



**Computer Algebra and
Dynamic Geometry
Systems
in Mathematics Education**
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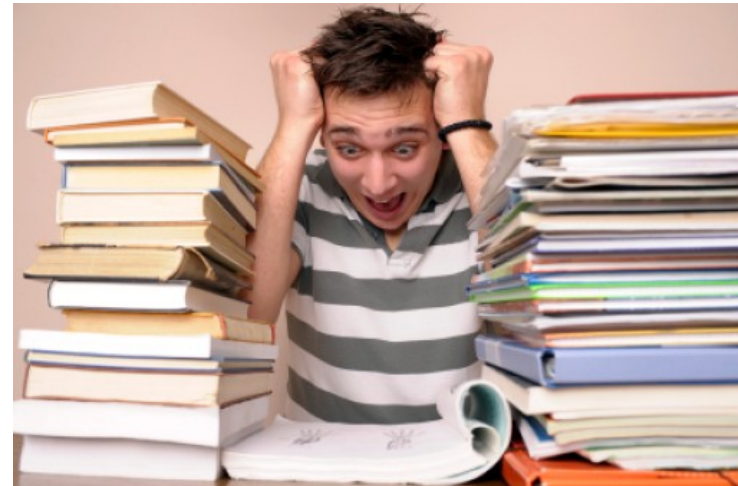
Teaching/learning geometric transformations in high school with DGS

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THE STARTING POINT

The **traditional treatment of geometric transformations** in high school teaching is generally by means of **formal definitions, examples, basic properties.**

The set of notions that is then presented to the students results to be **difficult to understand.**



Moreover, generally, the topic is treated as a **separate chapter, without applications**, and students can not see its **utility.**

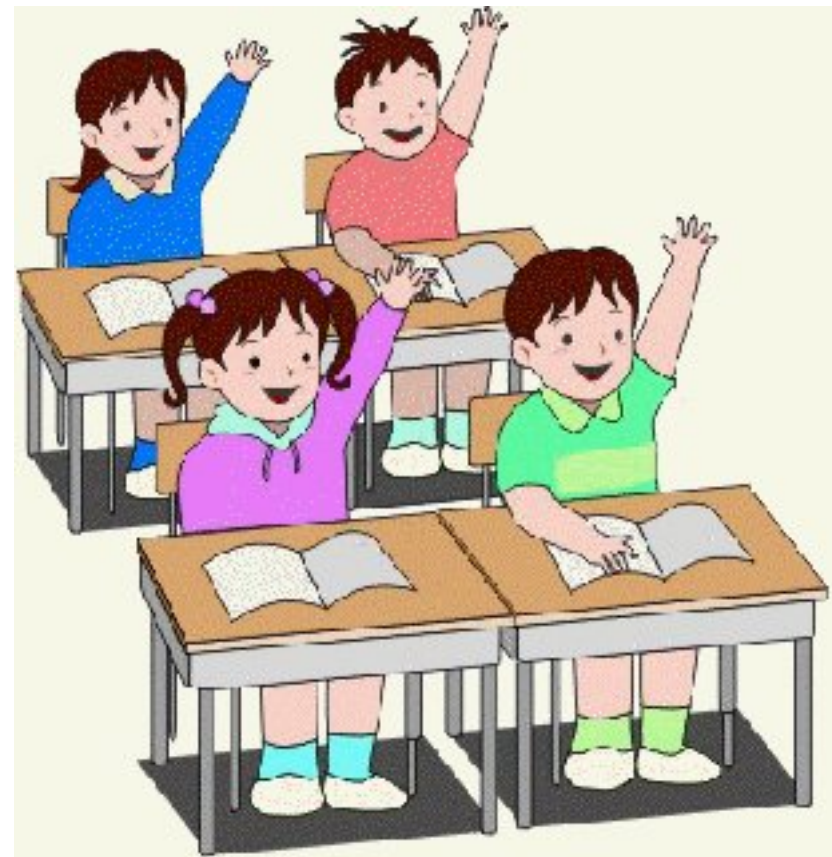


Firmly convinced that geometric transformations have a very important role in students education, we thought to design and elaborate an experimental activity on this topic, addressed to high school students in the first two years.

We decided to lead students to **discover definitions**, **properties** and **applications** of some fundamental geometric transformation.

To this end, on the strength of other previous experiences, we have built up some **mathematical laboratory activity** based on the use of a **Dynamic Geometry System (DGS)**.

It is necessary to find appropriate didactic strategies finalized to encourage **teachers** and **students** to not give up at first difficulties and to help students to overcome obstacles with proper activities that require an active involvement.




Tutors (4): two University professors and two high school teachers planned the activity

High school teachers (16):

prepared the teaching material together with the tutors and experimented it in class

Students (450):

who experienced the activity and provided feedback for its evaluation

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- **Step 1:** design of laboratories and preparation of materials necessary for the construction of laboratories.
 - **Step 2:** implementation of laboratories in class.
 - **Step 3:** comparative analysis of the obtained results.

We carried out **two mathematical laboratory activities** [curricula UMI]: the first addressed to **teachers** and the second one to **students**.

A mathematical laboratory activity is a "phenomenological space of teaching / learning of mathematics that is structured through the use of specific technological tools and articulated negotiation processes" (Garuti in Chiappini et al. 2007)

"learn by doing and seeing"

For such a purpose we arranged both phases so that first the teachers and then the students were just routed to become independent developing the activity.

Students activity was based on the use of a **DGS** - *Cabri Géomètre*.

More than ever, technology makes it possible to implement the use of the

"explore-discover-test-conjecture-proof " model

at every level, providing the possibility of increasing the inquiry-based approach in the way we teach and learn mathematics ...

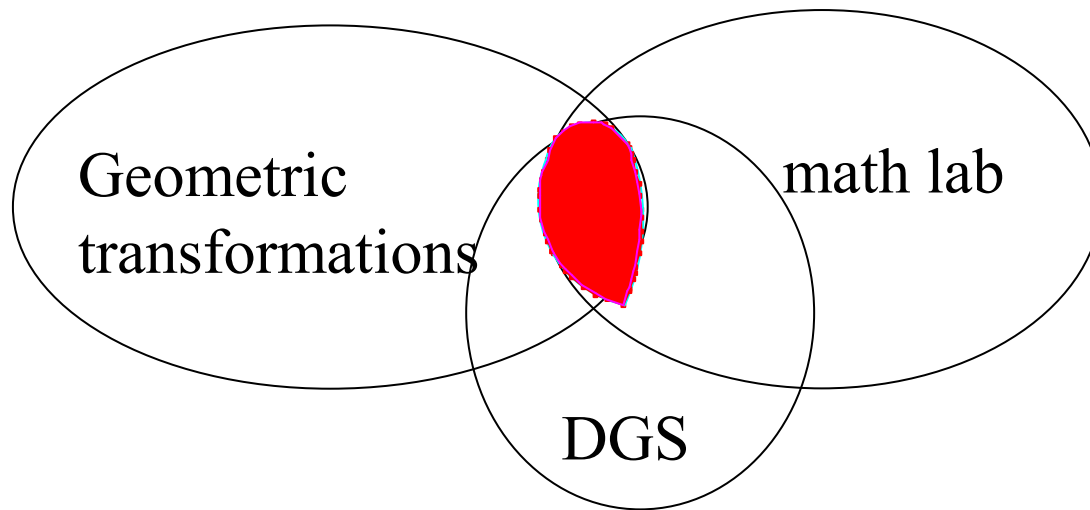
Moreover, DGS "has totally changed the way euclidean geometry can be studied. With a minimal introduction, students may explore and discover dynamically relevant properties rather than being told about them " (Quesada 2011).

DGS are nowadays very popular in schools, but unfortunately often underutilized.

- low availability of computer labs
- the unfamiliarity of the teachers to the teaching with technology.

So we choose to **involve teachers** in the construction of the activity to suggest them the lab methodology for class, but also to bring them to new technologies and teaching methods with them.

"Training of teachers with appropriate and sufficient proficiency is very important: teachers do not only need to learn how to use computers (or technology), but also how to incorporate computer when teaching" (M. Dogan 2011).



Content of the Activity: geometric transformations

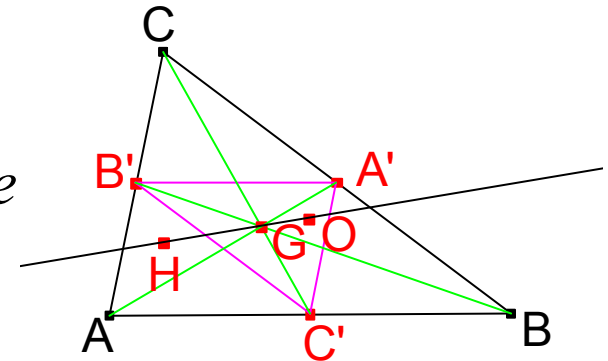
- some kind of isometries: *translations, central symmetries, axial symmetries*
- *homothetic transformations*

For each sort of transformation we highlighted some main properties: fixed points, fixed lines, properties of invariance

Significant applications

- the medial triangle of the triangle
- the Euler line
- the orthic triangle
- Feuerbach's circle

A triangle ABC corresponds to its medial triangle in the homothetic transformation with centre the centroid of ABC and ratio -0.5



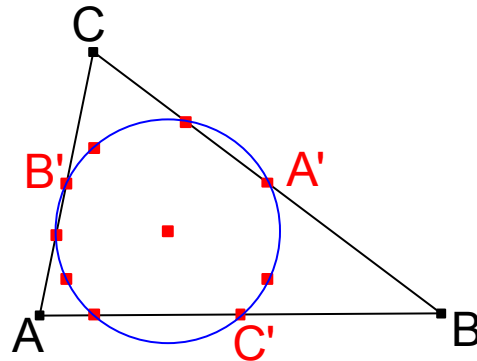
A triangle and its medial triangle have the same centroid G

The circumcentre of a triangle coincides with the orthocentre of the medial triangle

The orthocentre H of a triangle belongs to the line GO (line of Euler) and moreover G divides the segment OH in two parts such that $GH = 2OG$.

The orthocentre of a triangle coincides with the incentre of its orthic triangle

In a triangle ABC , the vertices of its medial triangle, the vertices of its orthic triangle and the Euler points (midpoints of AH , BH and CH) are all on a circle (circle of Feuerbach)



Step 0: The tutors choose the contents and methods of carrying out activities in school.

The tutors prepared some notes that were handed out to the teachers.

They also decided to structure the work through worksheets allowing students to work by them selves. These worksheet were built up during Step 1, together with the teachers and starting from a prototype worksheet prepared by the tutors.

<i>Construction</i>	Figures useful for the activity are built
<i>Exploration</i>	By “dragging” objects, the constructed figure is explored
<i>Observation</i>	Remarks raised in the previous phase are specified
<i>Verification</i>	The previous remarks are verified by means of DGS’ tools
<i>Conjecture</i>	Students are asked to propose a conjecture
<i>Validation Demonstration</i>	- Students are guided to prove (or disprove) the conjecture
<i>Theorem</i>	A theorem (just proved or to prove) is formulated by students
<i>Definition</i>	A definition is written or given by the students
<i>Sending to your notebook</i>	Students are invited to transcribe on a notebook, papery or electronic, a theorem, a definition, some comments ...

•Step 1: in the lab with teachers

16 teachers from 8 schools

6 scientifically-oriented high schools, 1 high school specialized in classical studies, 1 social science high school.

The teachers were all teachers of first two years of high school they all volunteered to participate at the activity

We met, all together, at the Department of mathematics and computer science, 7 afternoons for 2,5 hours each time

At the first meeting ... we discussed of the worksheets to give to students.

[CONSTRUCTION](#)

At the end of each meeting we decided what we would have done at the next meeting. Teachers were welcome to elaborate a worksheet proposal.

One of their proposal of the starting point for the following meeting, and all together we decided whether or not to change something.

Relevant was the contribution of some teachers: some strategies we adopted came out from the discussion and would have not came otherwise.



7 worksheets

Four worksheets on the geometric transformations:
translations, central symmetry, axial symmetry
homothetic transformation

Three worksheets on their applications: the medial
triangle, the orthic triangle, the Feuerbach circle

•Step 2: in the lab with students

Each teacher experimented the activity in her/his own class.

450 students were involved for 18 (3+15) hours

Also the tutors went to the schools every now and then.

Students reacted quite well to the course.

At first, they were scared, because “*it was a math project that was done after school hours*”. Afterwards, they volunteered with enthusiasm.



In the activity students explored, formulated conjectures, acquired concepts and operative capabilities.

The teacher was always there, ready to correct the path to follow with appropriate suggestions, to question the proposals that still need to be perfected using counter examples, to encourage them to continue, to praise them for every significant result.



In the end of the activity, students filled in an anonymous questionnaire, that have been very useful for evaluating the whole activity.

Step 3a: teachers voice

Log book: working modalities, organization of the activity, students reactions, results that have been reached from a motivational point of view (attitude, interest ...) and from a cognitive point of view.

Final questionnaire to evaluate the activity

Teachers appreciated realizing the worksheets with the tutors:

Collaborating with university professors to produce the working class material was fundamental, both in planning and in realizing the worksheets. Tutors helped us to set the worksheets in the most simple way for the students and to use a simple but at the same time rigorous language

As for the content

At first I was worried because geometric transformations are one of the topics where students encounter many difficulties. But from the first lesson I have observed that the students did not meet any difficulty, since they themselves were to discover the relationships and then to prove the various properties

Regarding the methodology used in the classroom

"I believe that the methodology used is very good with regard to the use of the worksheets that guide the student in geometric constructions and at the same time will help him in observations and deductions of geometrical properties, so to link theory with observable phenomena.

Furthermore, this methodology allows better collaboration between students".

"This way to work in class helped in the collaboration among student and with the teacher"

Involvement of students:

"The students have enthusiastically welcomed this activity, working together and comparing the obtained results collectively."

"The first meeting was a success, the three hours flew away, they found themselves to be the center of the project and the arguments were very attractive, the teacher was there only to help if needed. Each of them worked with great enthusiasm and playing an active role. The atmosphere was very serene and joyful.

It is improved the collaboration among students, in fact those most able were tutors for those who encountered some difficulties. The activity also allowed me to understand some of the dynamics, before unknown, within the class, and then to improve my relationship with students."

The use of Cabri:

"The use of the software Cabri was fundamental in the activity.

The only problem, in my opinion, is due to the fact that it is not a free software and therefore it can not be used by students outside the school lab: the limitations arose when the students wanted to develop work home. We recommend the use of a free software."

Validity of the teaching proposal

"The motivational attitude towards discipline and in particular towards geometry has changed significantly in a positive way," highlighted since the early meetings by "interest and curiosity about a new way of learning."

"Even the students who generally have difficulty in this case easily understood and assimilated the proposed issues."

Step 3b. Students' feedback

Questionnaire

Express your opinions on

- the topic of the course (how much you liked it, how hard it was, etc...);
- the use of Cabri II Plus during the activity and in general about its use for the study of geometry;
- the way the activity was conducted and in particular on the worksheets;

Has your attitude towards geometry changed after the activity?

Using Cabri was useful, new to me and very effective. It can be used to study geometry and I hope it can be used more often from students and teachers.

I never liked geometry, but I changed my mind during the course . I was very much interested.

Geometry we study on the books is different from dynamic geometry. Therefore I believe we should boost the study with computers

It was very interesting and I believe that it is a method that should be used in the next future for teaching.

Cabri

Helps to understand those theorems that are hard to read on the book

Allows to visualize what we theoretically study and this is what students need

Was very useful because we were aware of the picture we were creating and we did not stop at the static figure of the book

Is a very interesting program, but, above all, it helps in understanding lots of things that you can not show on the blackboard. I will use it at home as well!

Worksheets

Working with worksheets was perfect because, yes at first they looked repetitive and boring. But in the end they guide us, step by step, to finish our work and make us perfectly understand what we do.

Some students find the worksheets *hard* others *very easy to solve*

It is beautiful to discover yourself things that maybe we would have not understand from the teacher. At least once WE applied the theory and did the practice, by experimenting geometry ourself. And we liked it!



I changed the way I studied geometry: now it is interesting and fun

Conclusions

Teachers

Have been working on this project for one school year.

Never the less none of them GAVE UP!

More over, some of the teachers told us that they used the worksheets for doing the same topic in other classes.

Some worksheets were too easy

It is not easy to write worksheets for the WHOLE class.

Cabri vs a free DGS

In the end,

some teachers used the same math laboratory methodology with DGS for other geometry topics.

some students asked to be involved in future similar activities.

Хвала

Grazie

Thank you

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Хвала

THANK YOU

GRAZIE

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