Fuzzy and weighted automata: Determinization methods

ZORANA JANČIĆ
Department of Mathematics and Informatics
Faculty of Science, University of NIŠ
zoranajancic329@gmail.com

In this talk we give an overview of recent research on determinization of fuzzy and weighted automata.

Equivalence between recognizability of fuzzy languages over a locally finite complete lattice by fuzzy finite automata (FFA-recognizability) and deterministic finite automata (DFA-recognizability) was established by Bělohlávek [1]. A more general result was obtained by Li and Pedrycz [9], who studied fuzzy automata over a lattice ordered monoid $\mathscr{L}$, and proved that FFA-recognizability is equivalent to DFA-recognizability if and only if the semiring reduct $\mathscr{L}^*$ of $\mathscr{L}$ (with respect to the join and multiplication operations) is locally finite. Bělohlávek and Li and Pedrycz also gave a method for determinization of fuzzy automata, which results in a finite automaton if and only if $\mathscr{L}^*$ is locally finite. Another method, called the accessible fuzzy subset construction, has been developed in [4] for fuzzy languages over a complete residuated lattice $\mathscr{L}$. This method can result in a finite automaton even if $\mathscr{L}^*$ is not locally finite, and always gives a smaller automaton than the method by Bělohlávek and Li and Pedrycz. Certain criteria for finiteness of the resulting deterministic automaton have been obtained in [4, 5], where it has been also shown that this automaton is a minimal deterministic automaton recognizing all fuzzy languages which can be recognized by the original fuzzy automaton. The same construction is also applicable to determinization of fuzzy finite automata over a lattice ordered monoid, and even more generally, to determinization of weighted finite automata over a semiring.

In [6] we have developed another method, based on fuzzy transition sets, which gives an even smaller automaton than the accessible fuzzy subset construction. We have also shown that the well-known Brzozowski's minimization (canonization) algorithm for non-deterministic automata, based on two successive reversion and determinization operations, is also applicable to fuzzy and weighted automata.

It is worth noting that determinization of weighted finite automata over strong bimonoids has been also studied [2, 3]. A bimonoid is a structure which is not necessarily distributive and generalizes both lattices and semirings. Lack of distributivity makes it possible to define behavior of such weighted automata in several different ways, which requires distinct determinization methods for any of these behaviors. There are many natural examples of strong bimonoids and weighted automata over them. Among the most important ones are complete orthomodular lattices, which serve as a basis of quantum logic. Automata based on quantum logic, i.e., automata over a complete orthomodular lattice, may be viewed as a logical approach of quantum computation. An algorithm for determinization of automata based on quantum logic has been given in [7], and it has been since generalized in [2]. However, when applied to weighted automata over semirings this algorithm gives worse results than the above mentioned algorithms.

The talk reports a joint work with J. IGNJATOVIĆ (Niš) and M. ĆIRIĆ (Niš) [4, 5, 6].

REFERENCES

[1] R. Bělohlávek, Determinism and fuzzy automata, Information Sciences 143 (2002) 205–209.

[2] M. Ćirić, M. Droste, J. Ignjatović, H. Vogler, Determinization of weighted finite automata over strong bimonoids, (in preparation).

[3] M. Droste, T. Stüber, H. Vogler, Weighted finite automata over strong bimonoids, Information Sciences (submitted for publication).

[4] J. Ignjatović, M. Ćirić, S. Bogdanović, Determinization of fuzzy automata with membership values in complete residuated lattices, Information Sciences 178 (2008) 164–180.

[5] J. Ignjatović, M. Ćirić, S. Bogdanović, T. Petković, Myhill-Nerode type theory for fuzzy languages and automata, Fuzzy Sets and Systems (2009), doi: 10.1016/j.fss.2009.06.007.

[6] Z. Jančić, J. Ignjatović, M. Ćirić, Determinization of weighted and fuzzy automata, (in preparation).

[7] Y. M. Li, Finite automata based on quantum logic and their determinization, CoRR abs/0712.4341 (2007).

[8] Y. M. Li, Approximation and robustness of fuzzy finite automata, International Journal of Approximate Reasoning 47 (2008) 247–257.

[9] Y. M. Li, W. Pedrycz, Fuzzy finite automata and fuzzy regular expressions with membership values in lattice ordered monoids, Fuzzy Sets and Systems 156 (2005) 68–92.