Announcement for an upcoming Generation of TP-based educational math assistants

Comments on *eduTPS*: “Theorem-Prover based Systems for Education”

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Institute for Softwaretechnology
Graz University of Technology

Working group *eduTPS* at CADGME’12
June 22 - 24, 2012
Novi Sad, Serbia

This is **NOT** the topic . . .

Outline

Survey on Math. Software

In science of ...	SW-tools for ...	... standardized as ..
mathematics	geometry	CAD/CAM DGS
applied sciences	numerical computation	Spreadsheets (SSH)
math. education	symbolic computation	
	Computer Algebra Macsyma 1968	CAS
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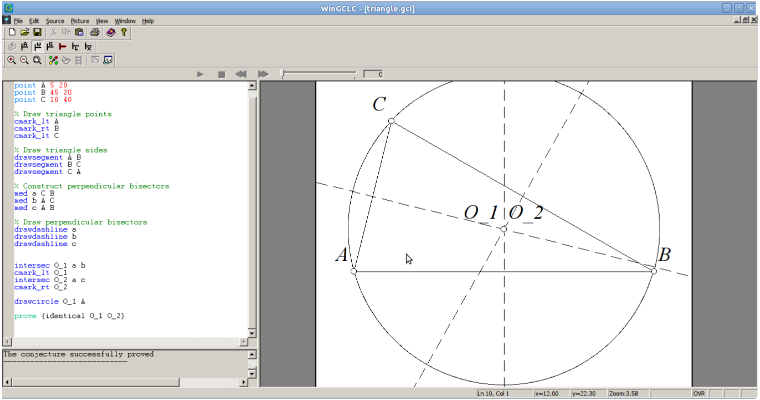
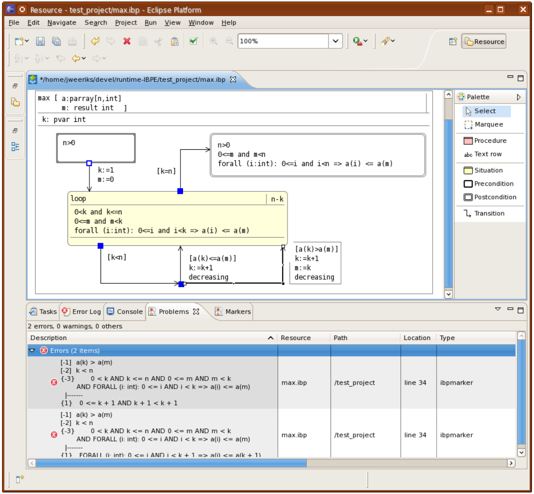
1. Survey on Mathematical Software
   - Three Examples for TP-based Math Assistants
2. Characteristics for a “New Generation”
   - Conceptual Foundations: Integration of Logics
   - Technological Features: Transparency, Flexibility
   - Expected Impact: Education, Research, Development
3. Conclusion — Invitation

GCLC, Univ. of Belgrade



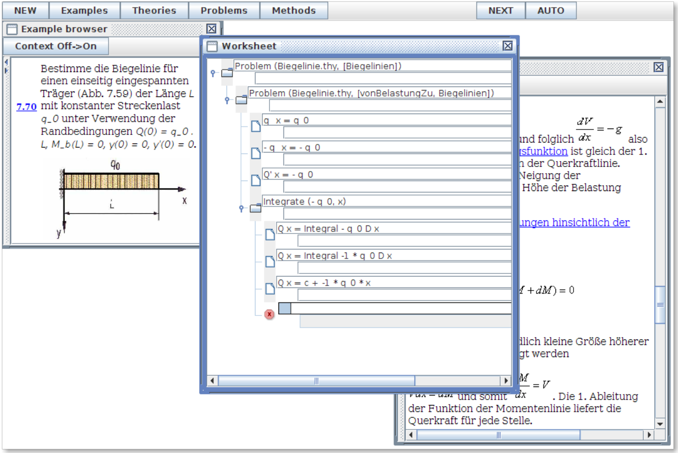
GeoGebra's "academic relative", see *prove {identical O_1 O_2}* at bottom left.

Socos, Abo Akademi Turku



For software “correct by construction” in education.

$\mathcal{ISAC}$, TU Graz & RISC Linz



For step-wise problem solving in applied mathematics.

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Conceptual Foundations

Mathematics is the science of *reasoning* ...

- each operation can be *proved*
- ...to *"prove"* is the essence which distinguishes math.

TP ((Computer) Theorem *Proving*) realises this essence.

Consequences for TP-based software:

1. TP provides a logical framework for CAS, DGS, ...
2. TP is **integrative** (rather than competitive)
3. TP covers an essential range of mathematics including STEM [1].

[1] STEM: "Science, Technology, Engineering and Mathematics"

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in PISA's competence model for mathematics:

1. **Communication**: *"... perceiving the existence of some challenge and recognizing a problem situation ..."*
2. **Mathematising**: *"... transforming a problem defined in the real world to a strictly mathematical form ..."*
3. **Representation**: *"... selecting, interpreting and using a variety of representations to capture a situation ..."*
4. **Reasoning** and argument: *"... logically rooted thought processes that check a justification that is given, ..."*
5. Devising **strategies** for solving problems: *"... critical control processes that solve problems ..."*
6. Using **symbolic**, formal and technical language and **operations**: *"... within a mathematical context ..."*
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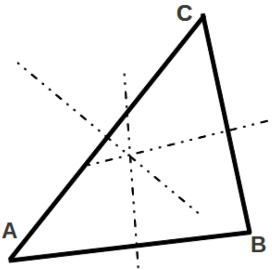
The perpendicular midlines of the sides in a triangle meet in one point.

We know !



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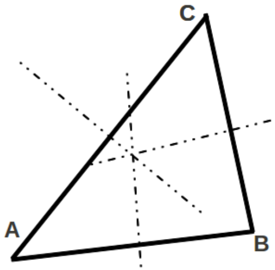

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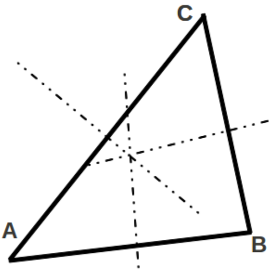

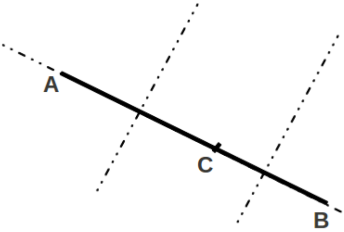

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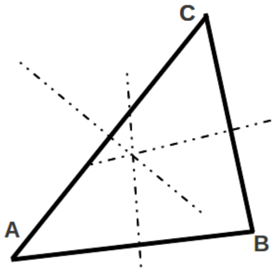

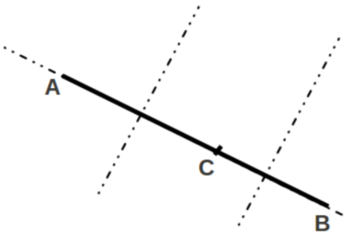

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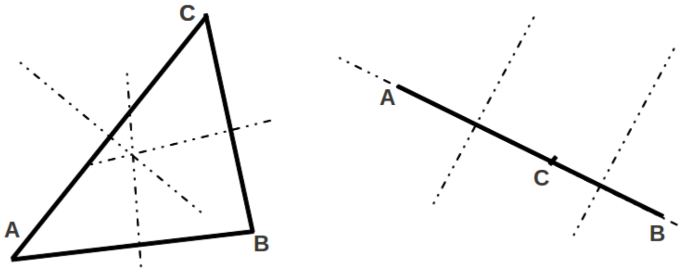

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These features arise from TP ...

1. check user input automatically, **flexibly** and reliably:
   Input establishes a *proof situation* (for *automated* proving) with respect to the logical context
2. give explanations on request by learners:
   All underlying mathematics knowledge is **transparent** due to the “LCF-paradigm” (not a program code!)
3. propose a next step if learners get stuck:
   “next-step-guidance” due to Lucas-Interpretation.

Thus featuring software support for:

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   All underlying mathematics knowledge is **transparent** due to the “LCF-paradigm” (not a program code!)
3. propose a next step if learners get stuck:
   “next-step-guidance” due to Lucas-Interpretation.

Thus featuring software support for:

- **step-wise solving math** problems in STEM
- learning interactively like with a **chess-program**
- ...

Outline

Impact expected . . .

- for **learners** in STEM from high-school to university:
  - *independent learning* in all phases of problem solving
  - construction of solutions with *"next-step-guidance"*
  - *flexible* access to knowledge in context of steps
- for **researchers** in science education,cognitive science
  - *logging of steps* for analysis of problem solving behav.
  - summative *assessment* of step-wise problem solving
  - mechanised *analysis of prerequisites* in curricula
- for **educational planners** and administrators
  - curriculum development on *mechanised knowledge*
  - cross-*institutional interfaces* are explicit
  - *summative assessment* of institutions
- for **developers** of systems, knowledge and dialogs
  - TP components contribute *support by automation* in checking user input, access to knowledge, . . .
  - *but* require adherence to TP standards and logics

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abstracted to logics
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Mechanised Foundation of Math

Renewed Pedagogy for the Future
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△
STEM
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inquiry-based learning
embodyment, social experience
Effective Practice in Math Education

Let's hope for joint inspiration how to relate the sides !

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TP Technology I

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