DIRECTED LINE SEGMENTS AND FREE VECTORS

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Abstract

Vector structures are one of the most powerful objects in mathematics that cover a wide variety of applications. They are used for describing multidimensional spaces, for defining variety of physical quantities that cannot be described only by numbers (like force for example) and they are inevitable in describing a motion of an object. In the first grade of a high school, the idea of a free vector is introduced in geometry as an equivalence class of equality in the set of directed line segments, so the most common image of a vector is “an arrow”. In order to convey the correct idea of a vector to the students and in order to demonstrate to them the purpose of studying this part of the curriculum, the concept of a free vector is related to motion and to reading maps. The class is designed in such a way as to involve the use of computers and the mathematical program package GeoGebra.

According to the math curriculum in Serbia, first grade grammar school students (aged 15-16) meet the topic of vectors for the first time. The concept of free vectors is introduced as an equivalence class of equality in the set of directed line segments. Vector quantities used to be one of the simplest parts of geometry curriculum. And then, the situation changed. Approximately ten years ago, the number of students having problems with vectors started increasing. It was not hard to notice the most common mistakes. First of all, they did not distinguish between vectors and line segments and often would not take a direction into account. This caused problems with addition and subtraction of vectors. The addition of two given vectors was not a problem in itself – students were able to find the sum of two given

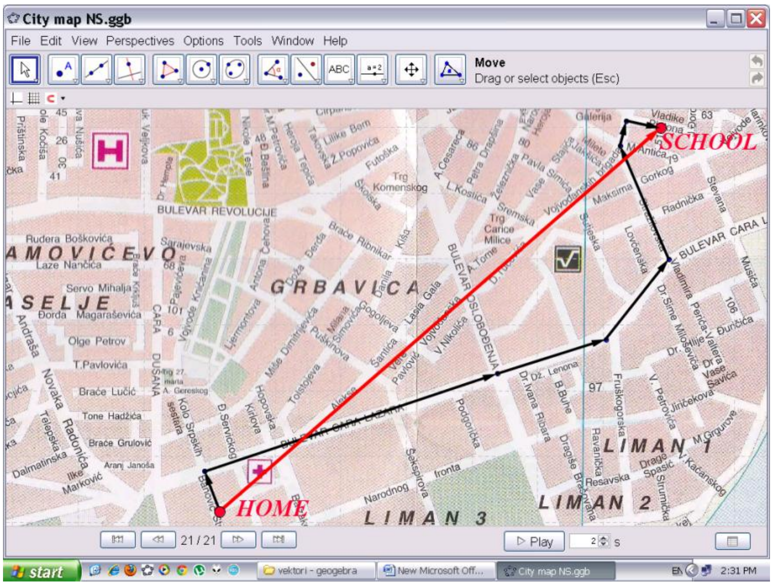

Figure 1: Journey representing the polygonal law

vectors using triangle or parallelogram law – but, if they had to use vectors to solve even simple problems in geometry - to prove a mid-point theorem, for example – they were not able to do it properly. Naturally, the next problem was short-term knowledge. As the concept of a vector was not grasped correctly from the very beginning, the problems started to accumulate in the third grade when Cartesian components and vector products were presented. On the other hand, vector quantities are extremely useful in physics, engineering and are very important for further mathematical education, so all these problems had to be solved. Having already had good experience with dynamic geometry and trigonometry by using GeoGebra package, I decided to try out a similar approach to teaching vectors. GeoGebra works in such a way as to allow inserting any image as a background. In one brief review, I put a city map as a background image in order to explain to my students the result of their everyday journey from home to school, pointing out the fact that the starting point i.e. HOME had to be the tail and finishing point i.e. SCHOOL had to be the tip of the resultant (figure 1), and that the resultant did not depend on summands, i.e. on the chosen way. What I wanted to achieve is to explain how polygonal law could be used to solve geometrical problems in which certain vector should be represented in terms of the given ones. Explaining the same thing in the same way by using only blackboard and colored chalks did not give a satisfying result, so I did not expect too much after this review. But surprisingly, by dragging "arrows" on the map and by finding the different ways from home to school the aim was achieved. It was obvious that many students were not able to create a mental image of the map so this brief visualization helped them. As this approach proved to be good, it required developing.

A vector is embodied in physics in a variety of ways: as a force, acceleration, a displacement, momentum etc. I chose displacement for this purpose for two main reasons: it

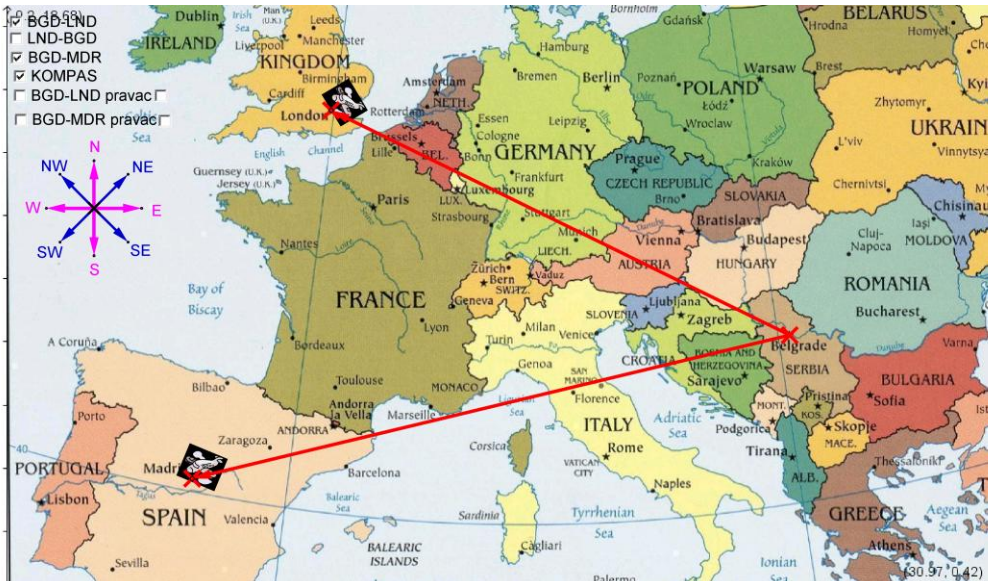

Figure 2

does not require much pre-knowledge and it naturally leads to translation. The class is designed in such a way as to define the directed line segments in the first step, then to define the relation of equality among them in the second, and finally to show that all equal directed line segments are representatives of the same vector, i.e. to introduce the concept of free vector.

In the first step, students observe two different flights on the inserted map of Europe: Belgrade-London and Belgrade-Madrid and compare them (Figure 2.). It is obvious that those flights have different lengths and are in different directions. Then, students are asked to find at least one displacement that is in the same direction as Belgrade-London. Surprisingly, almost all of them tried to find it on the same line. So I put a compass in, in order to make a notion of direction clear to them. Now it was obvious, that a parallel line was required, as the direction of observed displacement was west-northwest. The next task is to find the displacement which is in the same direction and has the same length, as observed one. There is one – Skopje-Paris. So, now there are two directed line segments $\overrightarrow{BL}$ (Belgrade-London) and $\overrightarrow{SP}$ (Skopje-Paris), that are in the same direction and have the same length. This leads to the second step – definition of an equality of two directed line segments. But before that, I like to use one of the properties of the GeoGebra package – I asked them to drag the

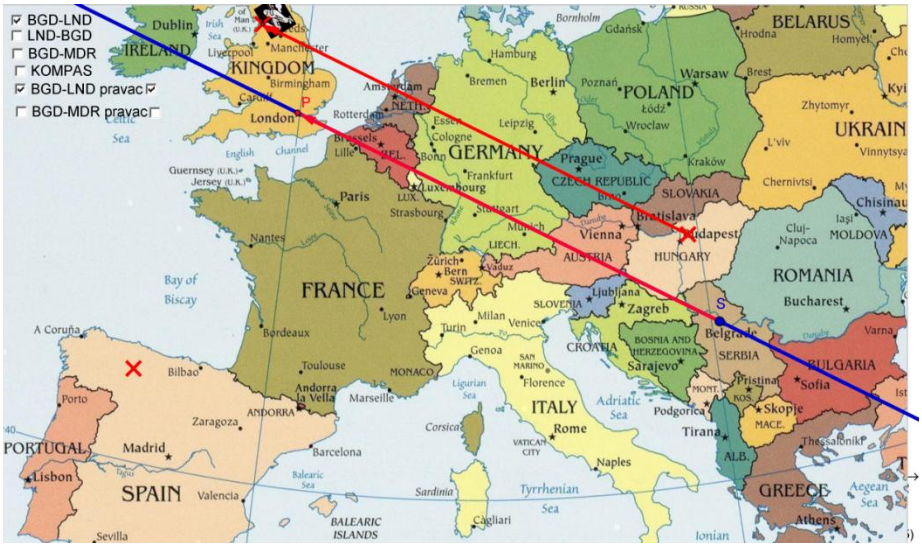

Figure 3

background image until the directed line segment $\overrightarrow{BL}$ takes the position of Skopje-Paris flight (Figure 3). This is to show that every displacement of certain direction and certain length could be described by only one directed line segment, so that is the reason why the equality relation is defined exactly in that way (Two directed line segments are equal if they are in the same direction and have the same length).

In the next step, students are asked to find all directed line segments that are equal to line representing the displacement London-Reading (or London-Oxford) on the map of The United Kingdom. There are few of them, so the whole set of those equal directed line segments is defined as a free vector (or vector), and all those directed line segments are the

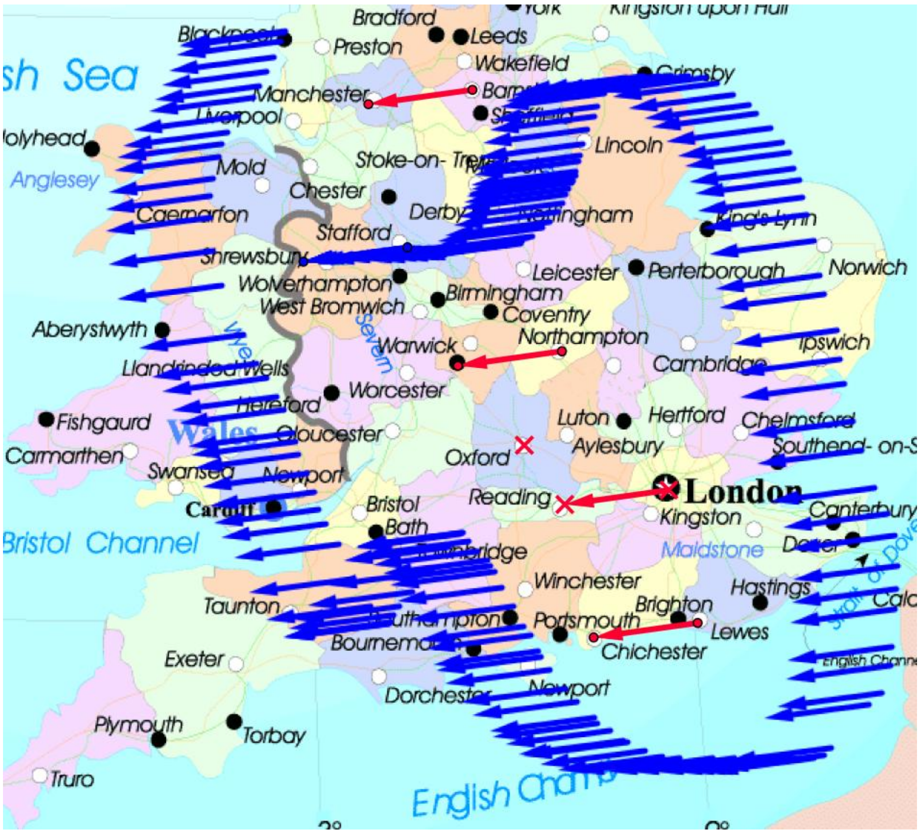

Figure 4

representatives of the same vector. Dragging that arrow and using the trace tool of GeoGebra package, proved to be extremely useful for the embodiment of the concept of free vector (Figure 4). Now it is obvious that they can use any representative of a certain vector according to the starting point, i.e. tail in the same way as they use rational numbers in addition (they use $\frac{1}{2}$ if they need to add it to $\frac{5}{2}$, for example, but if it need to be added to $\frac{7}{12}$, they would chose $\frac{6}{12}$, instead of $\frac{1}{2}$)

The next step is to introduce the notion of an opposite vector, to define the triangle, polygonal and parallelogram law of addition, to define subtraction of two given vectors, and finally to introduce a concept of a zero-vector. But according to my experience in teaching all those challenges seem to be pretty easy, after the notion of free vector has been grasped properly.

This is, of course, just one way of introducing the vectors to the first grade students, but it proved to be efficient, because by using GeoGebra dynamic environment, we managed to decrease the number of students that had difficulties with this topic.

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