Synchronizing inventory and transport within supply chain management (territorial approach)

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1 Introduction

Infora Research Group presented a problem of optimizing the distribution costs of cigarettes from a single production facility placed in Senta to approximately 15000 shops in Serbia. Current solution includes 5 warehouses (located at Novi Sad, Beograd, Požarevac, Kragujevac, Niš). The cigarettes are transported from Senta to warehouses with big rented trucks paid on the bases of distance from Senta.

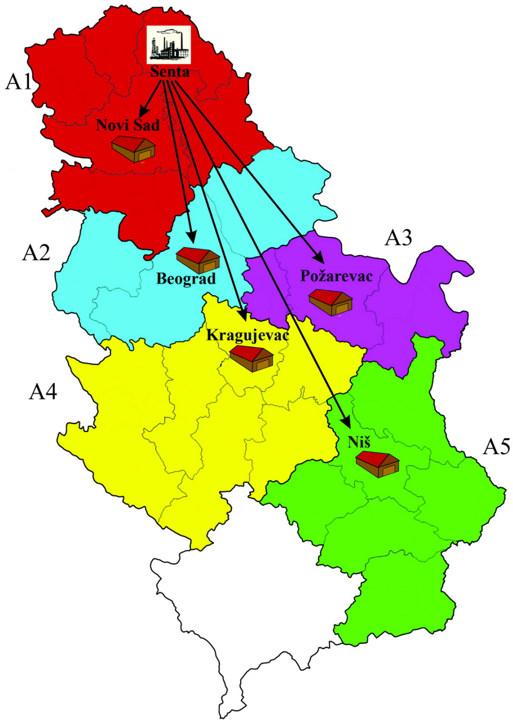

Figure 1: Placement of warehouses and division of territories into areas.

Cigarettes are stored in warehouses wherefrom smaller trucks, owned by distribution company, distribute them to cross-docking points where they are reloaded to several smaller vans which distribute them further to shops. Current number of cross-docking points covered by one warehouse is 5 in average. Vans can also be loaded at the warehouse with no extra renting cost and they supply shops near the warehouse.

The industry representative expected to get a mathematical model which can optimize the overall distribution costs by appropriate placement of warehouses and cross-docking points.

Major drawback was absence of reliable real data for van routes and shop demands. From the communication with the industry representative it turned out that there are big chances that reliable data on shop level could not be obtained in practise. Nevertheless, demand data for territories of the size of municipalities or similar could be obtained from historical data. Based on constraints mentioned above, our study group decided to develop the mathematical model which needs only demand data from a number of territories connected with the road network. The model is proposed with the current territorial division of Serbia into regions,

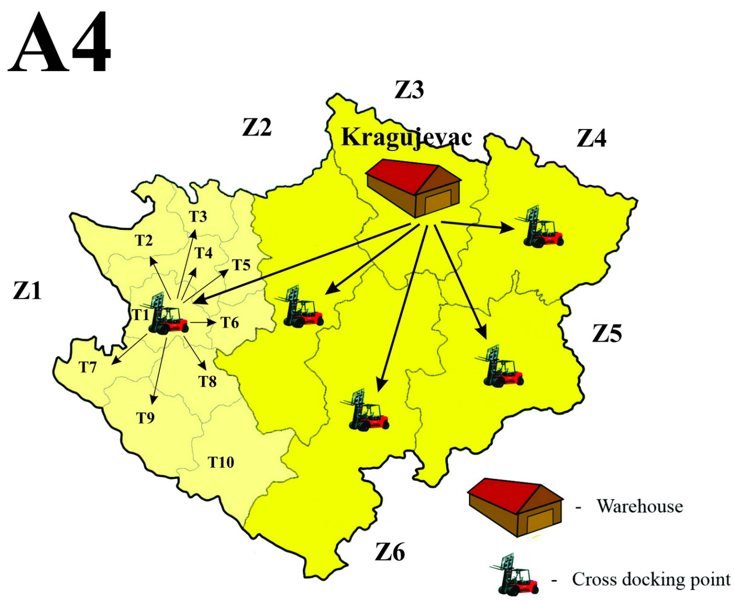

Figure 2: Placement of cross-docking points and division of territories into zones.

districts and municipalities in mind. Although municipalities are natural candidates for territories in the proposed model, smaller territories could sometimes be more appropriate.

2 Notation

- T - the set of territories. The whole market is divided into smaller territories T_i, for instance municipalities or smaller, such that demand data are known for that territory. Territories are organized as a graph with known distances between each two territories $d(T_i, T_j)$.
- W - the set of all possible places for warehouses W_k. For each possible warehouse place W_i, $T(W_i)$ denotes the territory on which it resides, R_{W_i} is overall renting cost for it over a unit time period. Renting cost may be a function of total demand covered by the warehouse.
- P - the set of all possible places for cross-docking points P_j. For each possible point P_j, $T(P_j)$ denotes the territory on which it resides, R_{P_j} is overall renting cost for it over a unit time period.
- D_{T_i} - known demand for each territory T_i over a unit period of time.

- L_i - overall distribution costs to individual shops on territory T_i under assumption that there is a cross-docking point on that territory.
- c_b, c_s, c_v - cost of transport per unit distance and unit quantity of goods for big trucks (from production facility to warehouses), small trucks (from warehouses to cross-docking points) and vans (from points to shops). Assumption is that all vehicles are reasonably well loaded so that this cost can be used as a constant in practise.

3 Modelling the total cost of distribution

For a fixed combination of warehouses $W_1, \ldots, W_{kmax}$ the set of all territories can be divided into areas $A_1, \ldots, A_{kmax}$ with respect to closeness to a warehouse based on known distances $d(T_i, T(W_k))$ (Fig. 1). It is assumed that all territories in area A_k are supplied through the warehouse W_k.

Within one area A_k there is a set of cross-docking points $P_1, \ldots, P_{jmax}$, each point P_j placed on territory $T(P_j)$. All territories from the area are divided into zones $Z_1, \ldots, Z_{jmax}$ with respect to closeness to points P_j based on known distances $d(T_i, T(P_j))$ (Fig. 2).

For fixed positions of warehouses and cross-docking points, and corresponding partition into areas and zones, total cost of distribution could be calculated in the following way. First, knowing the demands D_i for each territory T_i, by summation we obtain the total demands D_{P_j} for each zone Z_j with cross-docking point P_j, and total demands D_{W_k} for each area A_k with warehouse W_k.

On single territory T_i total cost of distribution is L_i in case there is a cross-docking point on T_i. In the other case, when the nearest point P_j is on another territory, cost can be approximated as $L_i + d(T_i, T(P_j)) \cdot D_{T_i} \cdot c_v$.

Total cost of distribution from one cross-docking point P_j on its zone Z_j is

$$C_{P_j} = R_{P_j} + \sum_{T_i \in Z_j} (L_i + d(T_i, T(P_j)) \cdot D_{T_i} \cdot c_v) = \sum_{T_i \in Z_j} L_i + C^*_{P_j}$$

where the renting cost $R_{P_j} = R_{P_j}(D_{P_j})$ can depend on the demand, and

$$C^*_{P_j} = R_{P_j} + \sum_{T_i \in Z_j} d(T_i, T(P_j)) \cdot D_{T_i} \cdot c_v$$

does not depend on local costs L_i.

Total cost of distribution from one warehouse W_k on its area A_k is

$$C_{W_k} = R_{W_k} + \sum_{P_j \in A_k} (C_{P_j} + d(T(P_j), T(W_k)) \cdot D_{P_j} \cdot c_s) = \sum_{T_i \in A_k} L_i + C^*_{W_k}$$

where the renting cost $R_{W_k} = R_{W_k}(D_{W_k})$ can depend on the demand, and

$$C^*_{W_k} = R_{W_k} + \sum_{P_j \in A_k} (C^*_{P_j} + d(T(P_j), T(W_k)) \cdot D_{P_j} \cdot c_s)$$

does not depend on local costs L_i.

Total cost of distribution is

$$C = \sum_{W_k} (C_{W_k} + d(T(Production), T(W_k)) \cdot D_{W_k} \cdot c_b) = \sum_i L_i + C^*$$

where

$$C^* = \sum_{W_k} (C^*_{W_k} + d(T(Production), T(W_k)) \cdot D_{W_k} \cdot c_b) \,.$$

Since the sum of local distribution costs $\sum_i L_i$ does not depend on number and positions of warehouses and cross-docking points, optimization of total cost function C is equivalent to optimization of C^*. The function C^* does not contain local costs L_i, so using this model these data are not needed.

Since the renting costs C_{W_k} for the warehouse W_k is allowed to depend on the total demand D_{W_k} it covers, costs of transport from central production facility in Senta to the warehouse, since it also depends on this demand, can be incorporated into the renting costs. This leads to the very similar model containing only warehouses and cross docking points, without the central production facility.

4 Optimizing the total cost of distribution

For optimization of the number and positions of warehouses W_k and cross-docking points P_j using the model described in previous section the data for local distribution costs L_i are not needed, only the data for demands D_i and territorial data of distances between territories. The territories could be organized as a weighted graph, each node representing one territory T_i with edge weights representing the distances. This graph could be assembled using the road map.

Optimization of the total cost function C^* can be organized in two levels. The first level optimizes the positions of warehouses on the map. For each proposed combination of warehouse positions, all territories are divided into areas A_k with respect to the nearest warehouse criteria. For each area A_k the second level of optimization provides the optimal number and positions of cross-docking points P_j on that area providing the minimal total cost $C^*_{W_k}$. These second level optimizations could be done in parallel. At the territory $T(W_k)$ where the warehouse is placed, the rental cost for cross-docking point should be put to zero, or this point should be fixed at warehouse position with other points being subject to optimization.

For this combinatorial optimization problem exact methods could be very time consuming, so heuristic approach could be more appropriate. Genetic algorithms, variable neighborhood search, simulated annealing, swarm optimization and other metaheuristics could provide good solutions. Constrains such as minimal or maximal demands and similar could be incorporated through penalty functions in the model.

The second level optimization problem is recognized in literature as a hub location problem ([1]). Similar class of problems ([2]) could be very efficiently solved using modern metaheuristics. Variable neighborhood search algorithm can solve problems with 100 nodes and 10 hubs for less than a second. The same problem can also be considered as the hub location problem with star network structure ([3]). Memetic algorithms (hybrid metaheuristic methods combining genetic algorithm and a local search procedure) are also very efficient for this type of problems ([4],[5]).

The model developed in previous section can be also considered as a hierarchical facility location problem ([6]). A mixed integer linear programming model for this problem is recently proposed in [7].

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