EUROPEAN CONSORTIUM FOR MATHEMATICS IN INDUSTRY

ECMI Modelling Week 2018
University of Novi Sad, Faculty of Sciences

MODELLING DRILLING CUTTINGS FORMATION

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Contents

1 Introduction

To make off-shore resources accessible, the analysis of processes during well drilling is needed to maximize the drilling speed while ensuring the safety of the equipment. The most influential system parameters are the bit diameter and type, the drilled soil type, the rotation speed of the drill string and pressure applied on the bit. For a given set of parameters the goal is to estimate the drilling speed and the distribution of the particle size in the drilling mud.
Due to the lack of data on the mentioned parameters, our goal is to verify that a model of the particle distribution during a well drilling process can be used to describe the phenomena observed in Yi et al. 2013 and Saasen, Dahl, and Jødestøl 2013.

2 Report content

Fragmentation is a continuous and irreversible kinetic process with important applications in nature and technology. These include liquid droplet breakup, polymer degradation, crushing or grinding of rocks, etcetera. Although there are two main approaches to describe this process. The first one is connected with fracture mechanics (for instance Rittinger's low). The second approach comes from statistics as an enormous amount of different abandoned factors which cannot be taken into account as for example local fluctuations of density that are varied even for one layer of the formations beds. A basic goal of paper of Cheng and Redner 1990 is to provide a general theoretical description of the evolution of the distribution of fragment sizes that arises from such breakup process. A basic theoretical approach to describe this process is by the rate equations. These are an approximation of a mean-field character, as fluctuations are ignored. Thus the mass is the only dynamical variable that characterizes a given fragment in the rate equation approach. At all times throughout the system fragments are assumed to be distributed homogeneously, i.e. there is perfect mixing, and the variability of cluster shapes is ignored as well. According to Cheng and Redner 1990 a primary goal of this paper is to insure simulation model for such breakup process.

For the discussion of the kinetics of continuous, irreversible fragmentation processes the results shown in Cheng and Redner 1990 are used. The presented results provide a scaling theory to describe the evolution of the cluster size distribution.

Based on the scaling theory by Cheng and Redner 1990 a numerical solution, using finite differences, is implemented in MATLAB. To prove that this model describes the drilling process accurately its results need to be compared with real data.
Since the problem focuses on off-shore drilling, accessing reliable data is extremely difficult. Therefore [cuttingsizedistribution2] published a new approach to measure the distribution of the particle size while the actual drilling takes place. The standard method for analyzing the characteristics of the debris still is to bring the material into a laboratory on shore and seize it with different mesh sizes. Using that standard method,

[cuttingsize1] published the particle size distribution of a drilling in a particular oilfield in several depths. This data is used to estimate the free parameters of the implemented MATLAB model.

2.1 Mathematical problem formulation

$$\frac{\partial m(x;t)}{\partial t} = -\lambda(x)m(x;t) + \int\limits_{x}^{M_0} f(x|y)\lambda(y)m(y;t)dy \tag{1}$$

As shown in equation 1 the drilling process can be described with a partial differential equation as a branching process.

$$\lambda(x) \text{ - Overall rate at which x breaks} \tag{2}$$

$$f(x|y) \text{ - Conditional probability that the breakup of y produces an x-mer} \tag{3}$$

Equation 1 can be summarized by two separate processes. The term $-\lambda(x)m(x;t)$ describes the breakup of x-mers, which therefore disappear. The term $\int_x^{M_0} f(x|y)\lambda(y)m(y;t)dy$ describes the gain of x-mers due to the breakup of particles with mass larger than x.

Further assumptions and descriptions for equation 1 are shown in table 1.

$m(x;0) = m_{init}(x)$	Initial mass distribution
$\int_0^{M_0} m(x,t)dx = M_0, \quad \forall t \geq 0$	Conservation of mass
$\lambda(x) = 1[x > x_{min}]\lambda_0 x^{\mu}, \ \ (\mu \geq 0)$	Intensity of fragmentation
x_{min}	Smallest particle that can be fragmentized
$f(x\|y) = \gamma \frac{x^{\gamma-1}}{y^{\gamma}}, \quad (\gamma > 1)$	Conditional probability density function (Describing how particles of mass y will be distributed after their break up)

Table 1: Assumptions

2.2 Implementation

Equation 1 was solved numerically in MATLAB. Both in time and space, the discretization was uniform in order to facilitate calculations. A typical amount of time steps was 100 and a typical amount of space steps was 10000. The equation 1 was solved by using a basic forward Euler method in time. This was chosen for simplicity as the equation is quite simple itself. The explicit steps in the implementation were as follows. At first there was an initial mass distribution $m(x;0)$, then for every time step the next distribution was computed as:

$$m(x_j, t_i) = m(x_j, t_{i-1}) + dt\frac{\partial m(x_j; t_{i-1})}{\partial t}$$

where dt is the size of the time step and $\frac{\partial m(x_j;t_{i-1})}{\partial t}$ comes from Equation 1, i.e.

$$\frac{\partial m(x_j; t_{i-1})}{\partial t} = -\lambda(x_j)m(x_j; t_{i-1}) + \int_{x_j}^{M_0} f(x_j|y)\lambda(y)m(y; t_{i-1})dy.$$

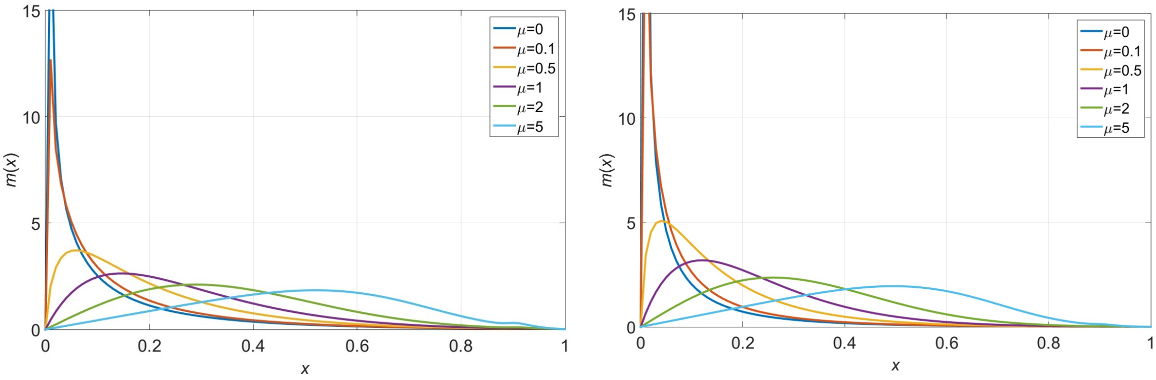
The integral in the above expression was calculated using the trapezoidal method.

3 Discussion and conclusions

The first thing one may note is that our solution i.e. using the forward Euler method, is known to be unstable for certain classes of differential equations and that an interesting thing to add would have been an implementation of for example a backward Euler method. In what follows we will disregard this.

First experiments were made to examine the parameters. The following figures show the mass distribution for different μ values. If not stated differently, x_{min} was 0.01 and $M_0 = 1$.

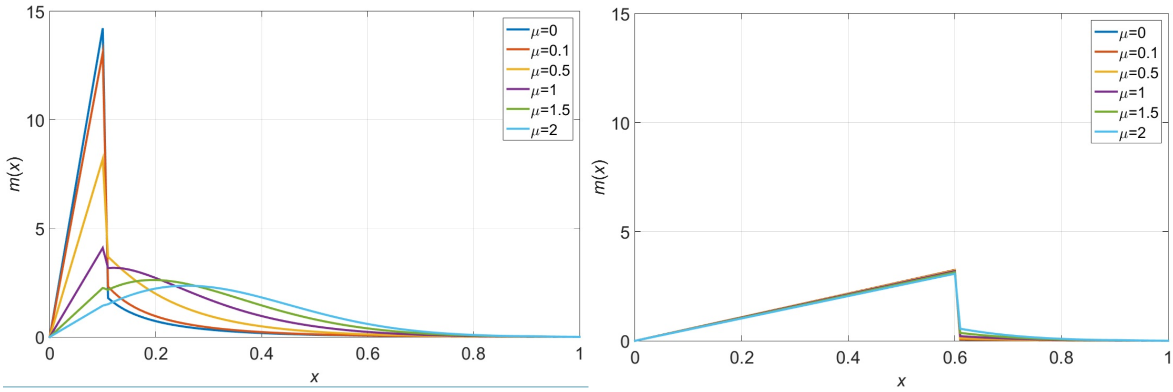
On Figures 1 and 2 the main conclusions regarding are parameters λ_0 and x_{min} are presented.



(a) $\lambda_0 = 40, t = 0.2s$ (b) $\lambda_0 = 1, t = 10s$

Figure 1: λ_0 changes the intensity linearly, it works as time

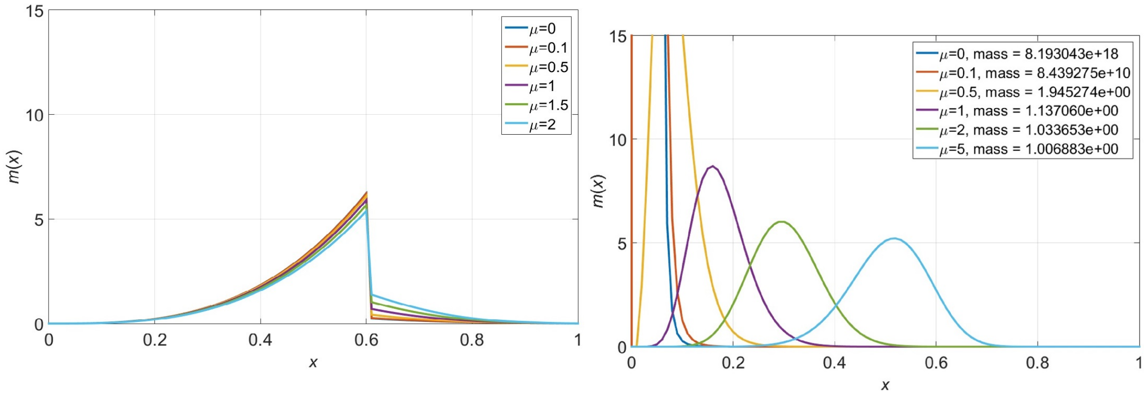
We weren't able to assess the meaning of parameters μ and γ when simulating for different values. Therefore, changing our strategy, we decided to compare the obtained simulations



(a) $x_{min} = 0.1, \gamma = 2, t = 10s$

(b) $x_{min} = 0.6, \gamma = 2, t = 10s$

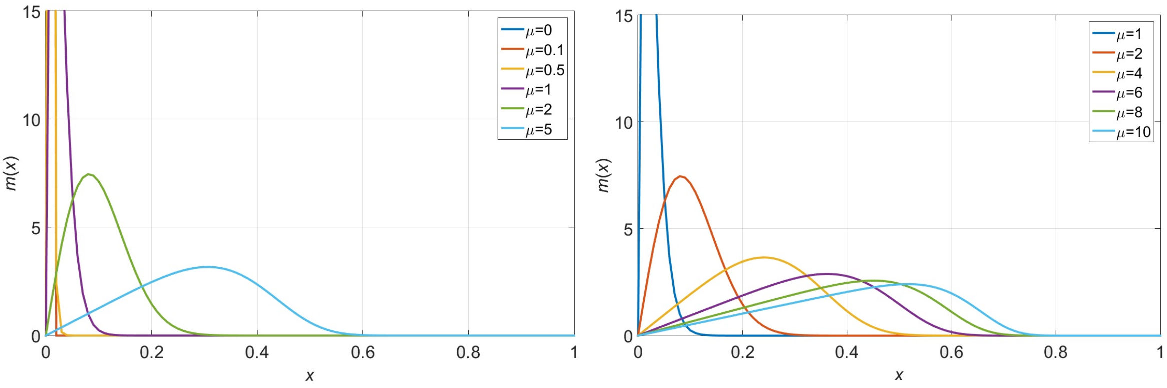
Figure 2: Large x_{min} will create a spike in the distribution, as the process does not divide particles smaller than x_{min}



(a) $x_{min} = 0.6, \gamma = 3, t = 10s$
the distribution is a power function before x_{min} in γ

(b) $\gamma = 10, t = 50s$
the simulation is not stable, mass created

Figure 3: The behavior of γ



(a) $t = 100s, \gamma = 2, \mu = 0 \dots 5$

(b) $t = 100s, \gamma = 2, \mu = 1 \dots 10$

Figure 4: Increasing μ will decrease the rate of the process (extremely long time taken here)

with the particle size distribution presented onYi et al. 2013. The methodology chosen was the following:

1. Simulate observations from the distribution suggested by Yi *et al.* regarding a certain depth, type of soil and bit;
2. Within the same shape as presented on the paper, plot an histogram of the simulated observations (*green*) and the histogram of the final particle size distribution of the simulation performed (*blue*).

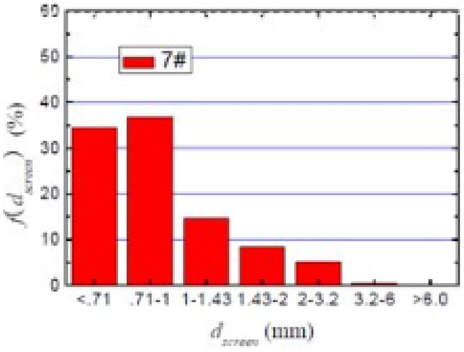
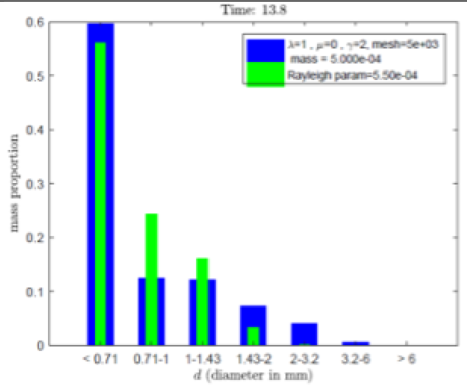
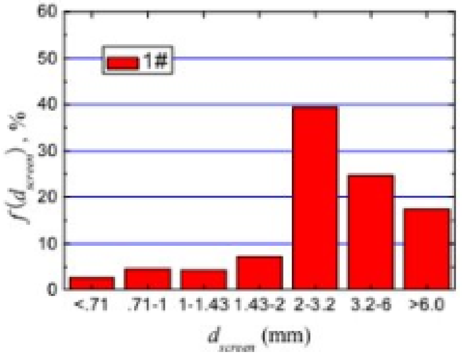
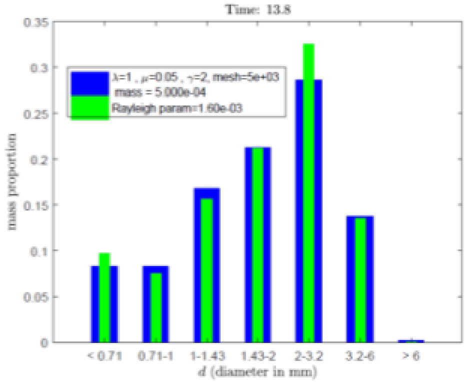
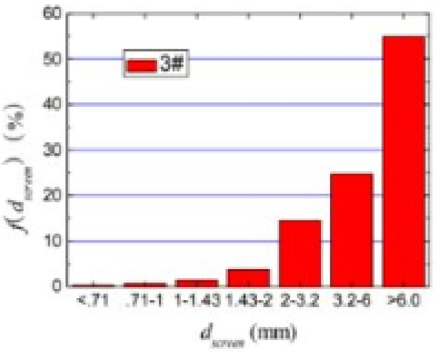
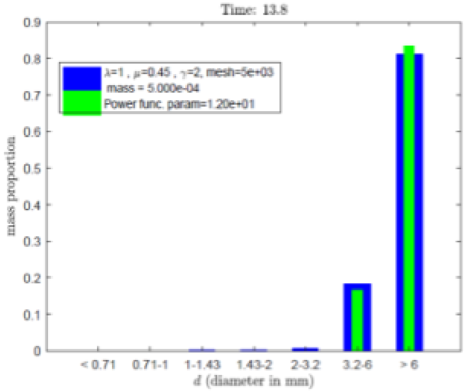
Depth	Results by Yi *et al.*	Simulations
<2000m		
2000-2800m		
>2800m		

Table 2: Parameter μ associated to depths $\leq$ 2800m and parameter γ to depths >2800m.

The main results obtained are presented on Table 2.

Although we obtained approximately similar results, we highlight the fact that the size particle scale chosen to report the results on Yi et al. 2013 wasn't the best one. A non uniform scale size might suggest a biased visualization such that the particle distribution is approximately the one mentioned in the study. Note that no statistical tests were performed to assess the goodness-of-fit of the simulations obtained neither of the suggested distributions by *Yi et al.*.

Hence, the conclusions stated above regarding parameters μ and γ might be valid under certain restrict assumptions, which accordingly to the scale proposed appear to be *valid*. Further experiments would need to be made in order to have more feasible comparisons.

4 Group work dynamics

The activity commenced with the presentation of the problem and some further explanations carried out by the group's instructor. It was then proceeded by the self-introduction of each member of the group.

After being oriented with the problem, we then decided to try to get a better understanding of our assigned task by reading through some relevant articles that were suggested by our instructor.

A brief discussion followed and each group member tried to implement a solution for the PDE using MATLAB. A big problem at this point was the unfamiliarity of all group members with PDE:s in general and numerical solutions of PDE:s in particular. The first solution that was implemented was that of Georg Bökman's, who then expounded his approach to our instructor and to the rest of the group.

Since our instructor approved of the results that were generated by his implementation, we started working in groups to further validate the model and to come up with a more in-depth analysis of both the problem and the solution. We divided our group into two. One group is responsible for identifying a more reliable data that can be tested at the model, and the other to find parameters that can reproduce the data that was already on hand.

The last 1.5 days were spent on creating the final presentation. The workload was split amongst the 5 members. While working on our respective task, we kept each other updated and informed via email. Thus ensuring that our final report covers our work and experience during the ECMI modelling week rigorously. We have also decided to equally split our talk during the presentation, with each member to discuss their respective task in the project.

Working in a group with five different nationalities (six, including our instructor) was both challenging and beneficial. For a group to tackle a certain issue whilst making sure that each member understands and are on the same page is challenging enough to discuss in their own native dialect/language. Therefore, it was a far greater challenge that we needed to discuss the issues using only our second common language, English.

And considering as well that we are not so familiar with each other yet since we have only been introduced a few days back.

This challenge entailed a great benefit for us, both as an individual member and as a part group. We get to learn ways on to how to express our views and to discuss an idea. Plus we got to know each other quite well in a limited amount of time.

5 Instructor's assessment

The regarded modeling problem proves to be a quite complex issue as it requires extensive background work and knowledge from different fields. Furthermore, there is not enough experimental data to evaluate the model thoroughly. These peculiarities formed the way the group worked during the week (literature review, implementation of a found approach, tuning etc.).

Although time was short and the participants had different educational backgrounds, the group worked towards their own approach and did it in a rather independent manner. The resulted model reflects many of processes involved in drilling and can be improved if sufficient resources and time are available.

References

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Saasen, Arild, Bjørn Dahl, and Kjell Jødestøl (2013). “Particle size distribution of top-hole drill cuttings from Norwegian sea area offshore wells”. In: *Particulate science and technology* 31.1, pp. 85–91.

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