On parallelogram laws for skew lattices

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During the last years, several authors were interested on the study of specific properties of the skew lattices that relate to fundamental properties of lattices. Unlike what happens in lattices in general, in skew lattices the properties of distributivity and cancellation are independent. In fact, cancellation is related to the property of symmetry that indicates the instances of commutativity in a skew lattice.

A skew lattice is a set S equipped with two associative, idempotent binary operations $\vee$ and $\wedge$ that satisfy the absorption laws

$$(b \wedge a) \vee a = a = a \vee (a \wedge b)$$

and their duals. The class of skew lattices forms a variety. The variety of cancellative skew lattices was studied in [3] and is part of a larger variety of skew lattices, the symmetric skew lattices. A skew lattice S is said to be strongly symmetric if it satisfies the identities $x \vee (y \wedge x) = (x \vee y) \wedge x$ and $x \wedge (y \vee x) = (x \wedge y) \vee x$. All cancellative skew lattices are strongly symmetric. Strongly symmetric skew lattices are symmetric.

When we consider a skew lattice S consisting of exactly two $\mathcal{D}$-classes $A > B$, an element $b \in B$ determines $A \wedge b \wedge A = \{a \wedge b \wedge a : a \in A\}$ defined as the coset of A in B. B is partitioned by cosets of A in B. Furthermore, the image set in B of any $a \in A$ is a transversal of cosets of A in B. Cosets permit us to give a characterization to strongly symmetric and symmetric skew lattices.

The study of parallelogram laws for cancellative skew lattices began in 2005, with Karin Cvetko-Vah [2]. In this talk we generalize this result and extend it to strongly symmetric skew lattices and symmetric skew lattices. Later we apply these results, relating the number of cosets of A in B defined as $[A : B]$. This permits us to establish several combinatorial properties that relate the number of elements of the D-classes themselves.

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