Modular fractal lattices and von Neumann frames

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Let L be a bounded lattice. If for each $a_1 < b_1 \in L$ and $a_2 < b_2 \in L$ there is a lattice embedding $\psi : [a_1, b_1] \to [a_2, b_2]$ with $\psi(a_1) = a_2$ and $\psi(b_1) = b_2$, then we say that L is a *quasifractal*; see [1]. If ψ can always be chosen an isomorphism or, equivalently, if L is isomorphic to each of its nontrivial intervals, then L will be called a *fractal lattice*; see [1] again. Jakubík and J. Lihová [7] proved that there is a proper class of quasifractals (in fact, chains) that are not fractals. Some open problems on (quasi)fractals will be mentioned in the talk.

For a ring R with 1 let $\mathcal{V}(R)$ denote the lattice variety generated by the submodule lattices of R-modules. The prime field of characteristic p will be denoted by F_p. Let $\mathcal{U}$ be a lattice variety generated by a nondistributive modular quasifractal.

The first target, see [2], is to prove that $\mathcal{U}$ is neither too small nor too large in the following sense: there is a unique $p = p(\mathcal{U})$, a prime number of zero, such that

- $\mathcal{V}(F_p) \subseteq \mathcal{U}$ (*"neither too small"*);
- $\mathcal{U}$ is Arguesian and, for any ring R, $\mathcal{V}(R) \subseteq \mathcal{U}$ implies $\mathcal{V}(R) = \mathcal{V}(F_p)$. (*"nor too large"*).

Von Neumann n-frames have been used in the heart of modular lattice theory for long, see Herrmann [4], Giudici [5] and Wehrung [8] for recent developments.

The second target is to *construct* a new frame, called *product frame*, from an "outer" frame and an "inner frame", and to give a *motivation* for the next talk by SKUBLICS, based on [3].

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