On the generation of clones containing near-unanimity operations

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Denote by M_n^d the set of clones on $A = \{0, ..., n-1\}$ which contain a $(d+1)$-ary near-unanimity operation. It is an easy consequence of the Baker-Pixley Theorem that any $C \in M_n^d$ is finitely generated and $|M_n^d| < \infty$. Hence, for $C \in M_n^d$, we can define

$$\lambda(C) := \min\{k \in \mathbb{N} |\ C \text{ is generated by its operations of arity} \leq k\}$$

and

$$\lambda_d(n) := \max\{\lambda(C) |\ C \in M_n^d\}.$$

In 1989, H. Lakser determined $\lambda_2(n)$ for $n \geq 5$ and gave an asymptotic formula for the case $d > 2$. An exact determination of $\lambda_d(n)$ for $d > 2$ and n sufficiently large was stated as an open problem and, to my knowledge, has not yet been answered.

A similar question arises when we talk about the graphics of the clones containing near-unanimity operations. For a clone C, denote by Γ_C^k the kth graphic of the clone. Note that, due to the Baker-Pixley Theorem, any $C \in M_n^d$ can be written as $\operatorname{Pol}\Gamma_C^k$ for some finite k. Set

$$\mu(C) := \min\{k \in \mathbb{N} |\ C = \operatorname{Pol}\Gamma_C^k\}$$

and

$$\mu_d(n) := max\{\mu(C) |\ C \in M_n^d\}.$$

In this talk, we determine $\lambda_d(n)$ and $\mu_d(n)$ for all $d \geq 2$ and n sufficiently large. Surprisingly, we obtain exact, coinciding formulas for both problems. The result can be formulated in the following theorem.

Theorem. $\lambda_d(n) = \mu_d(n) = (n-1)^d - 1$ *for all* $d \geq 2$ *and* n *sufficiently large.*