International Conference

Applications of Generalized Functions in General Relativity, Stochastics and Mechanics

27 October - 1 November, 2016.

Novi Sad, Serbia

Book of Abstracts

Organized by

Novi Sad Branch of the Serbian Academy of Sciences and Arts

and

Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad

Scientific and Organizing Committee

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Schedule

Thursday, 27 October (Department of Mathematics and Informatics)

10.00-18.00	Discussions	

Friday, 28 October (Serbian Academy of Sciences and Arts, Belgrade)

14.00-15.00	James Vickers	*Generalised Functions and Singularities of Einstein's Equations*

Saturday, 29 October (Novi Sad Branch of SASA)

9.30-10.00	M. Oberguggenberger, M. Schwarz	*Dynamic Methods for Stochastic Parameter Identification*
10.00-10.30	Tijana Levajković	*Equations Involving Malliavin Calculus Operators: Applications and Numerical Approximation*
10.30-11.00	Dora Seleši	*A Generalized Stochastic Harmonic Oscillator*
11.00-11.30	Coffee break	
11.30-12.00	Nenad Teofanov	*A Relaxation of Carleman Condition and some Applications*
12.00-12.30	Filip Tomić	*A New Type of Local Regularity with Applications to the Wave Front Sets*
13.00-14.30	Lunch for all participants at Novi Sad Branch of SASA	

Monday, 31 October (Novi Sad Branch of SASA)

9.30-10.00	Nenad Antonić	*An anisotropic version of the Schwartz kernel theorem and applications*
10.00-10.30	Marin Mišur	*On L^p Compactness of Commutator of Multiplication and Fourier Multiplier Operator*
10.30-11.00	Ivana Vojnović	*H-distributions with Unbounded Multipliers*

11.00-11.30	Jelena Stojanov	*Finslerian Frameworks*

(Department of Mathematics and Informatics, Room 60)

12.15-13.00	Michael Kunzinger	*Singularity Theorems in General Relativity*
13.00-14.30		Lunch break
15.00-15.30	Sanja Ružičić	*On the Uniqueness of Solution to Generalized Chaplygin Gas*
15.30-16.00	Miloš Japundžić	*Approximate Solutions of Time and Time-Space Fractional Wave Equations with Variable Coefficients*
16.00-16.30	Lj. Oparnica, D. Zorica	*Microlocal Analysis of Fractional Wave Equations*

Tuesday, 1 November (Department of Mathematics and Informatics)

10.00-18.00	Discussions

Abstracts of Talks

Nenad Antonić
University of Zagreb, Croatia

An anisotropic version of the Schwartz kernel theorem and applications

We define distributions of anysotropic order, and establish their immediate properties. The central result is the Schwartz kernel theorem for such distributions, which represents continuous operators from $\mathrm{C}^l_c(X)$ to $\mathcal{D}'_m(Y)$ by kernels, which are distributions of order l in x, but higher, though still finite order $d(m+2)$ in y. Standard proofs of that theorem all depend on the reflexivity and Montel property of considered spaces, which is not the case here.

This result allows us to obtain more precise results on H-distributions, a recently introduced generalisation of H-measures (Antonić and Mitrović, 2011), which are, therefore, distributions of order 0 (i.e. Radon measures) in $x \in \mathbb{R}^d$, and of finite order in $\xi \in \mathrm{S}^{d-1}$. Variants of H-distributions have been successfully applied to problems in velocity averaging (Lazar-Mitrović 2012) and compensated compactness with variable coefficients (Mišur-Mitrović 2015). Extension to Sobolev space setting is given in (Aleksić-Pilipović-Vojnović 2016).

This is a joint work with M. Erceg and M. Mišur.

Miloš Japundžić
University of Novi Sad, Serbia

Approximate Solutions of Time and Time-Space Fractional Wave Equations with Variable Coefficients

In literature, fractional equations with variable coefficients and nonlinear part depending on solution in general case have been solved approximately, usually applying different numerical methods, and on bounded domain. Here, we solve the problem on an unbounded domain since in many cases the domain of governing equation for certain physical phenomena is appropriate to be an infinite or semi-infinite interval and in such cases these problems require special treatment.

We consider the Cauchy problem for fractional wave equations with time derivative as Caputo fractional derivative of order $1 < \alpha < 2$ and space variable coefficients on an unbounded domain. The space derivatives that appear in the equations are of integer or fractional order such as the left and the right Liouville fractional derivative as well as the Riesz fractional derivative. In order to solve this problem we introduce and develop generalized uniformly continuous solution operators and use them to obtain the unique solution on a certain Colombeau space. In our solving procedure, instead of the originate problem we solve a certain approximate problem, but therefore we also prove that the solutions of these two problems are associated.

At the end, we illustrate the applications of the developed theory by giving some appropriate examples.

Michael Kunzinger
University of Vienna, Austria

Singularity Theorems in General Relativity

The singularity theorems of General Relativity are important milestones in the understanding of solutions to the Einstein equations. Initiated by R. Penrose and continued by S. W. Hawking, R. Penrose, G.F.R. Ellis, R. Geroch and others, the investigation of singularity theorems to this day constitutes a central research field in mathematical relativity. They show that under realistic assumptions on the spacetime (and independently of any symmetries) there necessarily exist incomplete geodesics, which may be interpreted as singularities.

One weakness of the classical singularity theorems is that they do not make any statement on the actual nature of the singularities themselves. In particular, they do not imply that the curvature blows up where a causal geodesic ceases to exist. Thus, in principle, they allow the possibility that the spacetime might be singular in the above sense merely due to the fact that the differentiability of the spacetime metric drops below $\mathcal{C}^{\in}$ (twice continuously differentiable).

Recent progress in low-regularity causality theory has allowed to show that, in fact, both the Penrose and the Hawking theorem remain valid for metrics of differentiability class $\mathcal{C}^{\infty,\infty}$, the maximal class in which the geodesic equation still has unique solutions. We will report on these developments and discuss open questions and further directions of research.

Tijana Levajković
University of Innsbruck, Austria

Equations Involving Malliavin Calculus Operators: Applications and Numerical Approximation

This talk is devoted to the study of several types of stochastic differential equations involving Malliavin calculus operators with applications. Particularly, we present applications for the stochastic linear quadratic optimal control problem with different noise disturbances, operator differential algebraic equations arising in fluid dynamics and stationary equations. In addition, we provide a numerical framework based on chaos expansion and perform numerical simulations.

Marin Mišur
University of Zagreb, Croatia

On L^p Compactness of Commutator of Multiplication and Fourier Multiplier Operator

We generalise results on compactness of commutators of multiplication and Fourier multiplier operators by H. O. Cordes (1975) in several directions with respect to the smoothness of multiplication function and by replacing the Fourier multiplier operator by a more general pseudodifferential operator. Our prime motivation has been a particular case known as the First commutation lemma - the basic tool for defining H-measures and H-distributions. We review and improve the known results both in the standard L^2 setting, as well as for general L^p, with $1 < p < \infty$. Furthermore, we extend these results to less regular simbols.

This is joint work with Nenad Antonić and Darko Mitrović.

Michael Oberguggenberger, Martin Schwarz:
University of Innsbruck, Austria

Dynamic Methods for Stochastic Parameter Identification

We propose a new approach to analyze the propagation of disturbances in engineering structures in the presence of spatially distributed random imperfections. The applicative targets are reliability analysis, damage detection, system identification, and calibration of models in the presence of randomly perturbed structures in elasticity and strength of materials. The set-up applies to wave propagation in random media, i.e., materials with stochastically varying properties. The approach is done by a top down method. Instead of solving the direct problem, one uses Stochastic Fourier Integral Operators to model both the propagation and the material properties.

Ljubica Oparnica, Dušan Zorica
University of Novi Sad, Serbia

Microlocal Analysis of Fractional Wave Equations

Fractional wave equations are obtained when instead of Hooke's law fractional order constitutive equation is considered while equations of motion and strain measure are left unchanged. Such equations describe wave phenomenas when viscoelasticity of a material or non-local effects of a material comes into an account. We determine the wave front sets of solutions to such equations.

For the space fractional wave equation we show that no spatial propagation of singularities occurs and for the (time) fractional Zener wave equation, we show an analogue of non-characteristic regularity, see [1]. For the Eringen fractional wave equation, which models elastic wave dispersion in small scale structures as micro and nanostructures, there is no spatial propagation of singularities, [2].

References

[1] G. Hörman, Lj. Oparnica, and D. Zorica. Microlocal analysis of fractional wave equations. *Z. Angew. Math. Mech.*, to appear 2016.

[2] G. Hörman, Lj. Oparnica, and D. Zorica. Eringen fractional wave equation: Solvability and microlocal analysis. work in progress.

Stevan Pilipović, Nenad Teofanov, Filip Tomić
University of Novi Sad, Serbia

A Relaxation of Carleman Condition and some Applications

We introduce a new type of local regularity and investigate it from the microlocal point of view. That lead us to the definition of new type of wave front sets, and we present their basic properties. On a more advanced level, we study propagation of singularities of solutions to linear PDE's and present our main result:

$$\mathrm{WF}_{0,\infty}(P(x,D)u) \subseteq \mathrm{WF}_{0,\infty}(u) \subseteq \mathrm{WF}_{0,\infty}(P(x,D)u) \cup \mathrm{Char}(P(x,D)),$$

where u is a Schwartz distribution, $P(x,D)$ is a partial differential operator with coefficients in suitable classes of smooth functions and $\mathrm{WF}_{0,\infty}$ is the wave front set described in terms of new regularity conditions.

Sanja Ružičić
University of Novi Sad, Serbia

On the Uniqueness of Solution to Generalized Chaplygin Gas

This talk is devoted to the study of problem of finding a unique solution to Riemann problem for generalized Chaplygin gas model. That is a model of the dark energy in Universe introduced in the last decade. It permits an infinite mass concentration so one has to consider solutions containing the Dirac delta function. Although it was easy to construct solution to this Riemann problem, the usual admissibility conditions, overcompressiveness, do not exclude unwanted delta-type waves when a classical solution exists. We are using Shadow Wave approach in order to solve that uniqueness problem since they are well adopted for using Lax entropy-entropy flux conditions and there is a rich family of convex entropies. Also, we are making comparison between two conditions frequently used for admissibility check: convex entropy-entropy flux pair (entropy condition) and overcompressibility. Through the analysis we investigated several cases where solution to Riemann problem is unique. At the end we propose the next steps in the analysis.

The talk is based on joint work with Marko Nedeljkov.

Dora Seleši
University of Novi Sad, Serbia

A Generalized Stochastic Harmonic Oscillator

We construct a new infinite-dimensional harmonic oscillator and other types of stochastic quantum operators on Hida's white noise space. In particular, we prove that there is an elegant relationship between the quasi-quantum operators which are building blocks of the stochastic harmonic oscillator, the Malliavin derivative, the Skorokhod integral and the multiplication operator. The new family of operators we obtain contains both the ordinary product and the Wick product as special cases. Due to the operator's construction on a symmetric Fock space, the results are not restricted to stochastic analysis but also describe the behavior of bosonic particle-systems in quantum theory.

Jelena Stojanov
University of Novi Sad, Serbia

Finslerian Frameworks

Finsler geometry extends the Riemannian one by considering metric structure defined over the tangent bundle of basic manifold. Anisotropic (directionaly dependent) metric endows space with wide range of geometrical objects and increases its applicability. Main concepts of Finsler space will be discused. Finslerian frameworks for dynamical systems and image processing will be presented. KCC theory based on Finslerian framework reveals stability of the system and its invariants. An example of numerically fitted Finsler metric structure for the Garner dynamical system of cancer cell population will be shown. Beltrami framework models image as surface and various surface evolution enable processing of the image. Anisotropic extension of Beltrami framework with background theory is developed and related to the theory of harmonic maps. Anisotropic Beltrami flow is determined and particular cases are discretized for application in image processing.

Nenad Teofanov
University of Novi Sad, Serbia

A Relaxation of Carleman Condition and some Applications

Carleman classes arose in connection to the Haramard uniqueness problem, and later served as a reservoir for test function spaces of ultradifferentiable functions. A well known example are Gevrey classes. It turns out that the Gevrey classes are connected to properties of certain differential operators, and in particular when discussing if the Cauchy problem is well posed. In this lecture we introduce and study spaces of ultradifferentiable functions which contain Gevrey classes. For the definition we use sequences of the form $\{p^{\tau p^{\sigma}}\}_{p\in\mathbf{N}}$, $\tau > 0$, $\sigma \geq 1$. This includes the Gevrey type regularity when $\sigma = 1$ and $\tau > 1$, and the analytic regularity when $\sigma = \tau = 1$.

We discuss the relation between our classes and those arising in the Braun-Meise-Taylor-Vogt theory, and prove that, for example, our spaces enjoy the inverse closedness property.
As opposite to regularity, we consider singular directions in phase space by introducing appropriate wave-front sets. We identify singular supports of certain ultradifferentiable functions as projections of intersections/unions of wave-front sets.
By using the technique of approximate solution we prove the usual microlocal embedding:

$$\mathrm{WF}_{0,\infty}(P(D)u) \subseteq \mathrm{WF}_{0,\infty}(u) \subseteq \mathrm{WF}_{0,\infty}(P(D)u) \cup \mathrm{Char}(P), \quad u \in \mathcal{D}'(\mathbb{R}^d),$$

where $P(D)$ is a partial differential operator with the characteristic set $\mathrm{Char}(P)$, and $\mathrm{WF}_{0,\infty}$ is the wave front set described in terms of new regularity conditions. The proof is particularly challenging when the coefficients in $P(D)$ are non-constant.

James Vickers
University of Southampton, UK

Generalised Functions and Singularities of Einstein's Equations

This talk will describe how singularities of Einstein's equations are treated within the theory of General Relativity. In other field theories, such as electromagnetism, an important role is played by point charges, line sources and surface charges. Mathematically such concentrated sources are described using the classical theory of distributions. However Einstein's equations are a system of non-linear PDEs so that one cannot use classical distribution theory. This talk will explain how a non-linear theory of generalised functions (Colombeau algebras) may be used to describe important physical solutions of Einstein's equations such as Cosmic Strings, shells of matter and impulsive gravitational waves. It will also give a recent new description of gravitational singularities in terms of the evolution of test fields rather than the traditional definition using world lines of test particles. It will end by relating this work to new developments in the theory of PDEs.

Ivana Vojnović
University of Novi Sad, Serbia

H-distributions with Unbounded Multipliers

H-measures were introduced independently by Tartar, [4] and Gérard, [3]. They are used to determine weather a weakly convergent sequence in $L^2(\mathbb{R}^d)$ converges strongly. Antonić and Mitrović in [2] introduced H-distributions, extension of H-measures to an $L^p - L^q$ setting for $1 < p < \infty$ and $q = p/p - 1$, . In [1], H-distributions are constructed for sequences in dual Sobolev spaces, $W^{-k,p} - W^{k,q}$. Test functions for H-measures and H-distributions are bounded Fourier multipliers.
Using theory of pseudo-differential operators we construct H-distributions for sequences in dual Bessel potential spaces, $H^k_q - H^{-k}_p, k \in \mathbb{R}$. In this case we consider classes of unbounded

test functions. Also, a necessary and sufficient condition is given so that the weak convergence of sequence in H_p^{-k} implies the strong one. Results are applied on a weakly convergent sequence of solutions to a family of pseudo-differential equations.

This is a joint work with Jelena Aleksić and Stevan Pilipović.

References

[1] Aleksić, J.; Pilipović, S.; Vojnović, I. *H - distributions via Sobolev spaces*, Mediterranean Journal of Mathematics, 2016., pp 1-14

[2] Antonić, N.; Mitrović, D. *H-distributions: an extension of H-measures to an* $L^p - L^q$ *setting.* Abstr. Appl. Anal. 2011, Art. ID 901084, 12 pp.

[3] Gérard, P. *Microlocal defect measures.* Comm. Partial Differential Equations 16 (1991), no. 11, 1761–1794.

[4] Tartar, L. *H-measures, a new approach for studying homogenisation, oscillations and concentration effects in partial differential equations.* Proc. Roy. Soc. Edinburgh Sect. A 115 (1990), no. 3-4, 193–230.