

# Generalized Functions Online Workshop

May 12<sup>th</sup>, 2023

## Book of Abstracts

**Elena Cordero**, 9:45 - 10:10

### **Symplectic Analysis of Time-Frequency Spaces**

We present a different symplectic point of view in the definition of weighted modulation spaces and weighted Wiener amalgam spaces. All the classical time-frequency representations, such as the short-time Fourier transform (STFT), the  $\tau$ -Wigner distributions and the ambiguity function, can be written as metaplectic Wigner distributions. Namely, time-frequency representations can be represented as images of metaplectic operators, which become the real protagonists of time-frequency analysis. In earlier works it was conjectured that any metaplectic Wigner distribution that satisfies the so-called shift-invertibility condition can replace the STFT in the definition of modulation spaces. We prove that shift-invertibility alone is not sufficient, but it has to be complemented by an upper-triangularity condition for this characterization to hold, whereas a lower-triangularity property comes into play for Wiener amalgam spaces. The shift-invertibility property is necessary: Rihacek and conjugate Rihacek distributions are not shift-invertible and they fail the characterization of the above spaces. We also exhibit examples of shift-invertible distributions without upper-triangularity condition which do not define modulation spaces. Finally, we provide new families of time-frequency representations that characterize modulation spaces, with the purpose of replacing the time-frequency shifts with other atoms that allow to decompose signals differently, with possible new outcomes in applications.

**Anabela Silva, 10:10 - 10:35**

**A study of a nonlinear fractional boundary value problem and the Adomian decomposition method**

In this talk, we consider a class of fractional boundary value problems with Caputo fractional derivative of order in  $(2; 3)$ . The existence and uniqueness of solutions are discussed and the Adomian decomposition method is proposed to obtain an approximation of the solution.

**Stevan Pilipović, 10:35 - 11:00**

**Short Time Fourier transform in spaces with Hörmander metrics**

We analyze the short time Fourier transform (STFT) in  $L^2(\mathbb{R}^{2n})$ , where  $\mathbb{R}^{2n}$  is endowed with the Hörmander admissible metric. Also we consider (STFT) and frames over  $\mathbb{R}^n$  with the Hörmander metric as well as the Wigner transform. This is done by the use of *Conf*-families

$$\varphi = \{\varphi_X, X \in \mathbb{R}^{2n} \text{ or } \mathbb{R}^n\}$$

The continuity properties of (STFT) on various spaces are also given.

**Dragana Jankov Maširević, 11:10 - 11:35**

**Modified Bessel functions and their connection with some probability density functions**

At the beginning we briefly define Bessel functions and, consequently, the modified Bessel functions. Applications of modified Bessel functions frequently occur in statistics, for instance, they are a constituting terms of the probability density function of the non-central  $\chi^2$  distribution and also the McKay Bessel distribution. Bearing in mind various applications of random variable distributed according to the McKay law in statistics and a great applications of the non-central  $\chi^2$  distribution, for example

in finance, estimation and decision theory, in mathematical physics and, among others, in communication theory, the appropriate cumulative distribution functions has been widely considered in mathematical literature. The aim of this talk is to present several new formulas for such cumulative distribution functions. The main advantages of these expressions are that they are given in terms of some special functions, which have computational advantages and a wide range of applications.

**Marianna Chatzakou**, 11:35 - 12:00

### **Logarithmic Sobolev inequalities on Lie groups**

We will discuss our results on a number of logarithmic inequalities on several classes of Lie groups, including our version of the Gross log-Sobolev inequality (with respect to a Gaussian, on the first stratum of the group, measure) which is shown in the setting of stratified Lie groups. Applications of these inequalities will also be presented. The talk will be based on joint works with A. Kassymov and M. Ruzhansky.

**Sekar Nugraheni**, 12:00 - 12:25

### **Natural definition of generalized holomorphic functions**

In this talk, we will introduce and discuss how to define a generalized holomorphic function (GHF) without already assuming the Cauchy-Riemann equations but using a natural definition of complex differentiability. The setting is that of the Robinson-Colombeau ring and generalized smooth functions, based on a more general "growth condition" formalized by the notion of gauge, compared to the usual Colombeau ring. We use two gauges to define a new notion of hyperlimit function and of little-oh to accomplish the goal. We expect that several classical theorems of differential and integral calculus can be extended from the ordinary holomorphic case to the generalized holomorphic framework, and we present the results obtained so far.

**Sumandeep Kaur, 12:25 - 12:50**

### **Prime common index divisor of certain fields**

Let  $K = \mathbb{Q}(\theta)$  be an algebraic number field with  $\theta$  a root of an irreducible quintic polynomial of the type  $x^5 + ax + b \in \mathbb{Z}[x]$ . If  $\text{ind } \theta$  denotes the index of the subgroup  $\mathbb{Z}[\theta]$  in  $A_K$  and  $i(K)$  stand for the index of the field  $K$  defined by

$$i(K) = \gcd\{\text{ind } \alpha \mid K = \mathbb{Q}(\alpha), \alpha \in A_k\}.$$

A prime number  $p$  dividing  $i(K)$  is called a prime common index divisor of  $K$ . In this talk, for every rational prime  $p$ , we provide necessary and sufficient conditions on  $a, b$  so that  $p$  is a common index divisor of  $K$ . In particular, we give sufficient conditions on  $a, b$  for which  $K$  is non-monogenic.

### **Panel discussion on the Gender gap in science**

14:00-15:00

The panel will be led by Professor Marie-Françoise Roy. She is one of the founders of European Woman in Mathematics (EWM), convener of EWM 2009-2013, Chair of CWM (IMU Committee for Women in Mathematics) 2015 -2022.

<https://www.mathunion.org/cwm/people/gallery/marie-francoise-roy>

Professor Marie-Françoise Roy will present results from the gender gap in science project "A Global Approach to the Gender Gap in Mathematical, Computing, and Natural Sciences: How to Measure It, How to Reduce It?" and will lead a discussion after that.

**Olena Atlasiuk, 15:45 - 16:10**

**On differential systems in Sobolev spaces with generic inhomogeneous boundary conditions**

We study linear systems of ordinary differential equations on a finite interval with the most general (generic) inhomogeneous boundary conditions in Sobolev spaces. We investigate the characteristic of solvability of inhomogeneous boundary-value problems, prove their Fredholm properties, and find the indices, the dimensions of the kernel, and the cokernel of these problems. Moreover, we obtained the necessary and sufficient conditions for continuity in the parameter of solutions to the introduced boundary-value problems in the Sobolev spaces. Some constructive examples are also presented.

**Hazal Yüksekaya, 16:10 - 16:35**

**A Kirchhoff-type wave equation with delay: asymptotic behaviour and global nonexistence**

In the eighteenth century, the first equations with delay were considered by brothers Leonard Euler and Bernoulli. By A. Myshkis and R. Bellman, systematical study started at the 1940s. Since 1960, there have been appeared many surveys on the subject. In the middle of 1990s, robust control of systems with uncertain delay was started and led to the "delay bloom" in the beginning of the twenty-first century. Time-delay systems are also named systems with aftereffect or dead-time, equations with deviating argument, hereditary systems, or differential-difference equations. They belong to the class of functional differential equations which are infinite-dimensional, as opposed to ordinary differential equations. In this work, we consider a Kirchhoff-type wave equation with delay. Under suitable conditions, we prove the asymptotic behaviour and global nonexistence results.

**Jasmina Veta Buralieva, 16:35 - 17:00**

### **Some Asymptotic Results for the Distributional Stockwell and Wavelet Transforms**

Asymptotic results characterizing the quasiasymptotic behavior of Lizorkin distributions in terms of their Stockwell transforms are provided. An Abelian-type result relating the quasiasymptotic boundedness of Lizorkin distributions to the asymptotic behavior of their Stockwell transforms is given. The distributional relation between Stockwell and wavelet transforms is proved, and several asymptotic results for the distributional wavelet transform are also presented.

**Irina V. Melnikova,**

### **Generalized Functions Associated with Stochastic Semigroups**

A wide class of processes arising in various fields of natural science, economics and social phenomena can be mathematically described by stochastic differential equations. Recently, great interest in the problems of financial mathematics has led to significant advances in this area. Simulation based on Levy processes allows one to study along with continuous, jump processes.

At the same time, both in applications and in fundamental science, often what is needed is not the random process itself, but its probability characteristics. The talk is devoted to properties of probability characteristics defined by stochastic Levy processes. It is shown that, in contrast to the PDEs for characteristics determined by Wiener processes, the equations related to Levy processes are pseudo-differential. Nevertheless, the Cauchy problem for the equations is well-posed. The generalized functions and semigroup technique underlies the study of problems for the obtained pseudo-differential equations.