# Generalized Functions Online Workshop

# May 12th, 2026

# Book of Abstracts

**Dragana Jankov Maširević,** 9:30 – 9:50

**A four-parameter Kaiser-Bessel distribution: special function representations and probabilistic aspects**

Motivated by the importance in signal processing of the generalized Kaiser-Bessel window functions, whose definition is based on the modified Bessel function of the first kind, and by the fact that their symmetric bell-shaped form can be normalised to define a probability density function, we consider the four-parameter extension $4KB(\vartheta)$, $\vartheta = (a, \alpha, \nu, \mu)$ of the Kaiser-Bessel distribution introduced by Baricz and Pogány.
We present a systematic analysis of this family, including explicit representations of the cumulative distribution function, fractional-order moments, and the characteristic function.
Further properties are studied, such as moment determinacy, unimodality, and infinite divisibility.

**Olena Atlasiuk,** 9:50 – 10:10

**Approximation problem for generic boundary-value problems in Sobolev spaces**

We study a wide class of linear inhomogeneous boundary-value problems for $r$th order ODE-systems depending on a parameter $\mu$ belonging to a general metric space $\mathcal{M}$. The solutions belong to the Sobolev spaces $\left(W_p^{n+r}\right)^m$, $n \in \mathbb{N} \cup \{0\}$, $m, r \in \mathbb{N}$, $1 \leq p \leq \infty$. The boundary conditions are of a most general form $By = c$, where $B$ is an arbitrary continuous operator from $\left(W_p^{n+r}\right)^m$ to $\mathbb{C}^{rm}$. Thus, they may contain derivatives of the unknown vector function of integer and/or fractional orders $\geq r$. We prove that the solutions of the original problems can be approximated in the space $\left(W_p^{n+r}\right)^m$, $p \leq \infty$, by solutions of ODE-systems with polynomial coefficients, right-hand sides of the equation, and multipoint boundary conditions, which are independent of the right-hand sides of the original problem.

**Milica Žigić,** 10:10 – 10:30

**On Function Spaces Beyond Classical Gevrey Classes**

We consider spaces of smooth functions defined by relaxing Gevrey type regularity and decay conditions. We show that these classes fit naturally within the general framework of the weighted matrices approach to ultradifferentiable functions. We examine equivalent definitions of Gelfand-Shilov type spaces associated with extended Gevrey regularity and establish their nuclearity. In addition to invariance under the Fourier transform, we provide symmetric characterizations of these spaces. We also study time-frequency representations of the introduced classes. Finally, we present a general construction of a smooth orthonormal wavelet which, together with its Fourier transform, belongs to the extended Gevrey class, yielding an example beyond all classical Gevrey classes.

**Sekar Nugraheni,** 10:30 – 10:50

**Generalized Holomorphic Functions**

In this talk, we present a generalized framework for complex analysis that uses the Robinson-Colombeau ring of generalized numbers and spaces of generalized smooth functions (GSF). Historically, generalized holomorphic functions have been defined a priori via the Cauchy-Riemann equations to circumvent the difficulties of pointwise evaluation inherent in Schwartz distributions. We propose a more natural approach, defining generalized holomorphic functions (GHF) via the limit of the incremental ratio.

Following these initial definitions, we will then advance GHF theory by introducing path integration. Using the integration theory of GSF, we

overcome the traditional restriction to compactly supported generalized points, enabling the definition of functions on arbitrary domains. We will demonstrate how this allows for the extension of classical results, including Morera's theorem, the Cauchy-Goursat theorem, Cauchy's integral formula, and the Riemann mapping theorem, to the generalized setting.
Finally, we will address the topological and non-Archimedean constraints that severely restrict ordinary series in the sharp topology of the Colombeau ring. To establish the equivalence between generalized holomorphic and generalized analytic functions, we reintroduce the theory of hyper-power series (summation over hyperfinite natural numbers). We will discuss the convergence properties of these series and how they facilitate the generalization of Liouville's theorem, Goursat's theorem, and the Paley-Wiener theorem, ultimately offering a promising pathway for future applications in solving nonlinear PDEs, such as generalizing the Cauchy-Kowalevski theorem.

## Panel discussion: "Inclusion and accessibility in STEM disciplines: strategies, tools, experiences, and perspectives"

11:00 – 12:00

In this panel we will discuss, with experts in the field, available tools and ongoing researches, aimed at implementing technological solutions and strategies which favour inclusion and accessibility within STEM studies, for people with disabilities and learning disorders. Some experiences in such research activities and in the support to impaired students will be presented, and further possible improvements will be proposed.
The panel will be led by Sandro Coriasco, President of IAGF, member of the group of the Laboratory "S. Polin" - Research and experimentation of new assistive technologies for STEM topics, Department of Mathematics "G. Peano", University of Turin.

**Abhishek Singh,** 13:30 – 13:50

**Abelian-Type Results for the Mexican Hat Wavelet Transform of Compactly Supported Distributions**

In this paper, we introduce a distribution space that extends the framework of the Abelian theorems to the Mexican hat wavelet transform (MHWT) of distributions. We establish two Abelian theorems for the MHWT applied to compactly supported distributions and for locally integrable functions, providing new insights into their asymptotic behavior.

**Ani Tumanyan,** 13:50 – 14:10

**On solvability of hypoelliptic operators in the whole space and boundary value problems in the half-space**

We study Fredholm solvability for hypoelliptic operators in the whole space and special boundary value problems in the half-space. For a class of regular hypoelliptic operators with special variable coefficients, a priori estimates are obtained and criteria for Fredholm solvability are established in multianisotropic weighted Sobolev spaces on $\mathbb{R}^n$. The essential spectrum of these operators is also described. Boundary value problems in the half-space for hypoelliptic operators with constant coefficients are investigated. Under certain conditions on the right-hand sides, correct solvability is established in multianisotropic Sobolev spaces on $\mathbb{R}^n_+$.

**Olga Sudaikova,** 14:10 – 14:30

**Weak asymptotics analysis of soliton dynamics for the Korteweg–de Vries equation**

The ansatz for the asymptotic solution of the Korteweg–De Vries equation is presented in works [1, 2]. In the weak sense, the ansatz has the following form [3]:

$$u(x,t,\varepsilon) = u_0(x,t) + g(t)\,\omega\left(a(t)\frac{x - \varphi(t)}{\varepsilon}\right) + e(x,t)\,\omega_0^-\left(\frac{x - \varphi(t)}{\varepsilon}\right) + O_{\mathcal{D}'}(\varepsilon^2)$$

where: $u_0(x,t)$ is a smooth background; $g(t)$ is the soliton amplitude; $\varphi(t) + \varepsilon\varphi_1(t)$ is the soliton trajectory; $e(x,t)$ is a function describing the soliton "tail"; $\omega(\eta) = \cosh^{-2}(\eta)$ is the soliton profile; $a(t)$ is a scaling factor determining the soliton width; $\omega_0^-$ is an approximation of the Heaviside step function; $\varepsilon$ is a small parameter tending to zero, the value of $O_{\mathcal{D}'}(\varepsilon^\alpha)$ is defined in [3].

In [3], by substituting the ansatz into the definition of the solution [2]

$$u_t + 6\,uu_x + \varepsilon^2 u_{xxx} = O_{\mathcal{D}'}(\varepsilon^\alpha),$$
$$u(u_t + 6\,uu_x + \varepsilon^2 u_{xxx}) = O_{\mathcal{D}'}(\varepsilon^\alpha),$$

at $\alpha = 2$, a system of equations was obtained (the same system as in [1, 2]) for the functions included in the ansatz, with the exception of the function $\varphi_1(t)$, which represents the correction to the soliton coordinate. In [4], based on this definition at $\alpha = 2$, the problem of soliton interaction was examined. However, the result in [4] is incomplete because, within the framework of the solution definition at $\alpha = 2$, the correction $\varphi_1(t)$, remains undetermined, yet the form of this correction is essentially the only significant result of nonlinear soliton interaction.

In our work, we refine the definition by setting $\alpha = 3$ and obtain, in addition to the well-known system derived in [3], the equation for the correction $\varphi_1(t)$. Thus, the new definition can be used for a more accurate description of soliton dynamics and interaction, including in non-integrable versions of the KdV equation.


[1] Maslov V. P. and Tsupin V. A., Necessary conditions for the existence of infinitely narrow solitons in gas dynamics (Russian) Dokl. Akad. Nauk SSSR, 1979, v. 246, 298-300; Soviet Phys. Dokl., 1979, v.24, N 5, 354-356.

[2] Maslov V. P. and Omelyanov G. A., Asymptotic soliton-form solutions of equations with small dispersion, Russian Math. Surveys, 1981, v. 36, N 3, 73-149.

[3] Danilov, V. and Omelyanov, G. and Shelkovich, V. Weak asymptotics method and interaction of nonlinear waves. In M. Karasev

(Ed.), Asymptotic Methods for Wave and Quantum Problems (AMS Translations, Series 2, Vol. 208, pp. 33-165).
[4] Danilov V.G. and Omelyanov G.A, Dynamics of propagation and interaction of infinitely narrow delta-solitons, Nonlinear Analysis: Theory, Methods and Applications, 61:8 (2005), 1245-1291.

**Jasmina Veta Buralieva,** 14:40 – 15:00

**Asymptotic results for the Stockwell and wavelet transforms of derivative Lizorkin distributions**

Several Abelian- and Tauberian-type results characterizing the quasiasymptotics of the $m$-th derivative of Lizorkin distributions in terms of their Stockwell and wavelet transforms are obtained. Moreover, some asymptotic results for the both transforms of the $m$-th derivative of the Dirac delta distribution are given.

**Astrit Ferizi,** 15:00 – 15:20

**Directional short-time fractional Fourier transform and asymptotic behavior of generalized functions**

Following an idea of Candès, Grafakos and Sansing introduced directionality into time–frequency analysis by composing the Radon transform with the short-time Fourier transform (STFT). However, their formulation lacked a reconstruction formula, which was later addressed by Giv through a modified definition. Motivated by this approach, we introduce the directional short-time fractional Fourier transform (DSTFRFT), a new integral transform that extends the capabilities of the short-time fractional Fourier transform (STFRFT) and provides a more efficiently tool for analyzing directional properties of signals. In addition, we develop a Schwartz distributional framework for its analysis
Within this framework, we establish several Abelian- and Tauberian-type results that, characterize the quasiasymptotic behavior of tempered distributions in terms of the asymptotic properties of their DSTFRFT. In particular, these results relate the quasiasymptotic behavior of Schwartz distributions to the boundary asymptotics of the DSTFRFT.
The talk is based on joint work with Katerina Hadzi-Velkova Saneva.

**Anabela S. Silva,** 15:20 – 15:40

**Analysis of Fractional Delay Differential Equations with Non-Singular Kernels**

Unlike classical fractional derivatives with singular power-law kernels, the Caputo-Fabrizio operator offers a smooth fading memory via an exponential kernel. This work extends the theory of this operator to systems with discrete delays. We develop an existence framework based on a stepwise contraction approach and explore the system's sensitivity to perturbations in the initial condition.