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The use of DGS to face optimization problems at secondary school

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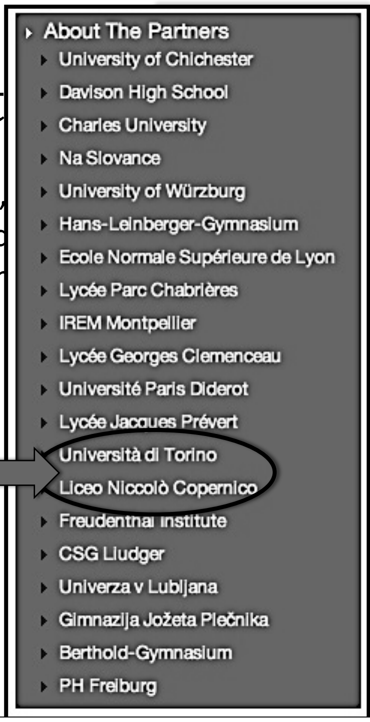
EdUmatics Project

- A European Comenius Project, different partners are involved university partners from sever



EdUmatics Project

- A European Comenius Project, in which many different partners are involved (20 school and

"wide concern that, although a range of technologies are available, the majority of teachers have not had any formal opportunities to learn about them and fewer teachers still have integrated the more complex technologies within their practices.

EdUmatics Project

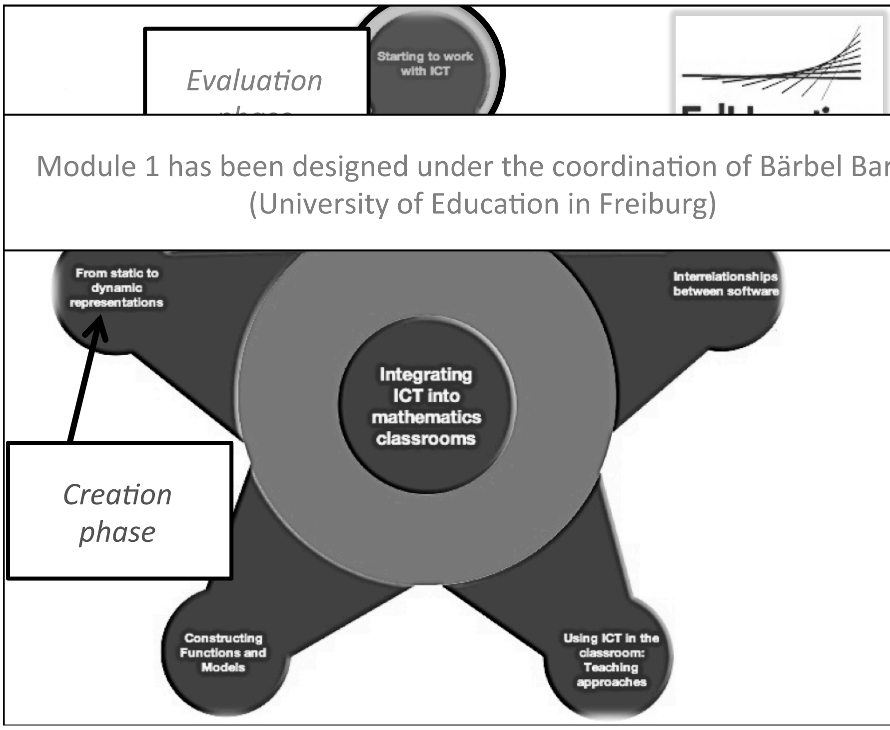
- A European Comenius Project, in which many different partners are involved (20 school and

"wide concern that, although a range of technologies are available, the majority of teachers have not had any formal opportunities to learn about them and fewer teachers still have integrated the more complex technologies within their practices.

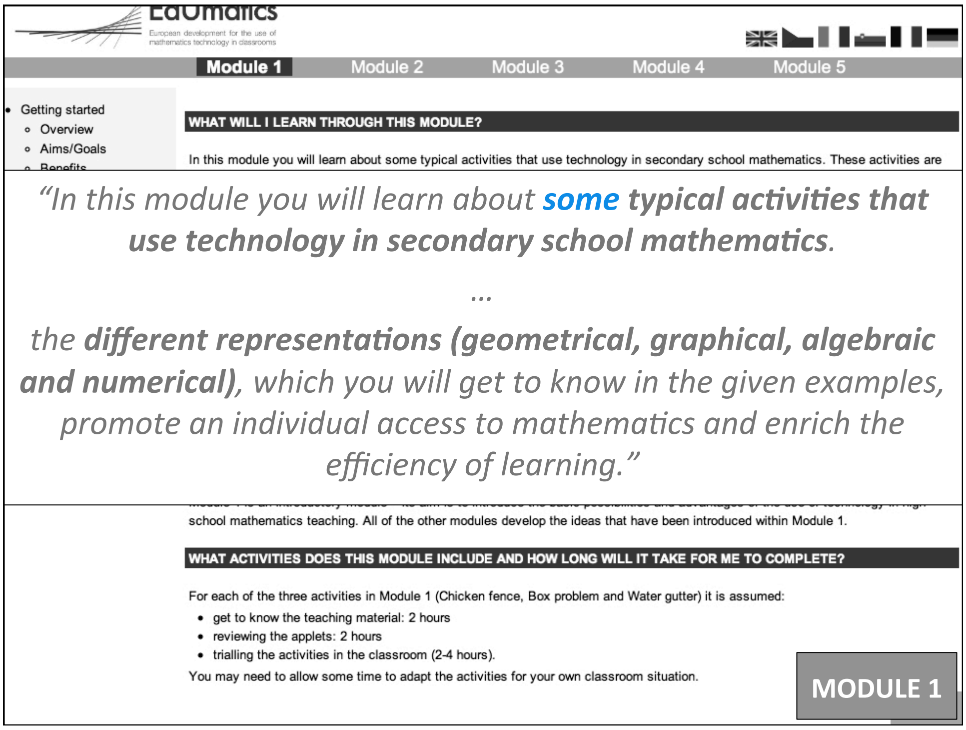
Furthermore, there is a growing desire to put the technology into the hands of the learners to enable them to become independent, fluent and confident users of technology in a range of mathematical scenarios that involve problem-solving and mathematical modelling."

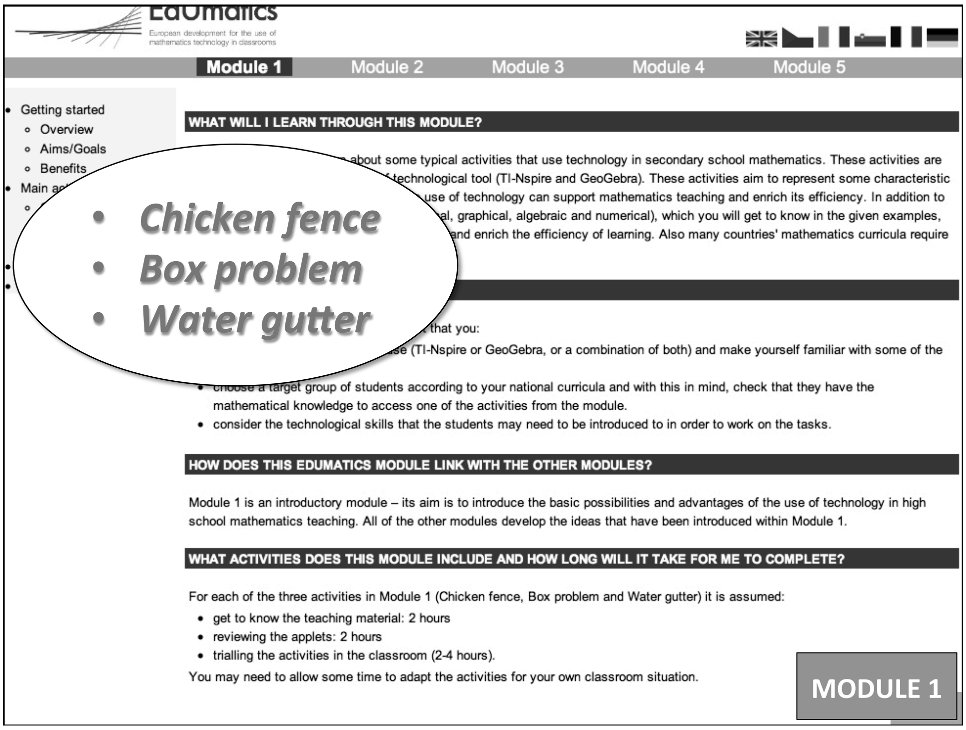
EdUmatics Project

- A European Comenius Project, in which many different partners are involved (20 school and university partners from seven EU countries)
- The EdUmatics project aims to provide secondary mathematics teachers with support to integrate technology within their classroom practice
- The resulting professional development modules offer a range of tasks and activities for both trainee and experienced teachers.



Module 1 has been designed under the coordination of Bärbel Barzel (University of Education in Freiburg)


Module 1
Module 2
Module 3
Module 4
Module 5
Getting started
Overview
Aims/Goals
WHAT WILL I LEARN THROUGH THIS MODULE?
In this module you will learn about some typical activities that use technology in secondary school mathematics. These activities are
"In this module you will learn about **some** typical activities that use technology in secondary school mathematics.
…
the different representations (geometrical, graphical, algebraic and numerical), which you will get to know in the given examples, promote an individual access to mathematics and enrich the efficiency of learning."
school mathematics teaching. All of the other modules develop the ideas that have been introduced within Module 1.
WHAT ACTIVITIES DOES THIS MODULE INCLUDE AND HOW LONG WILL IT TAKE FOR ME TO COMPLETE?
For each of the three activities in Module 1 (Chicken fence, Box problem and Water gutter) it is assumed:
get to know the teaching material: 2 hours
reviewing the applets: 2 hours
trialling the activities in the classroom (2-4 hours).
You may need to allow some time to adapt the activities for your own classroom situation.
MODULE 1


Module 1
Module 2
Module 3
Module 4
Module 5
Getting started
Overview
Aims/Goals
Benefits
WHAT WILL I LEARN THROUGH THIS MODULE?
Chicken fence
Box problem
Water gutter
mathematical knowledge to access one of the activities from the module.
consider the technological skills that the students may need to be introduced to in order to work on the tasks.
HOW DOES THIS EDUMATICS MODULE LINK WITH THE OTHER MODULES?
Module 1 is an introductory module – its aim is to introduce the basic possibilities and advantages of the use of technology in high school mathematics teaching. All of the other modules develop the ideas that have been introduced within Module 1.
WHAT ACTIVITIES DOES THIS MODULE INCLUDE AND HOW LONG WILL IT TAKE FOR ME TO COMPLETE?
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MODULE 1

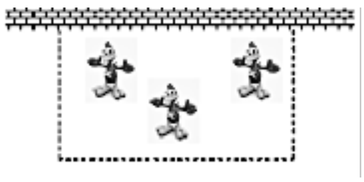
Methodology

- A scientifically-oriented high school (Liceo Scientifico)
- All the activities were carried out both at grade 11 and at grade 12
- Individual reasoning with paper and pencil
- Group work with a DGS: GeoGebra, and Excel
- Collective discussion
- Different methods were used: numerical, geometrical and graphical
- Group works and discussions were filmed through a video-camera

ACTIVITIES

Chicken fence

Farmer Ernestino wants to create a rectangle fence for his chickens.



Considering the wall as one side of the fence and that the whole fence is 10 meters long, what is the biggest area that the fence can occupy?

(isoperimetric problem)

ACTIVITIES

Box problem

A rectangular piece of cardboard (40 cm x 50 cm) is used to make a box without the lid in the following way: squares with the side *x* are cut out of each corner of the rectangle. The rest is folded up and stuck together to form a box.

Find the value of *x* to get the maximal volume of the box.

(3-dimensional extension of the chicken fence problem)

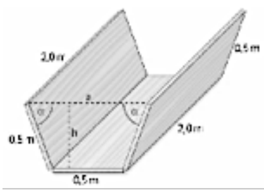
ACTIVITIES

Water gutter

A water gutter with optimal capacity shall be constructed from three boards with width 0.5 meters each (see illustration). The area of the cross section is given by the function A with:

$A(\alpha) = 0.25 \times (1+\cos(\alpha)) \times \sin(\alpha)$.

Find the angle α so that the water gutter has maximal capacity.



ACTIVITIES

Research

- Analysis of the processes going on in the classroom
- Use of video and written productions

- The role of the DGS
- In particular:
  - Its use to validate or confute conjectures, to elaborate strategies, and to check new methods
  - Its potential in overcoming obstacles or enhancing problem solving
  - Its limits respect to difficulties or misconceptions

ANALYSIS

Examples

- Potential and dynamicity of the DGS valued by the intervention of the teacher (I)

- Overcoming obstacles deriving from the use of the DGS (II)

- Choice of meaningful digits and the precision of the result (III)

ANALYSIS

I. Simone, Stefano and Davide

- **Grade 11**; numerical, geometrical and graphical methods are all requested.
- The students are using GeoGebra.
- They start from the geometrical method: they want to represent the rectangle and try to inscribe it in a semicircle.
- They look for a dynamic visualization, that is, a definition of the sides of the rectangle as variables through conditions on the diameter.

ANALYSIS

I. Simone, Stefano and Davide

- But the search does not work, and the students change to the numerical method, by the aid of the spreadsheet, Excel.
- They construct a three column-table: the values of the short side, the values of the long side, and the values of the area.
- They display the graph of the area as a function of the variable side (a parabola with downward concavity).

ANALYSIS

I. Simone, Stefano and Davide

- They go back to the geometrical method using GeoGebra.
- They are not able to define the two sides as variables, and they ask for the help of the teacher.

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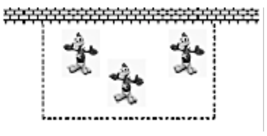
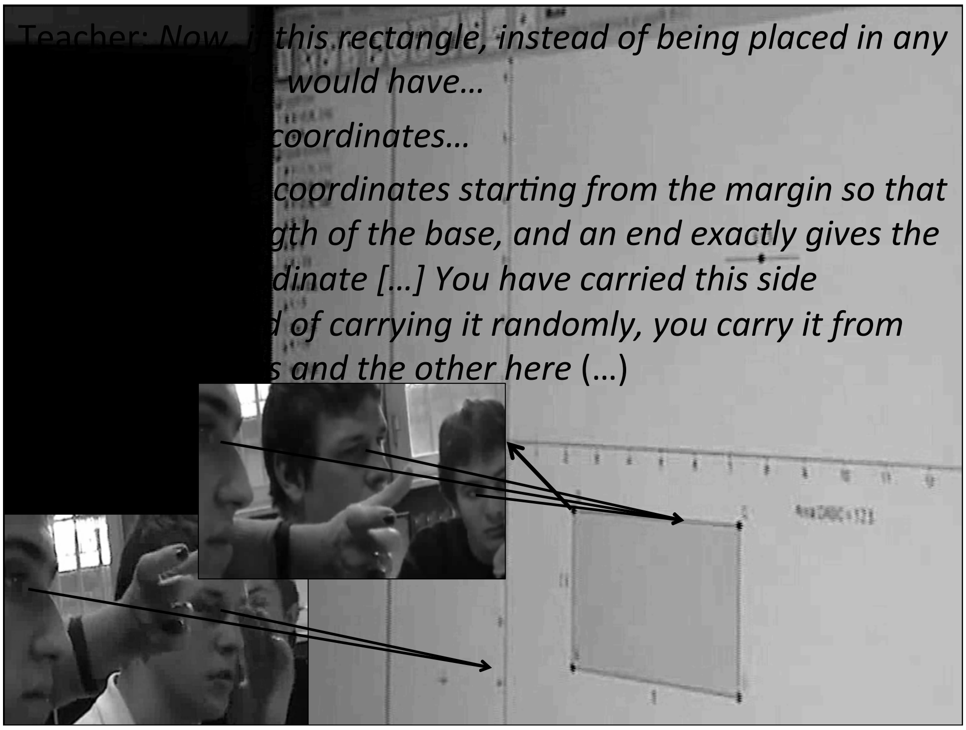
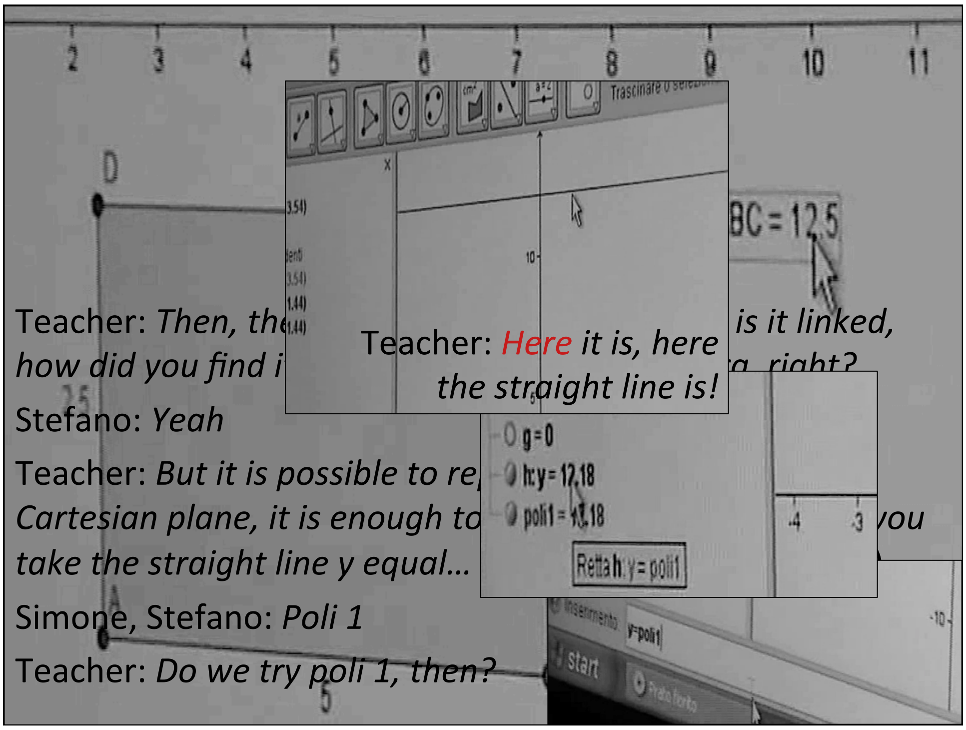
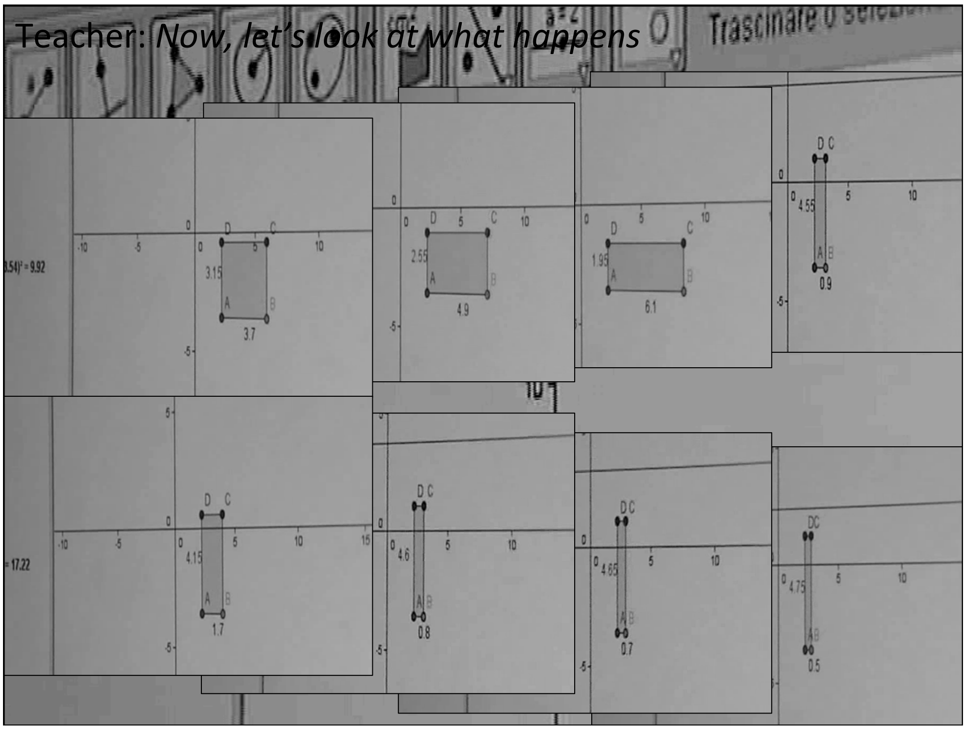
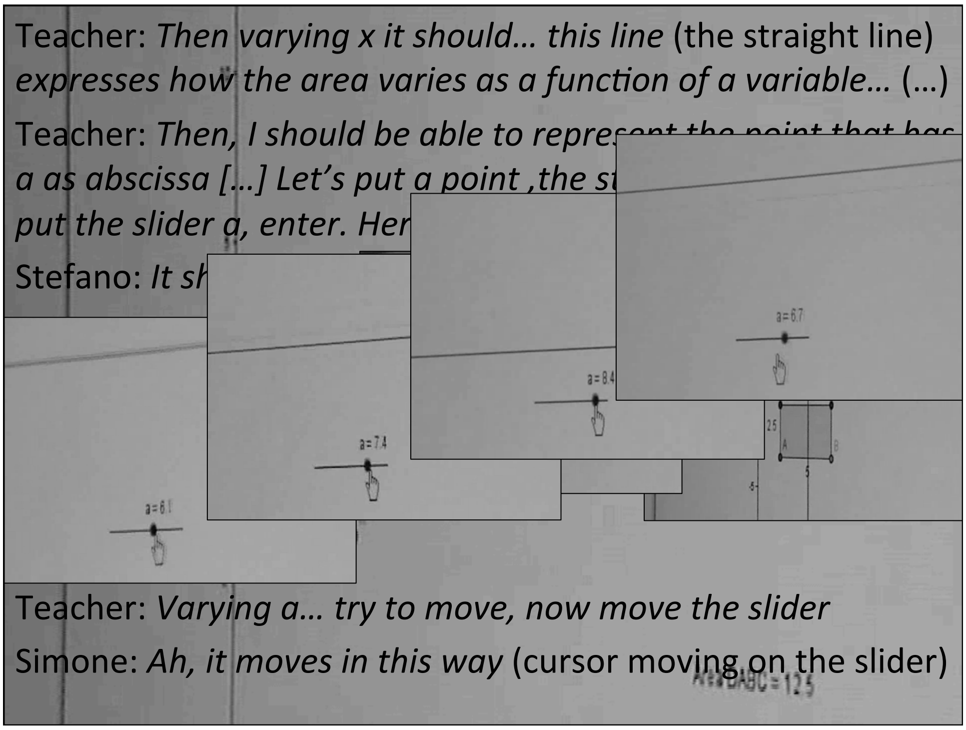


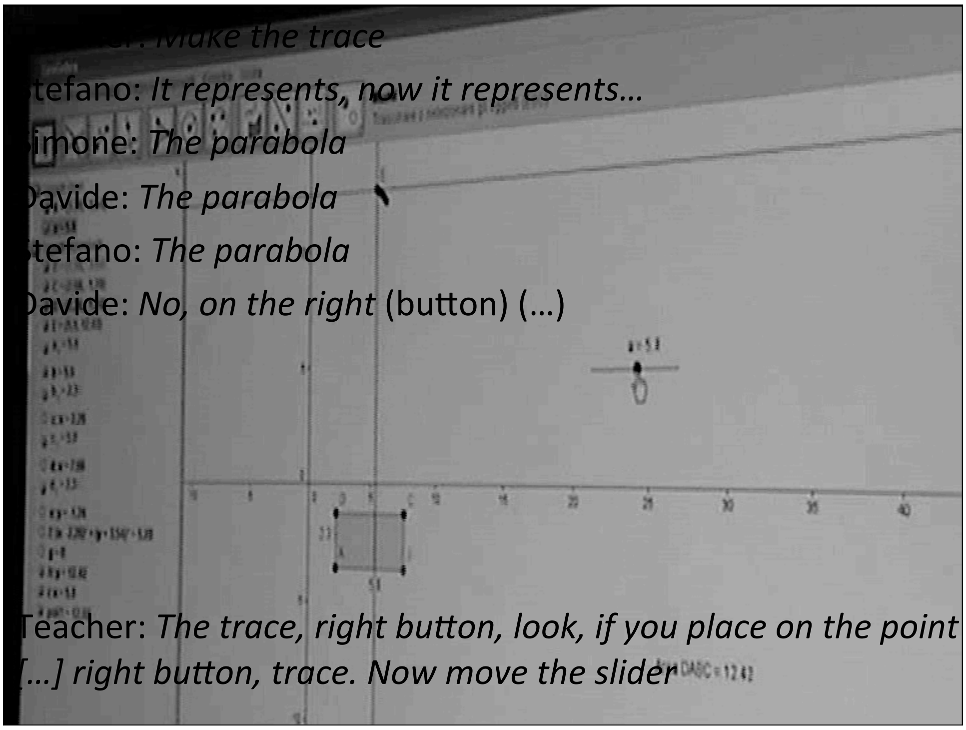
ANALYSIS

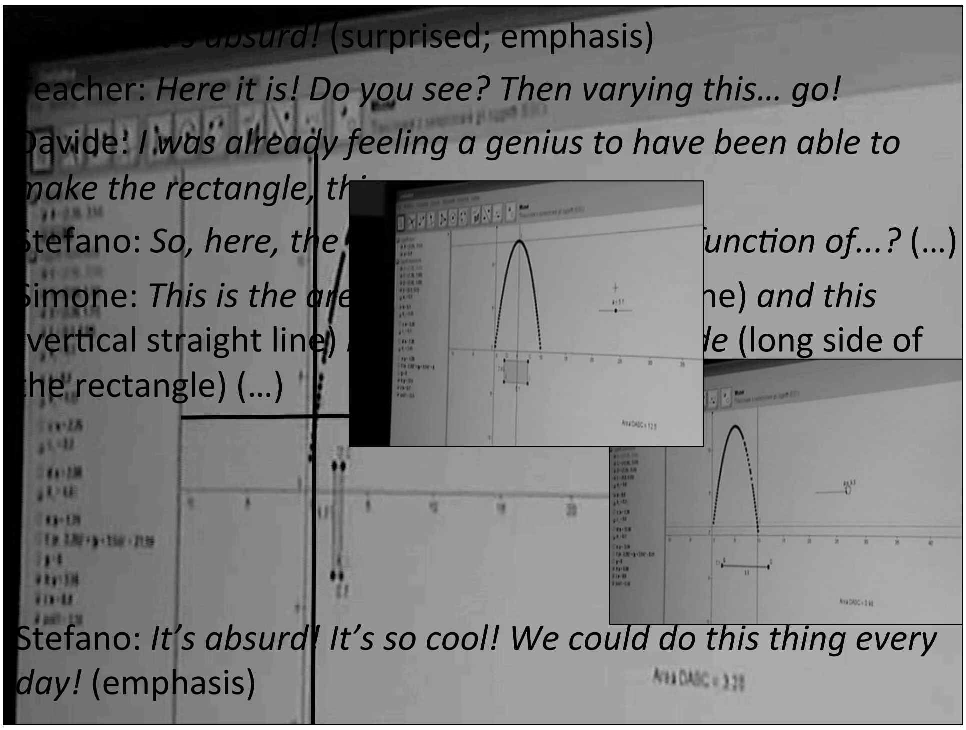



BC = 12.5
Teacher: Then, th
is it linked,
how did you find i
Teacher: Here it is, here the straight line is!
Stefano: Yeah
Teacher: But it is possible to re
Cartesian plane, it is enough to
you
take the straight line y equal…
g = 0
h: y = 12.18
Retta h: y = poli1
Simone, Stefano: Poli 1
Teacher: Do we try poli 1, then?
start


Teacher: Now, let's look at what happens

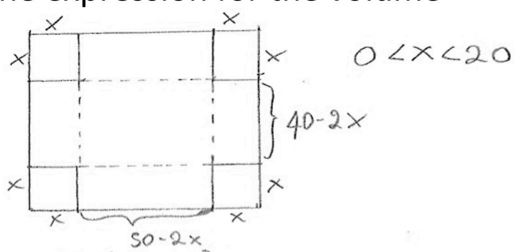

Teacher: *Then varying x it should… this line* (the straight line) *expresses how the area varies as a function of a variable…* (…)
Teacher: *Then, I should be able to repres*
a as abscissa […] Let's put a point ,the s
put the slider a, enter. Her
Stefano: *It sh*
a=6.1
a=7.4
a=8.4
a=6.7
Teacher: *Varying a… try to move, now move the slider*
Simone: *Ah, it moves in this way* (cursor moving on the slider)

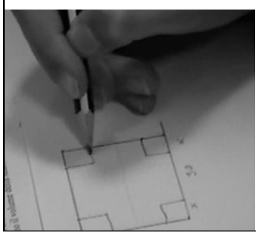

Make the trace
tefano: *It represents, now it represents…*
imone: *The parabola*
avide: *The parabola*
tefano: *The parabola*
avide: *No, on the right* (button) (…)
eacher: *The trace, right button, look, if you place on the point*
[…] right button, trace. Now move the slider



II. Adela, Giulia and Mattia

- **Grade 12**; graphical method is requested.
- The students are using GeoGebra.
- They have found the expression for the volume of the box.





$V = (40-2x)\cdot(50-2x)\cdot x$

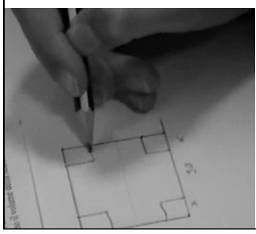
$V = (2000-80x-180x+4x^2)x$

$V_{(x)} = 2000x - 180x^2 + 4x^3$

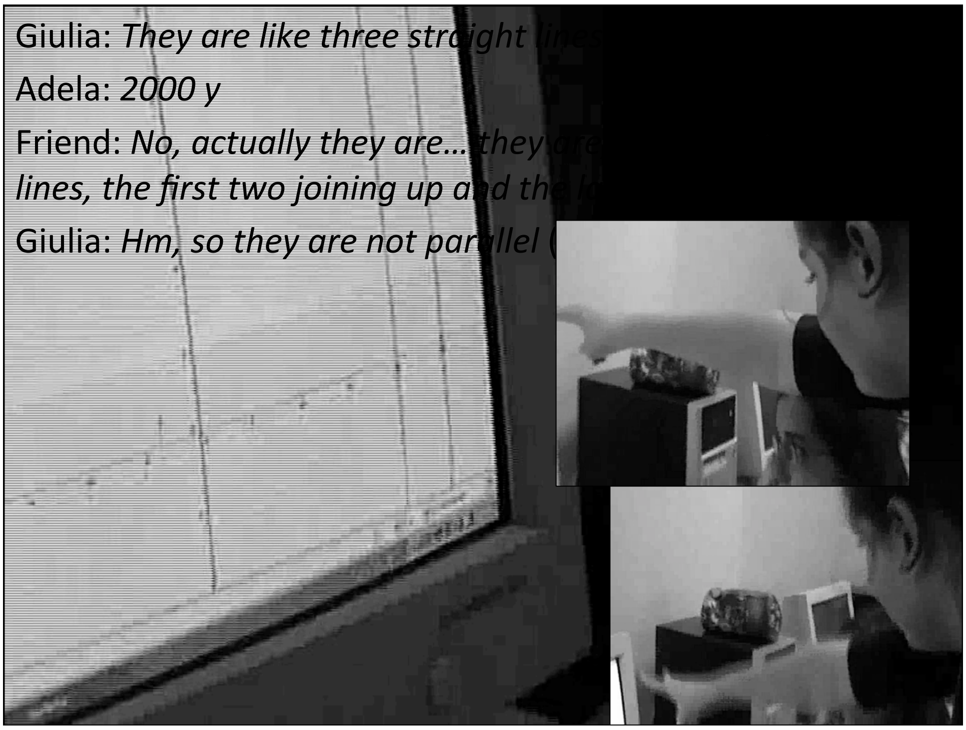
ANALYSIS

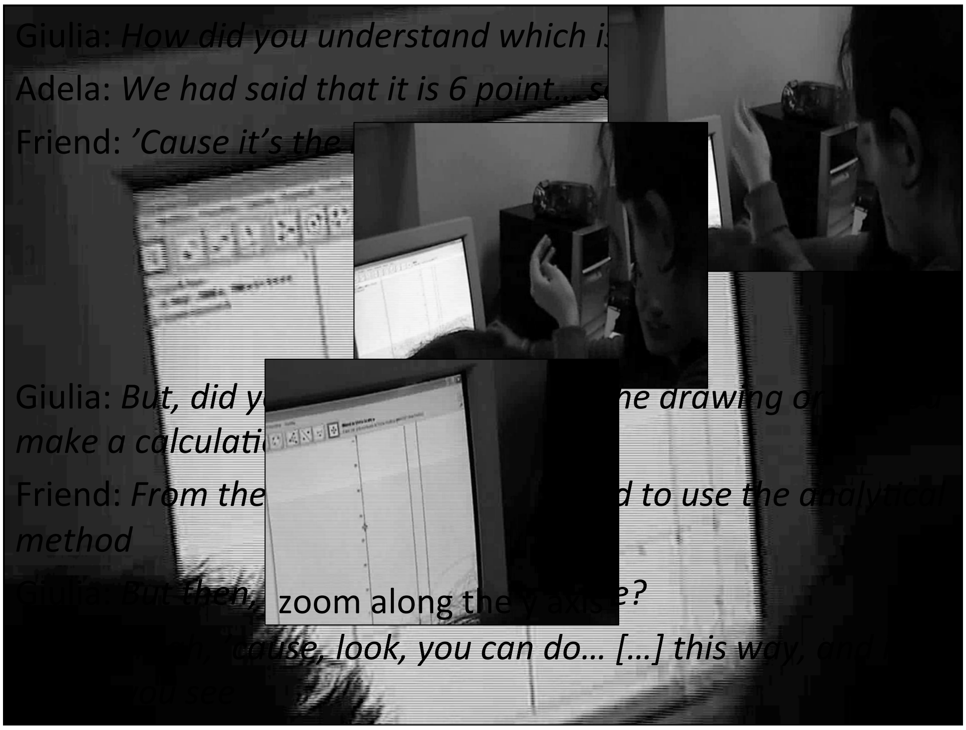
II. Adela, Giulia and Mattia

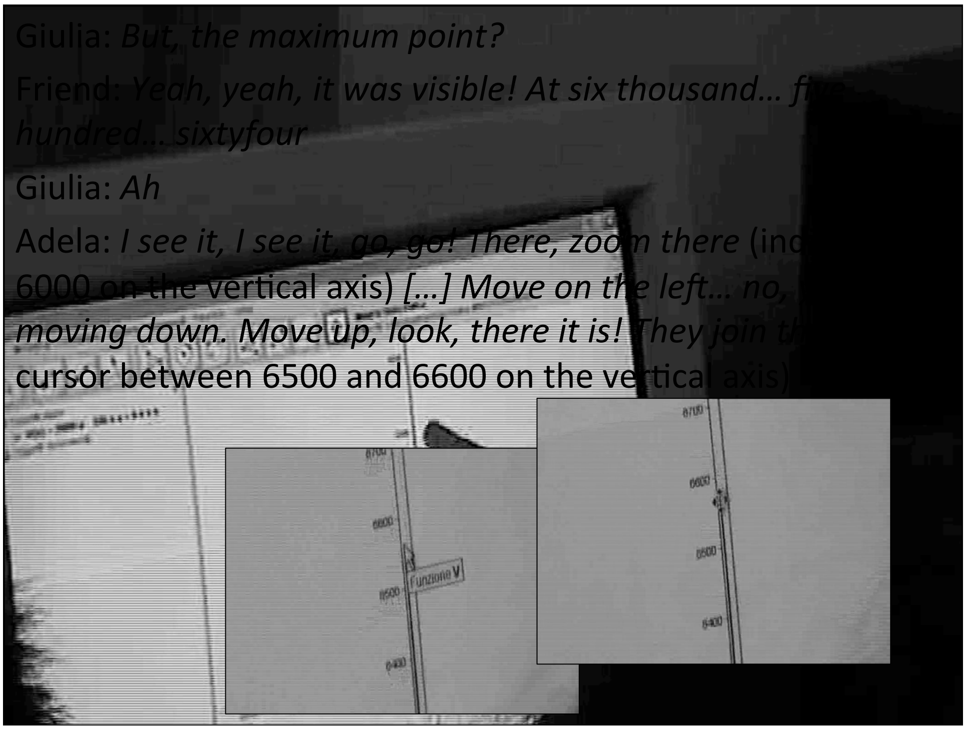
- They insert the expression in GeoGebra, but the screen does not display any graph.
- The students think to have been wrong: they retry to insert the function, they move the axes, and they get "*parallel straight lines*".
- The scale of the axes creates an obstacle, due to the high values of the volume.



ANALYSIS

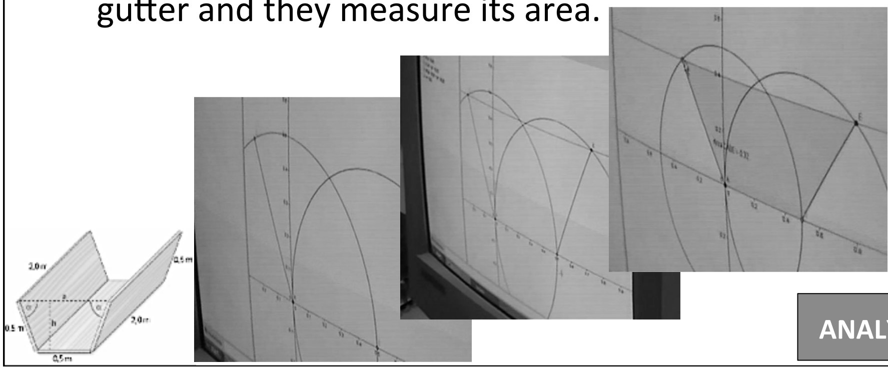



Giulia: *How did you understand which i*
Adela: *We had said that it is 6 point… s*
Friend: *'Cause it's the*
Giulia: *But, did y ne drawing o*
make a calculati
Friend: *From the d to use the analytical*
method
Giulia: *But then,* zoom along the y axis?
cause, look, you can do… […] this way, and
you see


Giulia: *But, the maximum point?*
Friend: *Yeah, yeah, it was visible! At six thousand… fi*
hundred… sixtyfour
Giulia: *Ah*
Adela: *I see it, I see it, go, go! There, zoom there* (ind
6000 on the vertical axis) *[…] Move on the left… no,*
moving down. Move up, look, there it is! They join th
cursor between 6500 and 6600 on the vertical axis)
6700
6600
6500
6400
Funzione V

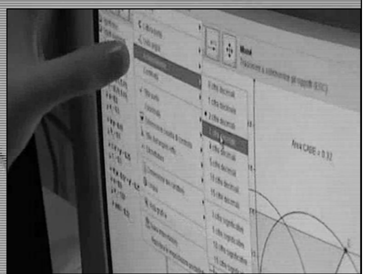
III. Andrea, Marco and Paolo

- **Grade 12**; geometrical method is requested.
- The students are using GeoGebra.
- They use two meeting circles to construct the trapezium that gives the section of the water gutter and they measure its area.

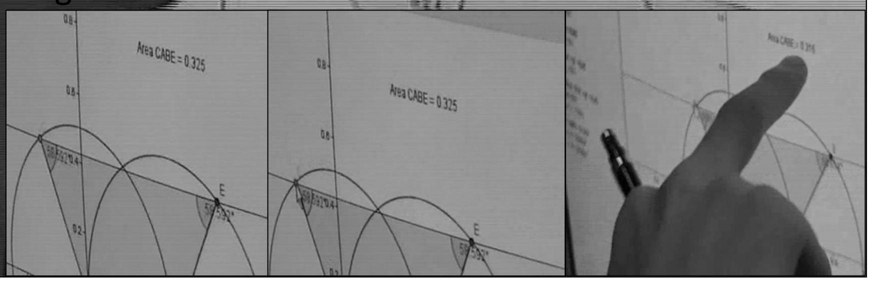


ANALYSIS

The students choose three decimal digits and measure the upper left angle.



Then, they move a point defining the trapezium to look for the maximum value of the area and the corresponding value of the angle.



Paolo: *Perfect, found!*

Marco: *The angle is 58.59 […] It changes, look, it changes*

Paolo: *It changes, but the area is small*
but if we put another digit… (…)

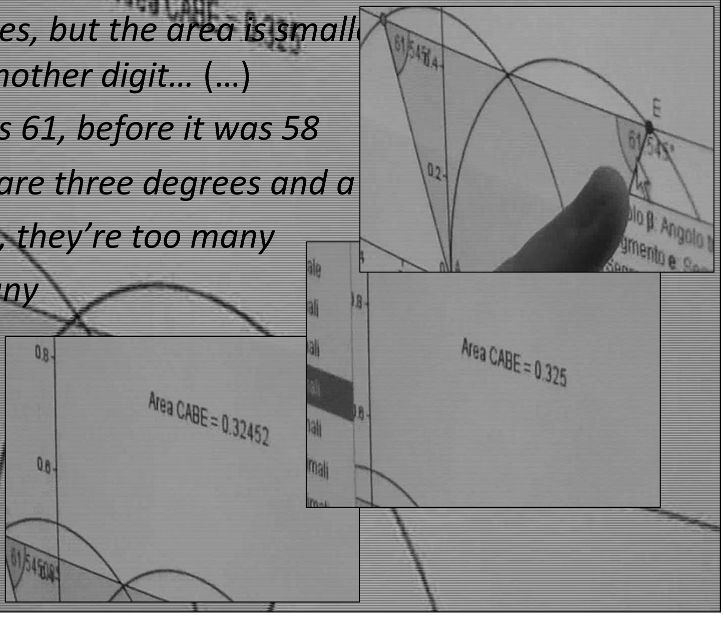
Marco: *Now it's 61, before it was 58*

Andrea: *There are three degrees and a*

Paolo: *In effect, they're too many*

Marco: *Too many*

Paolo: *I put… 5*

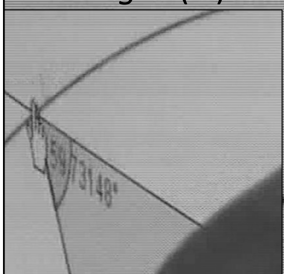


Paolo: *Here it is! It was 7 6! […] Go a little higher… perfect, leave it!*

Andrea: *59 degrees point 8!*

Paolo: *59.8 are almost 60 degrees*

Andrea: *Stop, 'cause if you move again, it changes* (…)



Andrea: *So, the angle is 59.8*

Paolo: *Now, let's try to take this angle, to replace it* (in the formula of the area) *and let's see if the area comes equal*

The DGS…

- The role of the teacher is fundamental to value the potential and to show the dynamicity of the DGS.
- The 'mobility' of the DGS allows for trial and errors, search for new methods, visualization, validation or refutation of conjectures, recognition of variability and invariance, transformational reasoning.
- The DGS can be used in an efficient manner to overcome obstacles and to discuss misconceptions.

Theoretical background

- Support of DGSs for dynamic thinking processes
  Arzarello, Ferrara & Robutti, Mariotti & Maracci, Sinclair
- Semiotic lenses (various signs not only words)
  Arzarello, Edwards, Nemirovsky, Radford, Sinclair
- Interactions students-DGS, teacher-DGS, teacher-students


Хвала
Thank you