Fuzzy and weighted automata: Bisimulation and structural equivalence

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In this talk we give an overview of our research on bisimulations for fuzzy and weighted automata [3, 6].

Bisimulation is a concept of structural equivalence that turned out to be a very useful tool in many areas of computer science and mathematics, such as modal logic, concurrency theory, set theory, formal verification, model checking, etc. Bisimulations have been mostly used to model equivalence between states of the same system, and to reduce the number of states of this system. In particular, many algorithms have been proposed to compute the greatest bisimulation equivalence on a given labelled graph or a labeled transition system, and the faster ones are based on the crucial equivalence between the greatest bisimulation equivalence and the relational coarsest partition problem.

Bisimulations between states of two distinct systems have been much less studied, probably due to lack of a suitable concept of a relation between two distinct sets which would behave like an equivalence. In most cases, bisimulations have been considered either as arbitrary relations (which turned out to be too general), or as functions (which are too special). However, a more suitable concept has appeared recently in [2], this is the concept of a uniform fuzzy relation. We define four kinds of fuzzy relations between fuzzy automata over a complete residuated lattice: forward, backward, forward-backward and backward-forward bisimulations, and we study those which are uniform. We show that a uniform relation φ between fuzzy automata $\mathscr{A}$ and $\mathscr{B}$ is a forward bisimulation if and only if its kernel and co-kernel are forward bisimulation fuzzy equivalences on $\mathscr{A}$ and $\mathscr{B}$, and φ induces an isomorphism between related factor fuzzy automata [3]. We also prove that fuzzy automata $\mathscr{A}$ and $\mathscr{B}$ are UFB-equivalent (i.e., there is an uniform forward bisimulation between $\mathscr{A}$ and $\mathscr{B}$) if and only if there exists a suitable isomorphism between factor fuzzy automata w.r.t. the greatest forward bisimulation fuzzy equivalences on $\mathscr{A}$ and $\mathscr{B}$. In fact, forward and backward bisimulation fuzzy equivalences on a fuzzy automaton are just the right and left invariant fuzzy equivalences, used in [4, 5, 9] for state reduction of fuzzy automata. In particular, two non-deterministic automata $\mathscr{A}$ and $\mathscr{B}$ are UFB-equivalent if and only if factor automata w.r.t. the greatest forward bisimulation equivalences on $\mathscr{A}$ and $\mathscr{B}$ are isomorphic. Let us note that uniform fuzzy relations are also used in study of equivalence between deterministic fuzzy recognizers [3, 7], and just for these automata the language (behavioral) equivalence can be modelled as structural equivalence.

Bisimulations for non-deterministic and fuzzy automata are defined by inequalities involving orderings of ordinary and fuzzy relations, which are defined through orderings in the related structures of truth values. These orderings are also needed to get the greatest bisimulation relations and the best reductions determined by them, and they are essential for some fundamental properties of ordinary and fuzzy relations. For example, ordinary and fuzzy relations can be represented by Boolean and fuzzy matrices and their composition can be modelled by the product of these matrices. In the general case, semirings are not required to be ordered, so in study of bisimulations for weighted automata over semirings certain difficulties are arising. However, we show how these difficulties can be overcome in the case of weighted automata over additively idempotent semirings [6]. The obtained results concerning weighted automata over additively idempotent semirings are also applicable to fuzzy automata over lattice-ordered monoids, distributive lattices, etc.

The talk reports a joint work with M. ĆIRIĆ (Niš), J. IGNJATOVIĆ (Niš) and M. BAŠIĆ (Niš) [3, 6].

REFERENCES

[1] M. Ćirić, J. Ignjatović, S. Bogdanović, Fuzzy equivalence relations and their equivalence classes, Fuzzy Sets and Systems 158 (2007) 1295–1313.

[2] M. Ćirić, J. Ignjatović, S. Bogdanović, Uniform fuzzy relations and fuzzy mappings, Fuzzy Sets and Systems 160 (2009) 1054–1081.

[3] M. Ćirić, J. Ignjatović, N. Damljanović, M. Bašić, Bisimulations for fuzzy automata, Fuzzy Sets and Systems (submitted for publication).

[4] M. Ćirić, A. Stamenković, J. Ignjatović, T. Petković, Factorization of fuzzy automata, in: E. Csuhaj-Varju, and Z. Ésik (eds.), FCT 2007, Lecture Notes in Computer Science 4639 (2007) 213–225.

[5] M. Ćirić, A. Stamenković, J. Ignjatović, T. Petković, Fuzzy relation equations and reduction of fuzzy automata, Journal of Computer and System Sciences (submitted for publication).

[6] N. Damljanović, M. Ćirić, J. Ignjatović, Bisimulations for weighted automata over additively idempotent semirings, (in preparation).

[7] J. Ignjatović, M. Ćirić, S. Bogdanović, Fuzzy homomorphisms of algebras, Fuzzy Sets and Systems 160 (2009), 2345–2365.

[8] J. Ignjatović, M. Ćirić, S. Bogdanović, T. Petković, Myhill-Nerode type theory for fuzzy languages and automata, Fuzzy Sets and Systems (2009), doi: 10.1016/j.fss.2009.06.007.

[9] A. Stamenković, M. Ćirić, J. Ignjatović, Reduction of fuzzy automata by means of fuzzy quasi-orders, Information Sciences (submitted for publication).