Recent results and applications of integral circulant graphs

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A graph is called circulant if it is Cayley graph on the circulant group, i.e. its adjacency matrix is circulant. A graph is called integral if all eigenvalues of its adjacency matrix are integers. Integral circulant graphs are extensively studied in the literature and there was a vast amount of research done on graph theoretical properties and classes of these graphs supporting perfect state transfer. Integral circulant graphs are the generalization of well-known class of unitary Cayley graphs.

In the recent work of Saxena, Severini and Shparlinski [6] it is stated that the integral circulant graphs are potential candidates for modelling quantum spin networks that might enable the perfect state transfer between antipodal sites in a network. We present a simple and general characterization of the existence of perfect state transfer in integral circulant graphs, in terms of its eigenvalues. Moreover, it is shown that in the class of unitary Cayley graphs there are only two of them allowing perfect state transfer.

We present the main result from [3] that there exist integral circulant graph with n vertices having perfect state transfer if and only if $4 \mid n$. We found several classes of integral circulant graphs having perfect state transfer for values of n divisible by 4. Moreover, we prove the nonexistence of perfect state transfer for several classes of graphs, including integral circulant graphs with exactly two divisors.

It is also interesting to know how far information can be potentially transferred between sites of the system modelled by the graph, or in other words the diameter of the graph. We showed that the diameter of integral circulant graphs is at most $O(\ln \ln n)$.

Motivated by the open question in [5], we focus on relevant properties of integral circulant graphs with application in quantum dynamics – such as clique and chromatic number. We completely characterize the clique and chromatic number for integral circulant graphs with exactly two divisors and give sharp bound in general case. In conclusion, we give some novel results about the automorphism group of these graphs.

The talk reports a joint work with A. ILIĆ (University of Niš).

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