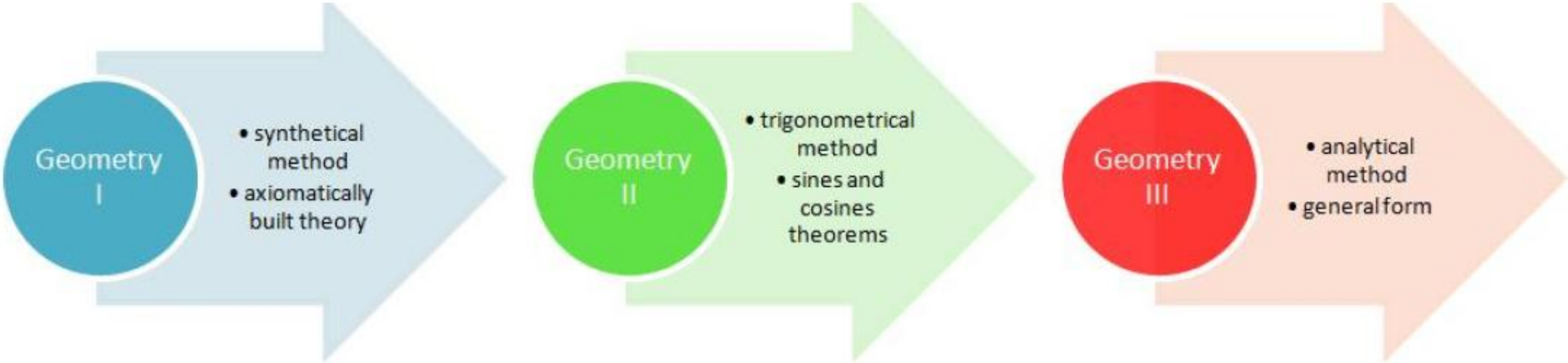
Teaching Geometry using a Computer

Elvira Ripco Šipoš

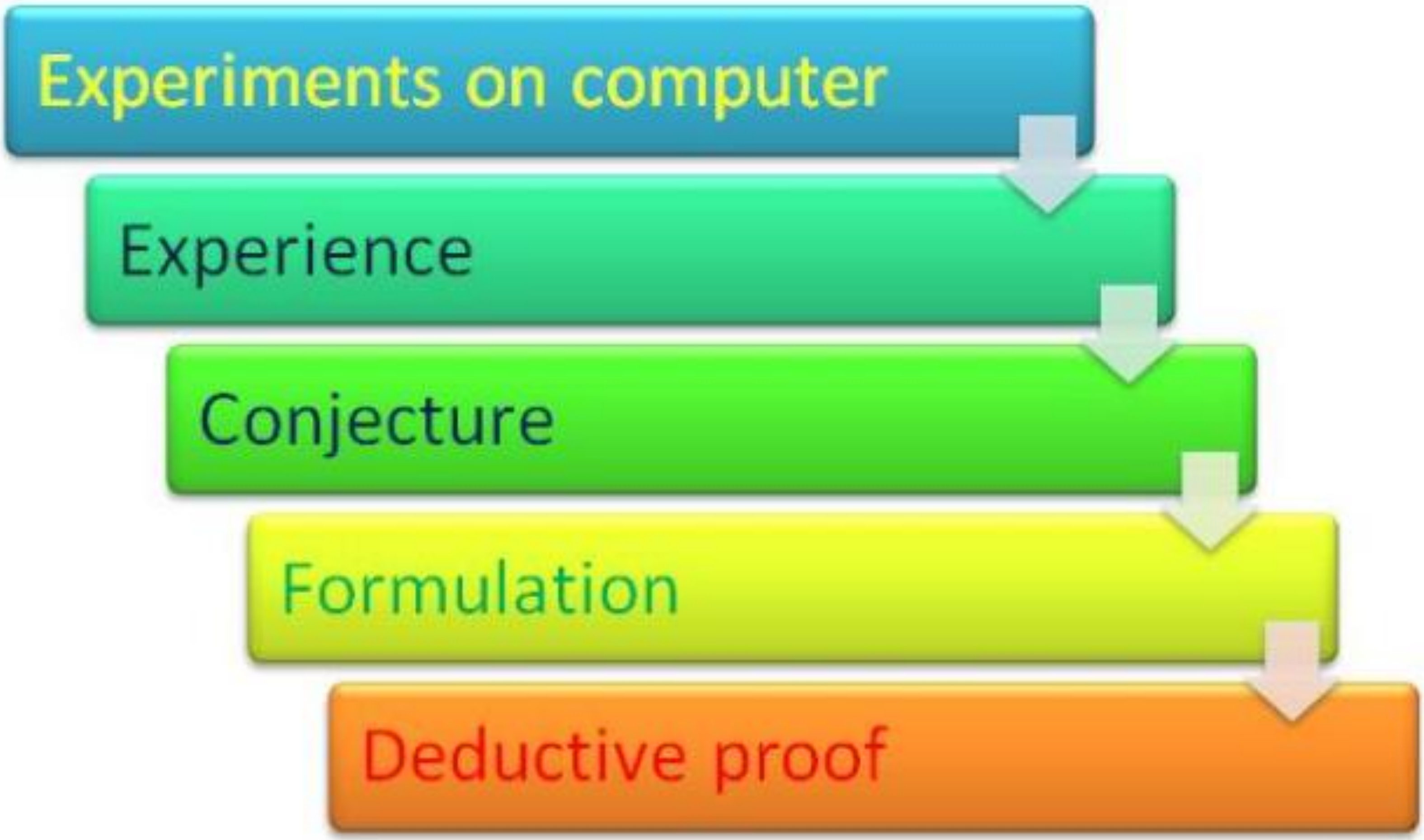
Bolyai Grammar School
Senta

- Introduction
- Matematical creativity and talented students
- Geometrija 1
- Geometrija 2
- Geometrija 3
- Apollonius' problems
- Is there geometry after Euclid?

Steps of spirality



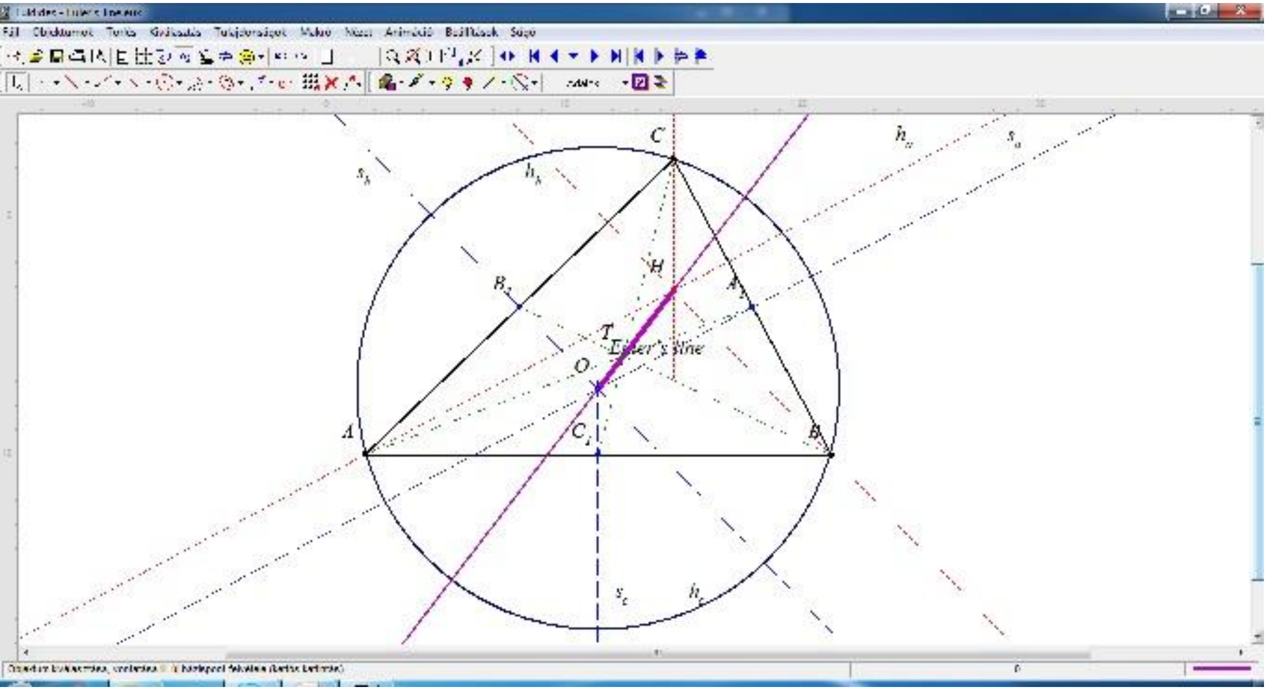
Teaching method



Matematical creativity and talented students

The pyramid of talented students in mathematics: Jane Piirto

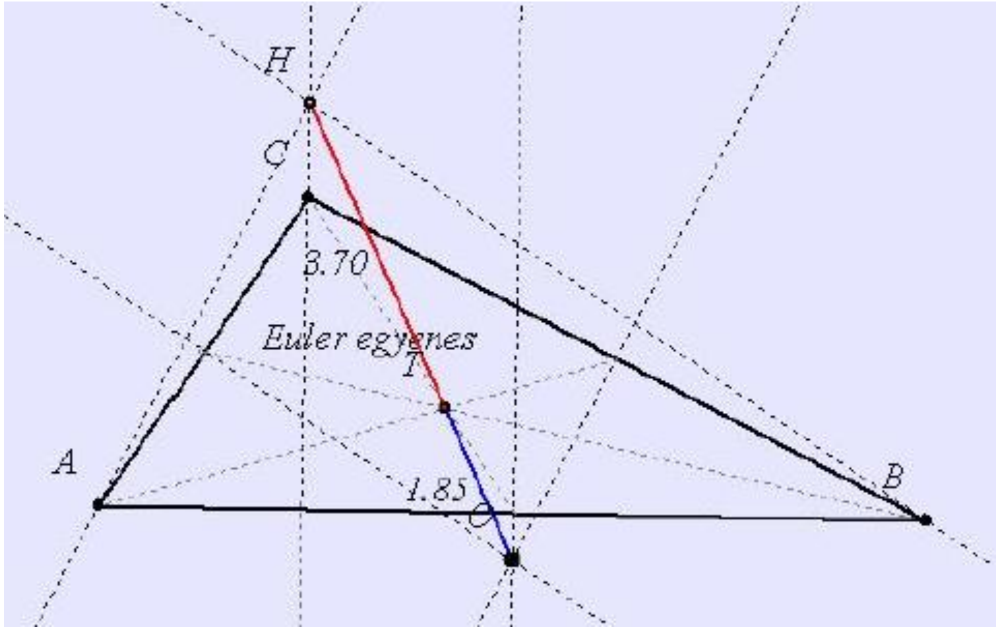
Geometrija 1
Planimetry



- Triangle centers (H orthocenter, T centroid, O circumcenter, S incenter)
- Euler's line
- Cognitive aim of teaching mathematics

Geometry 1
Planimetry

1. Visualization on computer

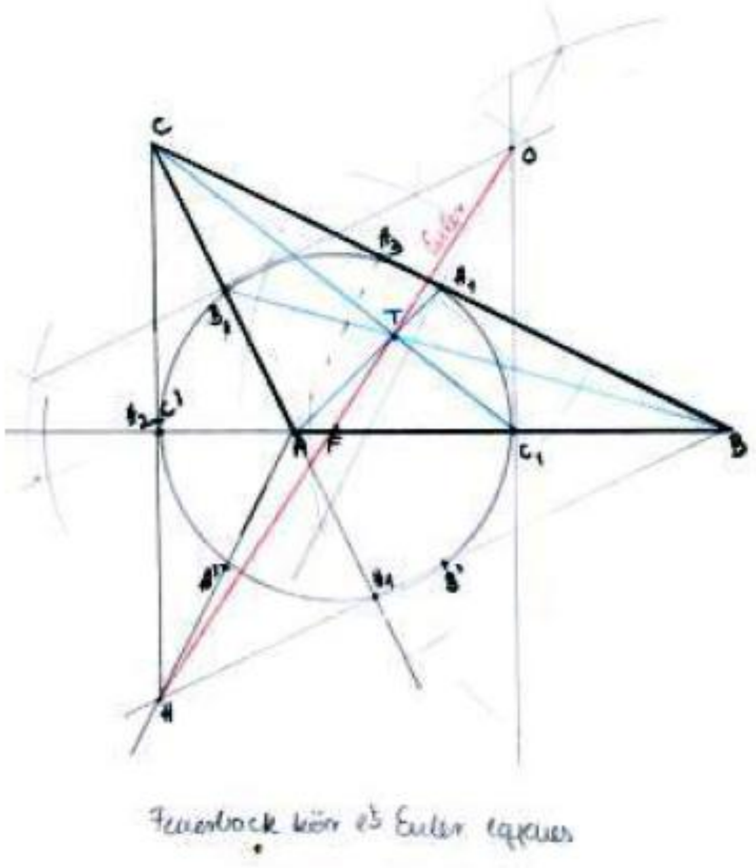


i) experiments on computer
ii) experiences
iii) conjecture
iv) formulation of the theorem
v) Deducitve proof
(affective end effective aim)

Geometry 1
Planimetry

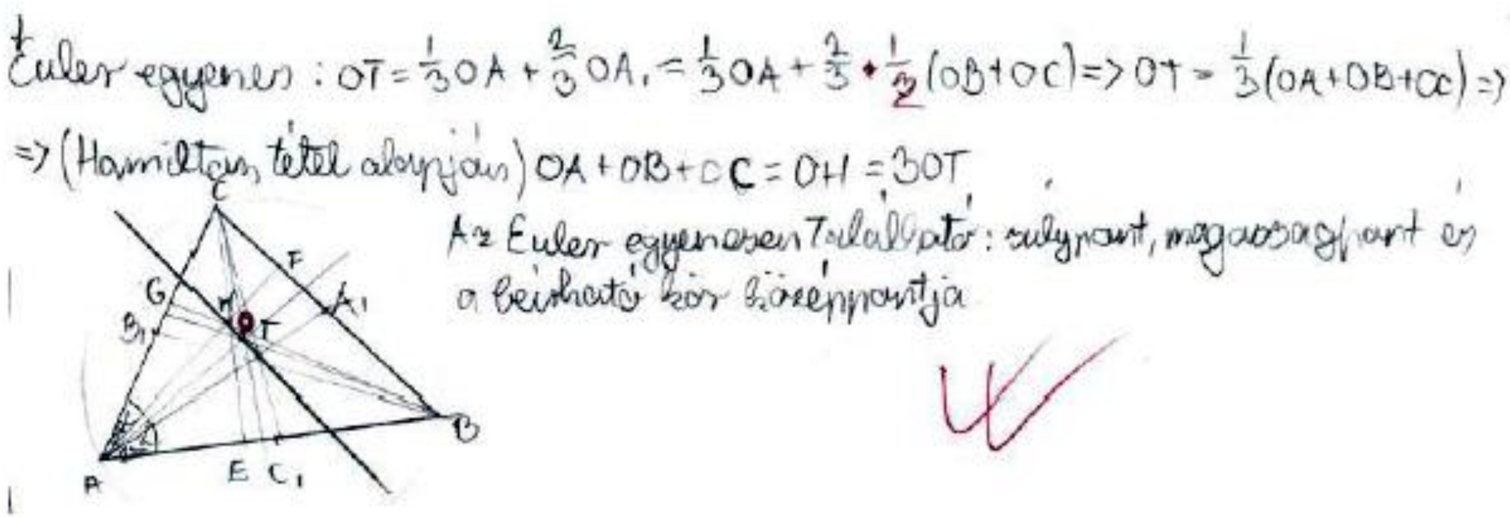
2. Construction on paper with technical pencil, rulers, bow

- Construction
- Control
- Psichomotorical aim



Geometry 1
Planimetry

3. Systematization and repetition (teorethical proof)



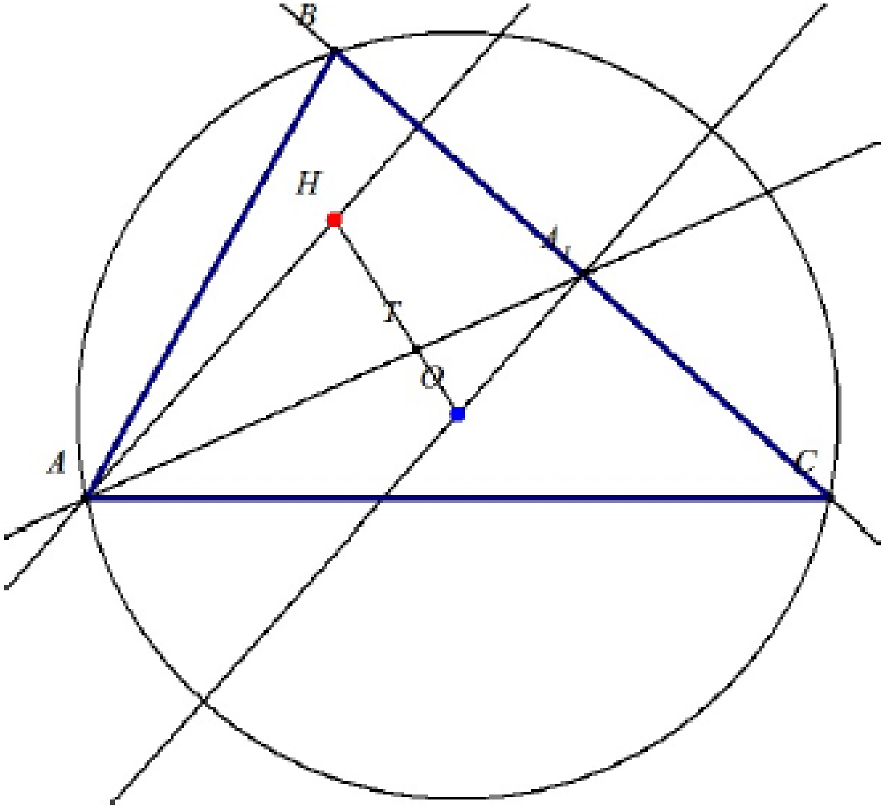
Euler egyenes: $OT = \frac{1}{3}OA + \frac{2}{3}OA_1 = \frac{1}{3}OA + \frac{2}{3}\cdot\frac{1}{2}(OB+OC) \Rightarrow OT = \frac{1}{3}(OA+OB+OC) \Rightarrow$

$\Rightarrow$ (Hamilton tétel alapján) $OA + OB + OC = OH = 3OT$

Az Euler egyenesen található: súlypont, magasságpont és a beírható kör középpontja

Triangle centers in constructions

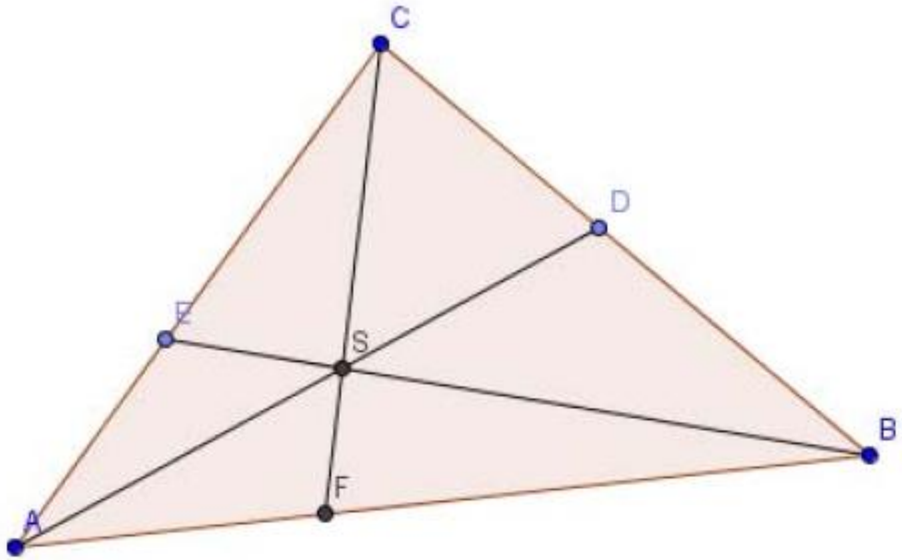
Given three points, one vertex, the orthocenter and the circumcenter of the triangle. Construct that triangle.



Didactically dual task

Geometry 2
Trigonometry

Ceva's theorem: Given a triangle ABC, and points D, E, and F that lie on lines BC, CA and AB respectively, than lines AD, BE and CF are concurrent if and only if



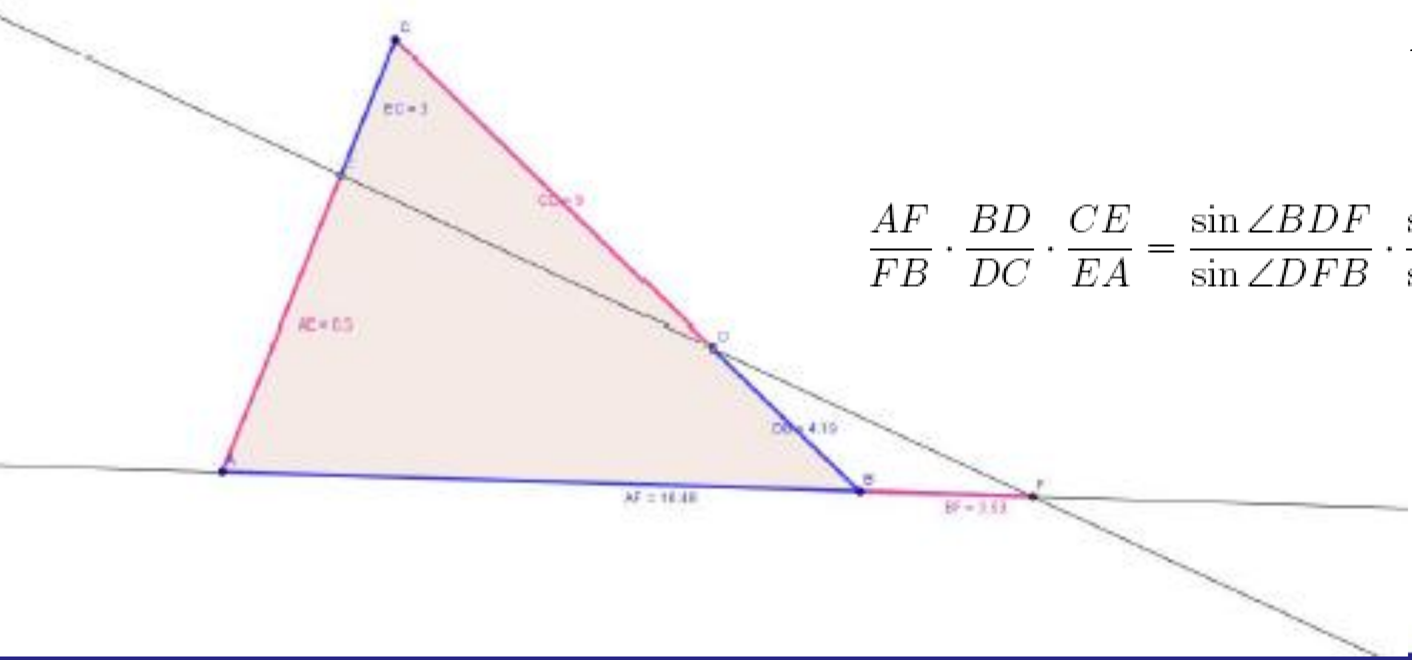
$$\frac{\overline{AF}}{\overline{FB}}\cdot\frac{\overline{BD}}{\overline{DC}}\cdot\frac{\overline{CE}}{\overline{EA}}=1$$

$$\frac{AF}{FB}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA}=\frac{\frac{AS\cdot\sin\angle ASF}{\sin\angle AFS}}{\frac{BS\cdot\sin\angle BSF}{\sin\angle BFS}}\cdot\frac{\frac{BS\cdot\sin\angle BSD}{\sin\angle BDS}}{\frac{CS\cdot\sin\angle CSD}{\sin\angle CDS}}\cdot\frac{\frac{CS\cdot\sin\angle CSE}{\sin\angle CES}}{\frac{AS\cdot\sin\angle ASE}{\sin\angle AES}}=1.$$

Geometry 2
Trigonometry

Menelaus' theorem: Given points A, B, C that form triangle ABC, and points D, E, F that lie on lines BC, AC, AB, points D, E, F are collinear if and only if

$$\frac{\overline{AF}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} = -1$$



$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = \frac{\sin \angle BDF}{\sin \angle DFB} \cdot \frac{\sin \angle CED}{\sin \angle CDE} \cdot \frac{\sin \angle DFA}{\sin \angle FEA} = 1$$

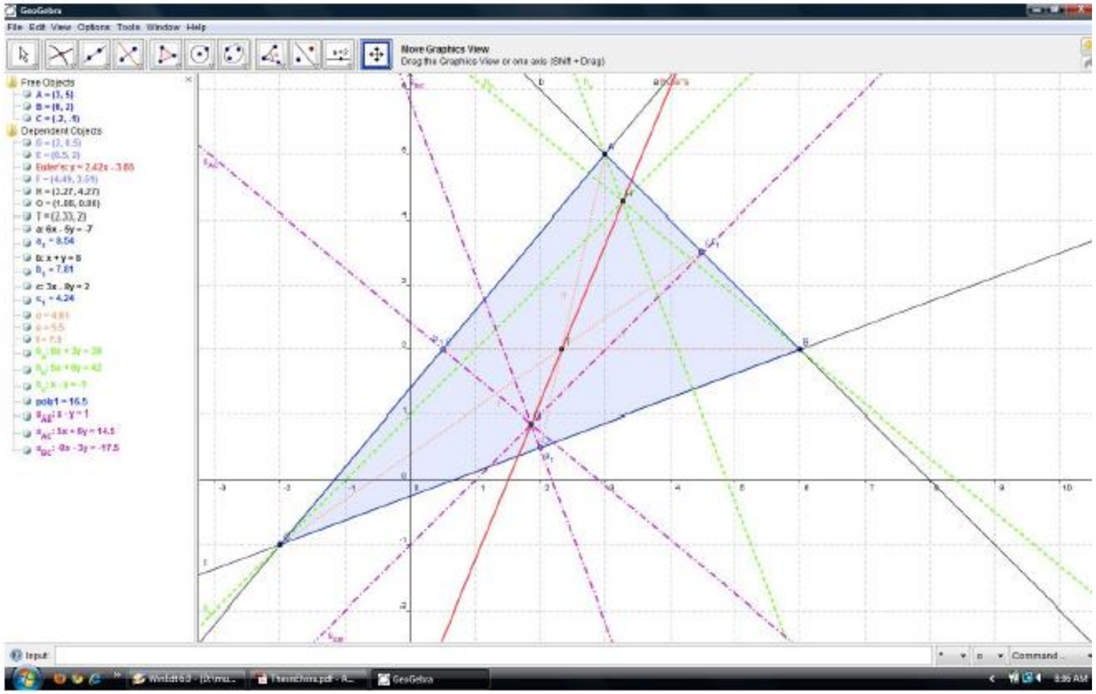
Geometry 3
Analitical geometry

Given three vertices of the triangle: A(3; 5), B(6; 2), C(−2;−1).
Find the equation of the Euler's line.

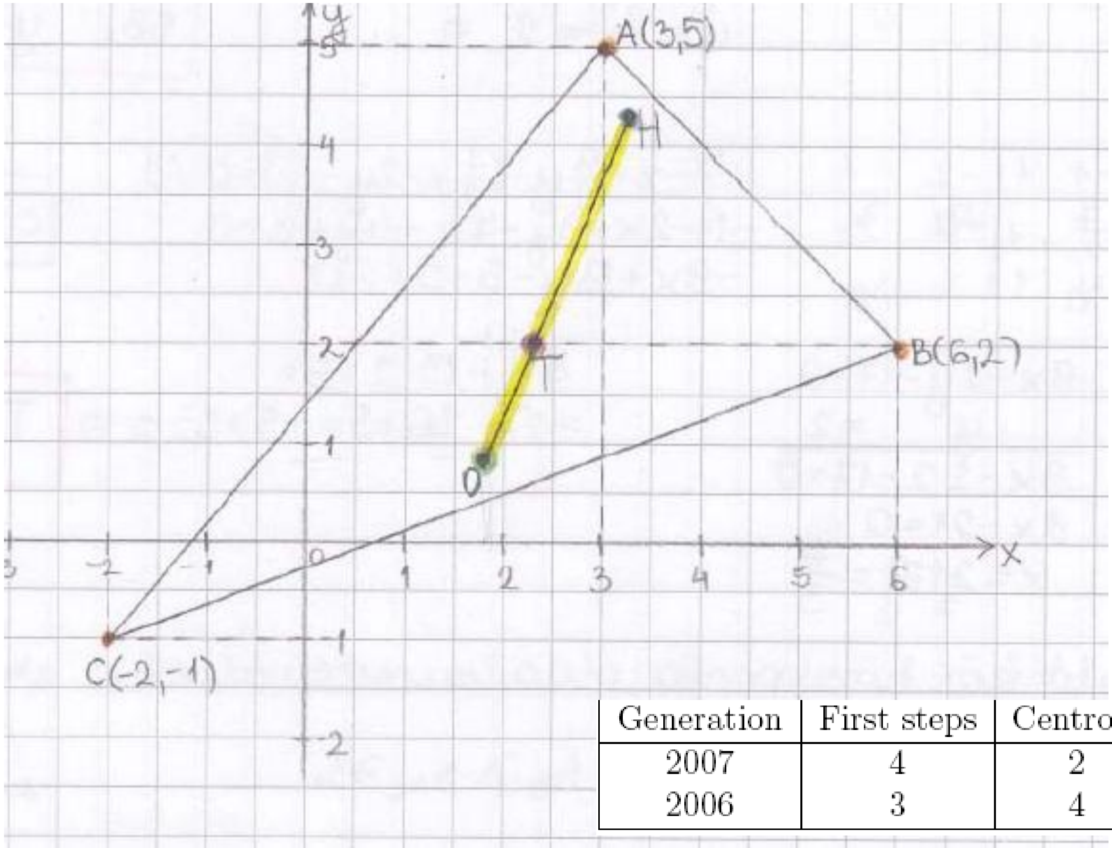
1. the equations of the lines AB, BC and AC;
2. the altitudes and the orthocenter of the triangle;
3. the medians and the barycenter;
4. the perpendicular bisectors and the circumcenter
5. the equation of the line HO (or HT or OT) and the verification of whether the third point belongs to that line;
6. the coordinates of vectors $\overrightarrow{HT}$ and $\overrightarrow{TO}$;
7. checking the ratio of vectors $\overrightarrow{HT}$ and $\overrightarrow{TO}$.

Geometry 3
Analitical geometry

Given three vertices of the triangle: A(3; 5), B(6; 2), C(−2;−1).
Find the equation of the Euler's line.



Geometry 3
Analitical geometry



Generation	First steps	Centroid	Circumcenter	Euler's line
2007	4	2	2	10
2006	3	4	5	8

Geometry 3
Analitical geometry

Euler's line

If triangle ABC has vertices $A(a_1, a_2)$, $B(b_1, b_2)$ and $C(c_1, c_2)$, then the edges of the triangle are:

$$AB = \mathrm{Det}\left[\begin{pmatrix} a_1 & b_1 & x \\ a_2 & b_2 & y \\ 1 & 1 & 1 \end{pmatrix}\right]$$

$$AC = \mathrm{Det}\left[\begin{pmatrix} a_1 & c_1 & x \\ a_2 & c_2 & y \\ 1 & 1 & 1 \end{pmatrix}\right]$$

$$CB = \mathrm{Det}\left[\begin{pmatrix} c_1 & b_1 & x \\ c_2 & b_2 & y \\ 1 & 1 & 1 \end{pmatrix}\right]$$

$$\mathrm{Solve}[(b_1 - c_1)\, a_1 + (b_2 - c_2)\, a_2 + n == 0,\ n]$$

$$\{\{n \to -a_1 b_1 - a_2 b_2 + a_1 c_1 + a_2 c_2\}\}$$

$$\mathrm{Solve}[(a_1 - c_1)\, b_1 + (a_2 - c_2)\, b_2 + m == 0,\ m]$$

$$\{\{m \to -a_1 b_1 - a_2 b_2 + b_1 c_1 + b_2 c_2\}\}$$

General form:

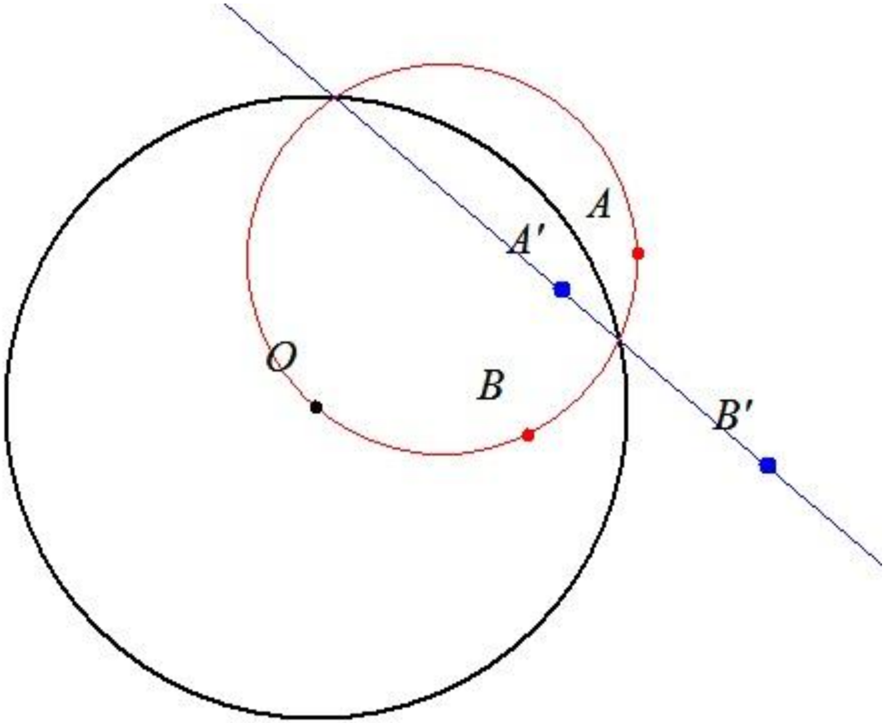
Given three vertices $A(a_1,a_2)$, $B(b_1, b_2)$, $C(c_1, c_2)$ of the triangle *ABC*.

Find the equation of the Euler's line

Geometry 1 examines :

- Isometric transformations
- Transformation of similarity
- Inversion
    - Apollonius’ problems
    - Poincaré’s model for non-Euclidean geometry

Inversion



1. The inverse of a circle (not through the center of inversion) is a circle.
2. The inverse of a circle through the center of inversion is a line.
3. The inverse of a line (not through the center of inversion) is a circle through the center of inversion.
4. A circle orthogonal to the circle of inversion is its own inverse.
5. A line through the center of inversion is its own inverse.
6. Angles are preserved in inversion.

Apollonius' problems

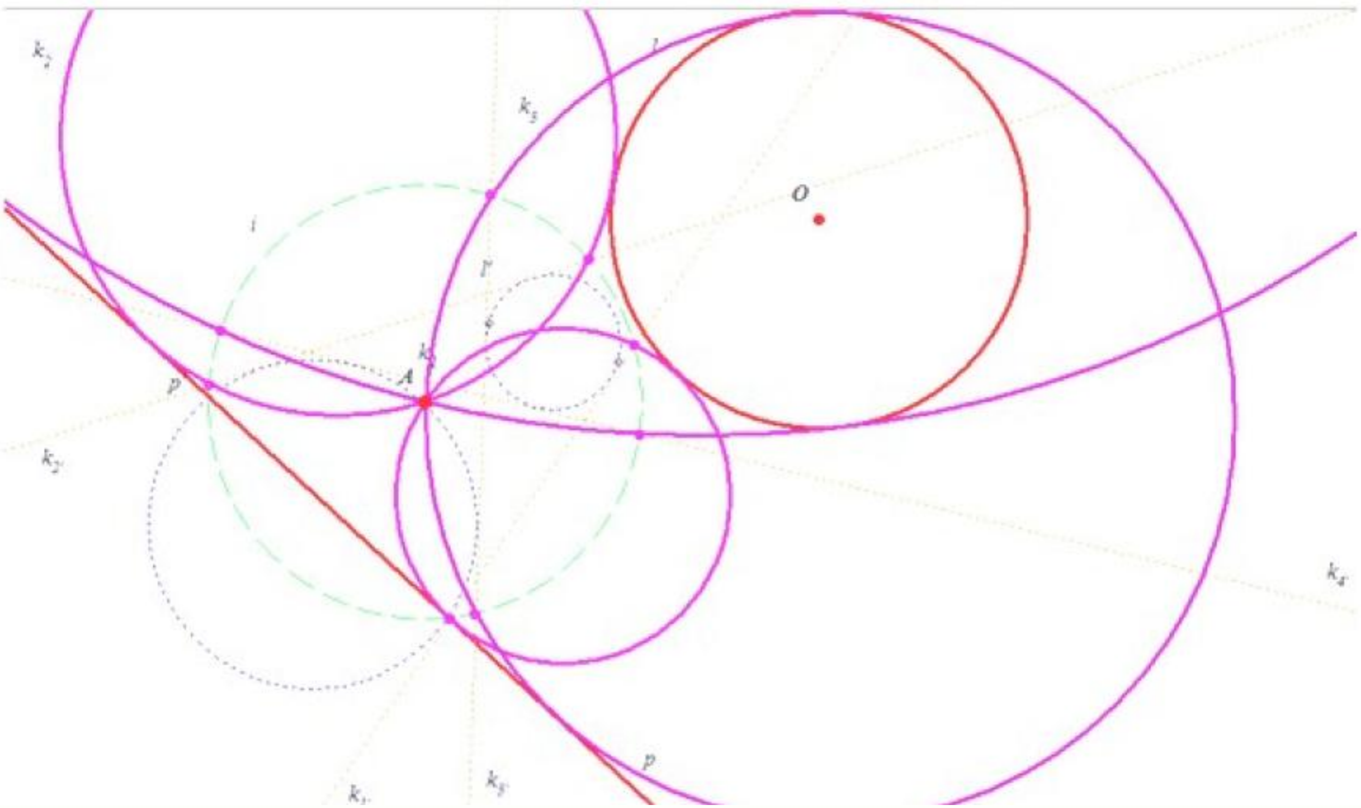
The basic problem is:

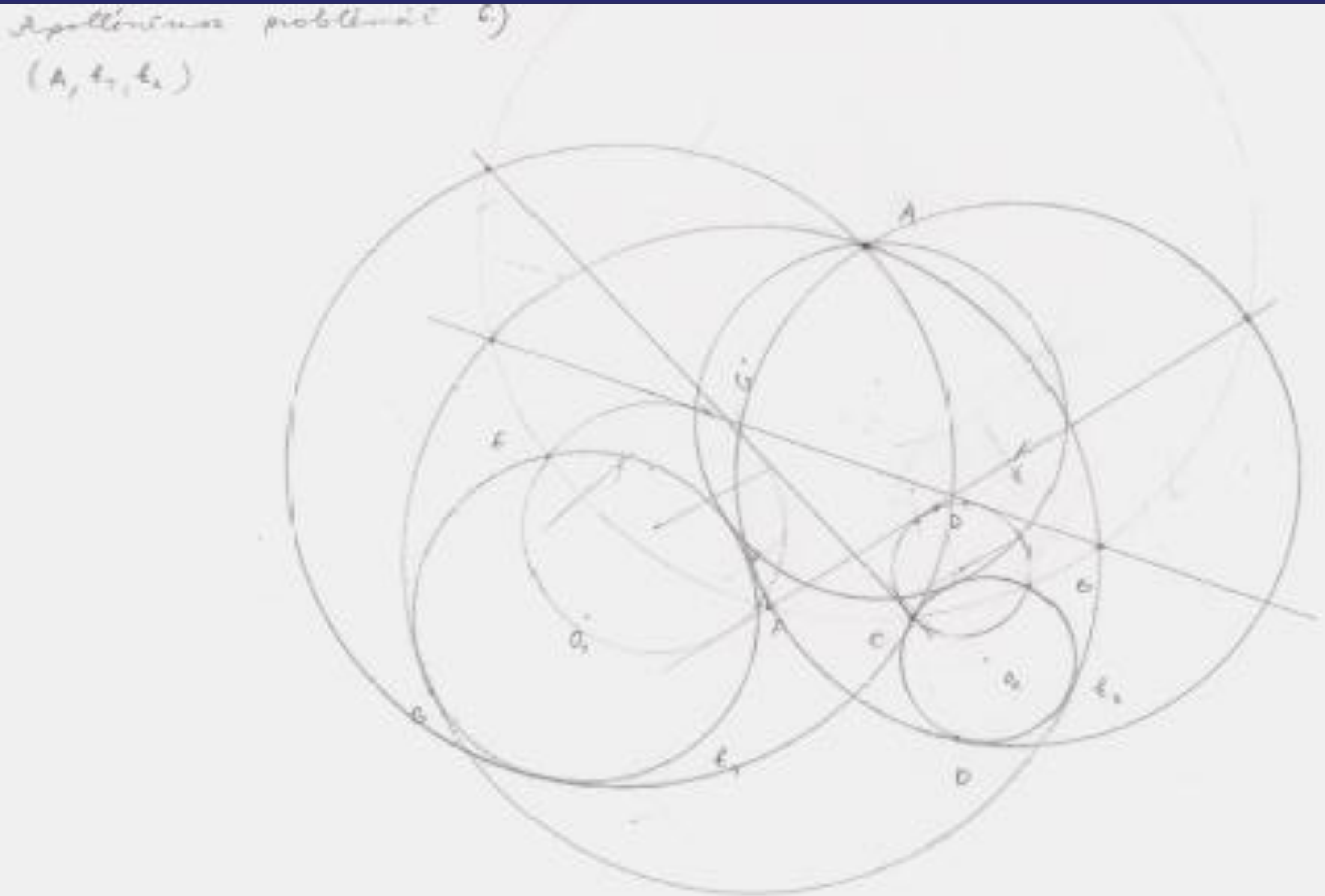
Given three objects, each of which may be a point, line, or circle, draw a circle that is tangent to each.

1. The circle crosses three given points (A, B, C).
2. The circle crosses two points and tangents a line (A, B, p).
3. The circle crosses two points and tangents a circle (A, B, k).
4. The circle crosses one point and tangents two given lines (A, p, q).
5. The circle crosses one point and tangents a lien and a circle (A, p, k).
6. The circle crosses one point and tangent two given circle (A, k_1, k_2).
7. The circle tangents three given lines (p, q, r).
8. The circle tangents two lines and a circle (p, q, k).
9. The circle tangents a line and two circles (p, k_1, k_2).
10. The circle tangents three given circles (k_1, k_2, k_3).

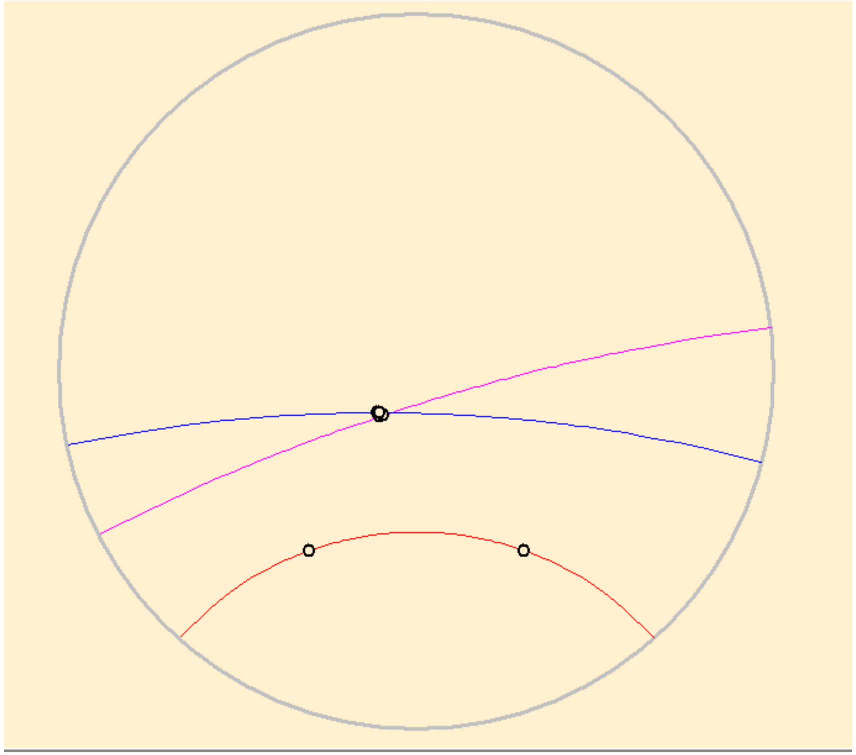
Apollonius' problems

5. THE CIRCLE ACROSS ONE POINT A TOUCHES THE GIVEN LINE p AND THE GIVEN CIRCLE l





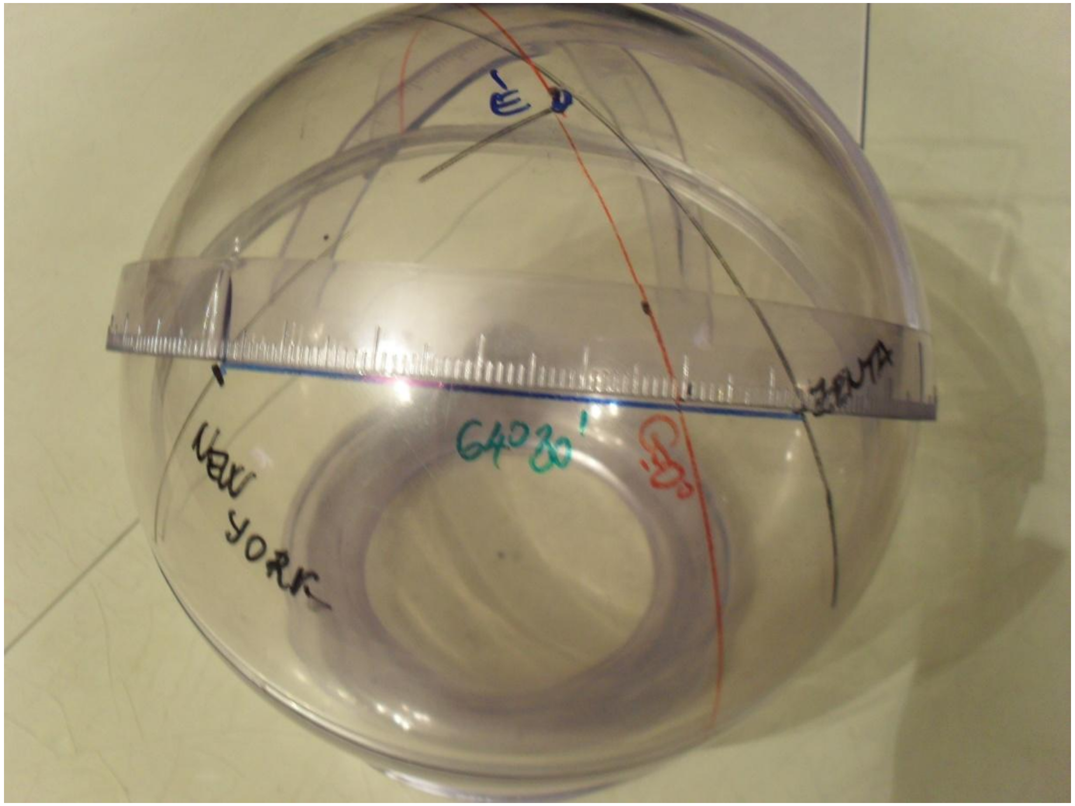
Non-Euclidean Geometry
Hyperbolic geometry



<u>Playfair's axiom:</u> Given a line and a point not on it, at most one parallel to the given line can be drawn through the point.

<u>Bolyai János-Nikolai Lobachevsky's axiom:</u>
Given a line and a point not on it, at least two parallel to the given line can be drawn through the point.

Non-Euclidean Geometry
Spherical geometry



Lénárt Sphere

Senta

Latitude 45° 55’ 45,48’’N

Longitude 20° 5’ 10,18’’ E

New York

Latitude 41 8′ 44″ N

Longitude 73 59′ 42″ W

Results and Comments

This visualization in experimental geometry helps to:

- develop/improve spatial and perception skills;
- increase intuitive skills, gain insight;
- predict theorems and the properties of geometrical figures, discovering new patterns and relations;
- increase divergent thinking and the checking of new ideas;
- recognize "visible" proofs and suggest approaches for formal proof;
- motivate students' active participation;
- increase the students' enthusiasm.

Results and Comments

The disadvantages of computer aided teaching are :

- a decrease of desire to prove the theorems;
- the deficiency of mathematical rigorousness since "everything is visible on the drawing";
- some students find the computer difficult to use therefore they become frustrated;
- it is still expensive for schools.

Thank You very much