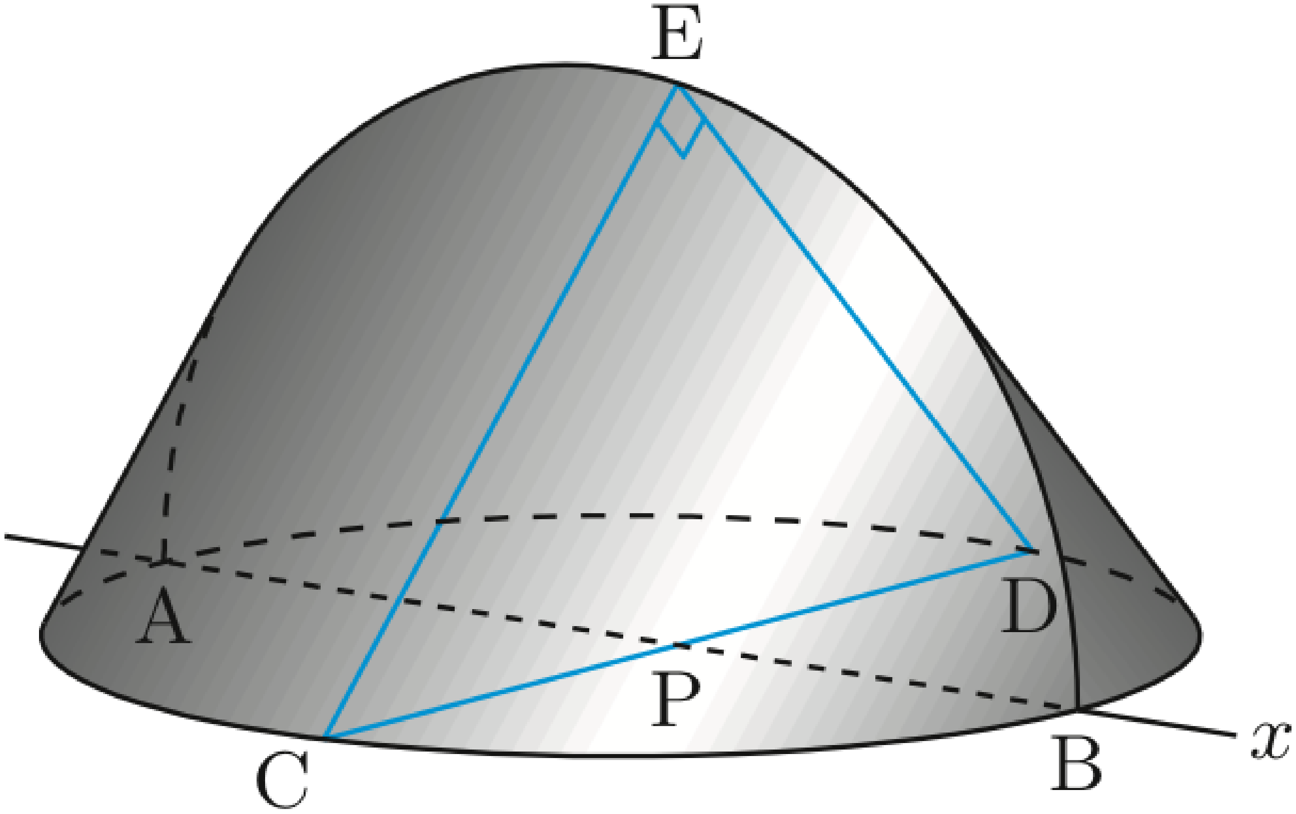


(a) The solid

(b) Its base

FIGURE 13

3D Figures(2)



Future Works

Developments of KETpic

KETpic is now developing
in various directions
with various teachers.

Developments of KETpic

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in various directions
with various teachers.

And

KETpic will certainly become a
comprehensive helper and
more useful helper
of teachers who make materials using TEX.