UNIVERSITY OF NOVI SAD
FACULTY OF SCIENCE
Department of Mathematics and Informatics

The 4th Novi Sad Algebraic Conference (NSAC 2013)
& the workshop "Semigroups and Applications 2013"

ABSTRACTS

Novi Sad, Serbia, June 5–9, 2013

The Organising Committee of NSAC 2013 & Semi2013:

- IGOR DOLINKA – co-chair
- PETAR MARKOVIĆ – co-chair
- MAJA PECH – secretary
- PETAR DJAPIĆ
- NEBOJŠA MUDRINSKI
- BORIS ŠOBOT
- BOJAN BAŠIĆ
- IVANA TOJIĆ
- ÉVA JUNGÁBEL

The Scientific Committee of NSAC 2013 & Semi2013:

- DRAGAN MAŠULOVIĆ (Novi Sad, SRB) – chair
- PAWEL IDZIAK (Krakow, POL)
- MILOŠ KURILIĆ (Novi Sad, SRB)
- MIKLÓS MARÓTI (Szeged, HUN)
- JOHN C. MEAKIN (Lincoln, USA)
- PREDRAG TANOVIĆ (Belgrade, SRB)
- FRIEDRICH WEHRUNG (Caen, FRA)

We gratefully acknowledge the support of

- The Ministry of Education, Science and Technological Development of the Republic of Serbia,
- The Secretariat of Science and Technological Development of the Autonomous Province of Vojvodina.

Typeset by Igor Dolinka.

CONTENTS

INVITED TALKS

Higher commutators, nilpotence, and supernilpotence

ERHARD AICHINGER
Institute of Algebra
Johannes Kepler University, LINZ (AUT)
erhard@algebra.uni-linz.ac.at

The n-ary commutator operation of a universal algebra associates a congruence $\beta := [\alpha_1, \ldots, \alpha_n]$ with every n-tuple $(\alpha_1, \ldots, \alpha_n) \in (\operatorname{Con} \mathbf{A})^n$. These commutator operations were introduced by A. Bulatov to distinguish between polynomially inequivalent algebras, and their properties in Mal'cev algebras were investigated by N. Mudrinski and the speaker. Using commutator operations, a different concept of nilpotence can be defined: an algebra is defined to be *supernilpotent* if for some $n \in \mathbb{N}$, $[1, \ldots, 1] = 0$ (n repetitions of 1). For finite Mal'cev algebras, being supernilpotent is equivalent to $\log(\mathbf{F}_{V(\mathbf{A})}(n))$ being bounded from above by a polynomial in n.

We will review some basic results on higher commutators and supernilpotent Mal'cev algebras, discuss results by J. Berman, W. Blok, and K. Kearnes that link supernilpotence to nilpotence, provide a generalization of one of these structural results to infinite expanded groups, and use these results to establish that the clone of congruence preserving functions of certain algebras is not finitely generated.

Prime Maltsev conditions

LIBOR BARTO
Department of Algebra
Charles University, PRAGUE (CZE)
libor.barto@gmail.com

The lattice L of interpretability types of varieties can be introduced in a number of equivalent ways, for instance:

- First we define a quasiorder $\leq$ on the class of varieties and then we identify varieties which are equivalent with respect to this quasiorder. The obtained partially ordered set turns out to be a lattice – the lattice L of interpretability types of varieties. The quasiorder $\leq$ is interpretability: we define $\mathcal{V} \leq \mathcal{W}$ if we can assign terms of $\mathcal{W}$ to basic operational symbols of $\mathcal{V}$ so that variables are mapped to the same variables and all identities valid in $\mathcal{V}$ are preserved.
- We define a quasiorder $\leq$ on the class of all clones and then we obtain the lattice L in the same way as above, that is, by identifying equivalent clones. We put $\mathbf{A} \leq \mathbf{B}$ if there exists a clone homomorphism from $\mathbf{A}$ to $\mathbf{B}$, equivalently, if $\mathbf{B}$ can be obtained from $\mathbf{A}$ by taking powers, subclones, quotient clones and by adding operations.

The position of a variety $\mathcal{V}$ (or a clone $\mathbf{A}$) in L determines how nice the variety is with respect to Maltsev conditions – the higher $\mathcal{V}$ is, the stronger Maltsev condition it satisfies. Some important classes of varieties (or clones) like the class of all congruence permutable (distributive, modular) varieties are filters in L.

The talk reports on a recent progress on the following question: Which important Maltsev filters are prime?

This is a joint work with JAKUB OPRŠAL (Charles University, Prague).

Permutation groups and transformation semigroups

PETER J. CAMERON
School of Mathematics and Statistics
University of ST ANDREWS (GBR)
pjc@mcs.st-andrews.ac.uk

I will talk about the way in which our knowledge of groups can help us study semigroups; in particular, new results about permutation groups which were motivated by applications to transformation semigroups. This is joint work with JOÃO ARAÚJO and others.

In the first part of the talk, I discuss some recent results on the semigroups $\langle a, G\rangle \setminus G$ and $\langle g^{-1}ag : g \in G\rangle$, where G is a permutation group and a a map which is not a permutation. Typical questions are: when are these semigroups equal? When, for given groups G and H, do they coincide? When do they have nice properties such as regularity or idempotent-generation? These lead to questions about new concepts in permutation group theory such as λ-homogeneity (where λ is a partition), (k, l)-homogeneity where $k < l$, and the k-universal transversal property.

The second part is a brief report on the synchronization project, the attempt to answer the question: for which permutation groups G is it true that $\langle a, G\rangle$ contains a map of rank 1 for any non-permutation a? The obstruction to this property turns out to be endomorphisms of very special graphs; but these lead to hard geometric and combinatorial problems about permutation groups.

Adequate and Ehresmann semigroups

VICTORIA GOULD
Department of Mathematics
University of York, Heslington, YORK (GBR)
victoria.gould@york.ac.uk

The semigroups of the title are classes of semigroups containing a semilattice of idempotents known as *projections*, but which need not be regular. *Adequate* semigroups form a quasi-variety of bi-unary semigroups, that is, semigroups possessing, in addition to the basic binary operation, two basic unary operations which we denote by $a \mapsto a^+$ and $a \mapsto a^*$. An inverse semigroup S is adequate if we define $a^+ = aa^{-1}$ and $a^* = a^{-1}a$, for any $a \in S$. On the other hand, any cancellative monoid M is adequate if we put $a^+ = 1 = a^*$ for all $a \in M$. Indeed, an adequate semigroup with one idempotent is precisely a cancellative monoid. Less trivially, we have recently shown that the free idempotent generated semigroup $\mathrm{IG}(Y)$ over a semilattice Y is adequate.

Ehresmann semigroups form the variety of bi-unary semigroups generated by the quasi-variety of adequate semigroups. We remark that adequate and Ehresmann semigroups may also be approached via the relations $\mathcal{R}^*$, $\mathcal{L}^*$, $\widetilde{\mathcal{R}}_E$ and $\widetilde{\mathcal{L}}_E$, which are natural extensions of Green's relations $\mathcal{R}$ and $\mathcal{L}$.

An inverse semigroup, regarded as an adequate semigroup, satisfies the identities

$$xy^+ = (xy)^+x \text{ and } y^*x = x(yx)^*.$$

An adequate semigroup satisfying these identities is said to be *ample*, and an Ehresmann semigroup satisfying them is known as *restriction*; such semigroups have been widely investigated, with techniques often parallel to those for inverse semigroups.

Adequate or Ehresmann semigroups, not satisfying the ample identities, do not behave like inverse semigroups. In particular, the structure of free algebras is not related to semidirect products, and neither can we use semidirect products to produce so-called 'proper covers'. Their investigation requires alternative approaches. The pioneer in this direction was Lawson; in [6] he showed that the category of Ehresmann (adequate) semigroups and morphisms is isomorphic to the category of (strongly cancellative) Ehresmann categories and inductive functors. Somewhat later, Gould and Gomes studied fundamental Ehresmann semigroups, developing an analogue of the Munn semigroup T_E [3].

The structure of the free adequate semigroup was discovered by Kambites [5], in terms of bi-rooted labelled trees. These may be regarded as an 'inflation' of Munn trees - allowing further collapse yields Munn trees, and a natural morphism to the free ample semigroup, which is embedded in the free inverse monoid [2].

We present our recent work which constructs an Ehresmann semigroup $\mathcal{P}(T,Y)$ from a monoid T acting by order preserving maps on a semilattice Y. Our semigroups satisfy a notion that we call 'T-proper'. We show that $\mathcal{P}(X^*,Y)$ is adequate and every Ehresmann semigroup has a cover (i.e. a projection separating preimage) of the form $\mathcal{P}(X^*,Y)$. Moreover, the free adequate semigroup on a set X is of the form $\mathcal{P}(X^*,Y)$. The construction of $\mathcal{P}(T,Y)$ is rather far from that of a semidirect product, and is inspired by that in the one-side case [4], which itself was influenced by an early result of Fountain [1].

This is joint work with MÁRIO BRANCO, GRACINDA GOMES (University of Lisbon) and DANDAN YANG (University of York).

REFERENCES

[1] J.B. Fountain, 'Free right h-adequate semigroups' in *Semigroups, theory and applications* (Oberwolfach, 1986), 97–120, Lecture Notes in Math., 1320, Springer, Berlin-New York, 1988.

[2] J. Fountain, G.M.S. Gomes and V.Gould, 'Free ample monoids', *I.J.A.C.* **19** (2009), 527–554.

[3] G. Gomes and V. Gould, 'Fundamental Ehresmann semigroups', *Semigroup Forum* **63** (2001), 11–33,

[4] G. Gomes and V. Gould, 'Left adequate and left Ehresmann monoids II', *J. Algebra* **348** (2011), 171-195.

[5] M. Kambites, 'Free adequate semigroups', *J. Australian Math. Soc.* **91** (2011), 365-390.

[6] M.V. Lawson, 'Semigroups and ordered categories I. The Reduced Case', *J. Algebra* **141** (1991), 422–462.

Infinite monoids as geometric objects

ROBERT GRAY

School of Mathematics

University of East Anglia, NORWICH (GBR)

Robert.D.Gray@uea.ac.uk

I shall describe ongoing joint work with MARK KAMBITES (University of Manchester) on the development of geometric methods for finitely generated monoids and semigroups. We study a notion of quasi-isometry between spaces equipped with asymmetric, partially defined distance functions (so called, semimetric spaces) and hence between finitely generated semigroups and monoids via their directed Cayley graphs. I shall give an overview of some basic concepts and results from this theory, and show how these ideas may be applied to investigate quasi-isometry invariants of finitely generated monoids.

Free adequate semigroups

MARK KAMBITES
School of Mathematics
University of MANCHESTER (GBR)
Mark.Kambites@manchester.ac.uk

Adequate semigroups form a class of semigroups in which cancellation properties of elements in general are strongly governed by cancellation properties of idempotents, and in which idempotents commute. Introduced by Fountain in 1979, they simultaneously generalise inverse semigroups and cancellative monoids, and their theory might be viewed as abstracting the common behaviour of these two well-studied, but rather disparate, classes of semigroups. I will describe an explicit geometric realisation of the free objects in the quasivariety of adequate semigroups, as sets of trees under a natural multiplication. This approach is inspired by and related to, but rather different from, Munn's celebrated description of free inverse semigroups.

Finitely based finite algebras

KEITH KEARNES
Department of Mathematics
University of Colorado, BOULDER (USA)
kearnes@euclid.colorado.edu

I will discuss the question of whether a finitely generated variety of algebras with finitely many subdirectly irreducible members is finitely axiomatizable.

Malcev families of quasivarieties closed under join or Malcev product

RALPH MCKENZIE
Department of Mathematics
Vanderbilt University, NASHVILLE (USA)
Ralph.N.McKenzie@vanderbilt.edu

We show that if K and L are quasivarieties of idempotent algebras satisfying $\mathcal{P}$ where $\mathcal{P}$ is any of the properties next listed, then the Malcev product of K and L satisfies $\mathcal{P}$, and therefore the variety generated by $K \cup L$ satisfies $\mathcal{P}$. These properties are: "has a Taylor term", "has a cube term", "has meet-semi-distributive congruence lattices", "has semi-distributive congruence lattices", "has n-permuting congruences, for some integer $n > 1$", "has a non-trivial congruence identity".

On the other hand, we exhibit examples of finite idempotent algebras **A** and **B**, each of which generates a variety satisfying $\mathcal{Q}$, while $\mathbf{A} \times \mathbf{B}$ does not, where $\mathcal{Q}$ is any one of: "has a Malcev term", "has Jónsson operations", "has Day operations".

These are joint results with RALPH FREESE (Honolulu).

Sierpiński rank of groups and semigroups

JAMES D. MITCHELL
School of Mathematics and Statistics
University of ST ANDREWS (GBR)
jdm3@st-and.ac.uk

The Sierpiński rank of a semigroup S is the least $n \in \mathbb{N}$ such that every countable subsemigroup of S is contained in an n-generated subsemigroup. This property is named for Sierpiński who showed that the Sierpiński rank of the semigroup Ω^{Ω} of functions from an infinite set Ω to itself is 2. The Sierpiński rank of a countable semigroup is just the least size of a generating set, and we will be more interested in uncountable semigroups. Analogous notions of Sierpiński rank can also be defined for inverse semigroups and groups. There are many examples of semigroups with finite Sierpiński rank in the literature. For example, semigroups of: partial functions, injective functions, surjective functions, binary relations, partitions, or partial bijections on an infinite set; continuous functions on the closed unit interval, the Cantor space, the Hilbert cube, the endomorphism semigroup of the countably infinite random graph, the universal poset, semilattice, distributive lattice, Boolean algebra, and more... There are significantly fewer groups which are known to have finite Sierpiński rank: the symmetric group, homeomorphisms of the Cantor space, $\mathbb{Q}$, $\mathbb{R} \setminus \mathbb{Q}$ are some examples.

In this talk, I will discuss the semigroups and group that have been most recently shown to have finite Sierpiński rank and I will relate the property of having finite Sierpiński rank to some other notions such as universal sequences and Bergman's property.

This is joint work with J. HYDE, J. JONUŠAS and Y. PÉRESSE (St Andrews).

Topological methods in model theory

LUDOMIR NEWELSKI
Institute of Mathematics
University of WROCŁAW (POL)
Ludomir.Newelski@math.uni.wroc.pl

I will survey the development of topological methods in model theory. I will discuss Morley rank and forking in stable theories and also various ways to generalize it to the unstable theories. I will focus on topological dynamics. Topological dynamics yields general counterparts of the notion of a generic type in a stable group.

Recent trends in model theory

ANAND PILLAY
Department of Pure Mathematics
University of Leeds, LEEDS (GBR)
pillay@maths.leeds.ac.uk

I will give a general introduction to model theory and stability, and mention some key themes as well as connections to algebraic geometry and number theory.

The 42 reducts of the random ordered graph

MICHAEL PINSKER
Équipe de Logique Mathḿatique
Université Diderot - Paris 7, PARIS (FRA)
marula@gmx.at

The *random ordered graph* is the up to isomorphism unique countable homogeneous structure that embeds all finite linearly ordered graphs. I will present a complete classification of all structures which have a first order definition in the random ordered graph. This classification includes in particular all structures definable in the order of the rationals (previously classified by Cameron '76), the random graph (Thomas '91) and the random tournament (Bennett '97). We obtained the result by the recent method of canonical functions, which I will outline.

This is joint work with M. BODIRSKY and A. PONGRÁCZ.

Free idempotent generated semigroups

Nikola Ruškuc
School of Mathematics and Statistics
University of St Andrews, ST ANDREWS (GBR)
nik@mcs.st-and.ac.uk

Let S be a semigroup, and let E be its set of idempotents. The structure of the set E can naturally be described as a bi-ordered set, a notion arising as a generalisation of the semilattice of idempotents in inverse semigroups. The free-est idempotent generated semigroup with this bi-order of idempotents is given by the presentation

$$\mathsf{IG}(E) = \langle E \mid \mid e \cdot f = ef \ (e, f \in E,\ \{e, f\} \cap \{ef, fe\} \neq \emptyset)\rangle.$$

(Here $e \cdot f$ is a product of two generating symbols, while ef stands for their product in S, which is an idempotent as a consequence of the condition $\{e, f\} \cap \{ef, fe\} \neq \emptyset$.) Given the controlling position that idempotents have in a free idempotent generated semigroup, it is natural to ask after the maximal subgroups in this semigroup. For instance, it is known that if S is a completely 0-simple semigroup, then all the maximal subgroups of the corresponding free idempotent generated semigroup are free (Pastijn).

Recently there has been a lot of work in describing maximal subgroups of free idempotent generated semigroups, e.g. by Brittenham, Dolinka, Gould, Gray, Margolis, Meakin, Yang and myself. In this talk I will attempt to go 'behind the scenes' of two such results, and convey the idea as to how they work. The first computes the maximal subgroups in the free idempotent generated semigroups arising from a concrete natural family of semigroups:

Theorem 1 (Gray, Ruškuc 2012)**.** *Let T_n be the full transformation semigroup, let E be its set of idempotents, and let $e \in E$ be an arbitrary idempotent with image size r ($1 \leq r \leq n-2$). Then the maximal subgroup H_e of the free idempotent generated semigroup $\mathsf{IG}(E)$ containing e is isomorphic to the symmetric group S_r. (For $r = n-1$ the group H_e is free, and for $r = n$ it is trivial.)*

The second shows that even very specialised class of semigroups such as bands can yield all groups as maximal subgroups of the corresponding free idempotent generated semigroups:

Theorem 2 (Dolinka, Ruškuc 2013)**.** *Let G be a group. Then there exists a band B_G such that $\mathsf{IG}(B_G)$ has a maximal subgroup isomorphic to G. Furthermore, if G is finitely presented, then B_G can be constructed to be finite.*

I will hope to bring out the common combinatorial viewpoint underlying both proofs, which might be of use in subsequent attempts to compute $\mathsf{IG}(E)$ for further specific classes of semigroups. I also hope that these viewpoints and methodology will feed into attempts to address the next major topic in this area, that of the word problem, and conditions under which it is decidable.

Quasivarieties and hyperplane arrangements

BENJAMIN STEINBERG
Department of Mathematics
City College of New York, NEW YORK (USA)
bsteinberg@ccny.cuny.edu

Tits introduced a multiplication on the faces of a hyperplane arrangement providing a structure of a left regular band. It is easy to see that a finite semigroup embeds in a hyperplane face monoid if and only if it belongs to the quasivariety generated by the 3-element monoid consisting of an identity and 2 left zeroes. We prove this quasivariety has no finite basis for its quasi-identities but has a polynomial time membership algorithm. Note that it is known that every normal band has a finite basis for its quasi-identities and that there are uncountably many quasivarieties of left regular bands.

This is joint work with STUART MARGOLIS and FRANCO SALIOLA.

Dualizable algebras

ÁGNES SZENDREI
Department of Mathematics
University of Colorado, BOULDER (USA)
szendrei@euclid.colorado.edu

I will discuss a sufficient condition which ensures that a finite algebra in a residually small variety with a cube term is dualizable. Known results on dualizable groups and rings follow as special cases.

Epigroup varieties with modular subvariety lattices

MIKHAIL V. VOLKOV
Institute of Mathematics and Computer Science
Ural Federal University, EKATERINBURG (RUS)
Mikhail.Volkov@usu.ru

A semigroup S is called an *epigroup* if, for any $a \in S$, there exists a positive integer n such that a^n is a *group element*, that is, belongs to a subgroup of S. Epigroups occur in the literature under various names; the term *epigroup* which is shorter and more flexible was suggested by Shevrin who also promoted the idea of viewing epigroups as semigroups with an additional unary operation. Indeed, it is well known (and easy to verify) that, for each element a of an epigroup S, there exists a unique maximal subgroup H of S which contains all but a finite number of powers of a. Denote the identity element of this group H by a^0; then $aa^0 = a^0a \in H$. Let $\overline{a}$ denote the inverse of $aa^0 = a^0a$ in H. This defines a new unary operation $a \mapsto \overline{a}$ on every epigroup. Recall that the class of all groups and the class of all completely regular semigroups—considered as unary semigroups with their natural unary operation—form varieties of epigroups and, as such, they have modular subvariety lattices. (For groups the modularity of the subvariety lattice is obvious while for completely regular semigroups it is a highly non-trivial result obtained independently by Pastijn and Petrich–Reilly at the beginning of the 1990s.)

We have found a complete classification of epigroup varieties with modular subvariety lattices thus solving a problem proposed by Shevrin in 1994. It the talk we outline the main ingredients of the classification and some of its consequences.

This is joint work with B. M. VERNIKOV, V. YU. SHAPRYNSKIĬ, and D. V. SKOKOV.

Graphs, polymorphisms, and multi-sorted structures

ROSS WILLARD
Department of Pure Mathematics
University of WATERLOO (CAN)
rdwillar@uwaterloo.ca

Given a relational structure **A**, the polymorphism algebra of **A** is the algebra having the same universe as **A** and whose operations are all homomorphisms $\mathbf{A}^n \to \mathbf{A}$. Finite algebras arising in this way have been studied for a long time, for example when **A** is a poset. In recent years, interest in the Constraint Satisfaction Problem Dichotomy Conjecture has shifted focus to the structures themselves, with the corresponding polymorphism algebras playing a supporting role.

In this talk I will survey some of the basic questions about finite relational structures that are currently under investigation, present some partial answers for bipartite graphs, and explain why it is useful to embrace multi-sorted structures.

CONTRIBUTED SHORT TALKS

Gauss' Lemma and valuation theory

PHẠM NGỌC ÁNH
Renyi Institute of Mathematics
Hungarian Academy of Sciences, BUDAPEST (HUN)
anh.pham.ngoc@renyi.mta.hu

This is a joint work with M. SIDDOWAY (Colorado College, USA).

We give a unified treatment of Gauss' Lemma by emphasizing its role in valuation theory. As an obvious consequence, one gets immediately the theorem of Ohm-Jaffard-Kaplansky on lattice-ordered abelian groups, Eisenstein's Criterion over arbitrary commutative rings, in particular, over Dedekind domains.

Pseudovarieties generated by Brauer type monoids

KARL AUINGER
Faculty of Mathematics
University of VIENNA (AUT)
karl.auinger@univie.ac.at

The pseudovariety generated by the series $\{\mathfrak{B}_n \mid n \in \mathbb{N}\}$ of all *Brauer monoids* is the pseudovariety $\mathbf{M}$ of all finite monoid while the pseudovariety generated by the series $\{\mathfrak{J}_n \mid n \in \mathbb{N}\}$ of all *Jones monoids* (also called *Temperly–Lieb monoids*) is the pseudovariety $\mathbf{A}$ of all finite aperiodic monoids. I shall give some idea of the proof of these results — based on wreath product decomposition and Krohn–Rhodes theory. The fact that the Jones monoids $\mathfrak{J}_n$ form a generating series for the pseudovariety $\mathbf{A}$ can be viewed as a solution to a problem raised by J.-É. Pin at the Szeged International Semigroup Colloquium in 1987. For the latter, the relationship between the Jones monoids $\mathfrak{J}_n$ and the monoids $\mathscr{O}_n$ of all order preserving mappings of a chain of length n is discussed.

Semigroups and geometric spaces

JENNI AWANG
School of Mathematics and Statistics
University of ST ANDREWS (GBR)
jenni@mcs.st-and.ac.uk

Many properties of groups can be found geometrically, through examining their Cayley graphs, which are well-understood metric spaces. Semigroups too, may be investigated via their Cayley graphs. Gray and Kambites in [1, 2, 3] mainly consider directed Cayley graphs, which are semi-metric spaces that carry directional information, and are less well understood than metric spaces. In our approach, we construct a simplified Cayley graph as follows: given a semigroup S, we define $\dagger(S)$ to be the Cayley graph of S from which we have removed all labels, edge directions, multiple edges and loops. Having lost all the directional information, is it still possible to recover any properties of the semigroup from these graphs? We take a combinatorial approach to investigate semigroups via these graphs. Geometric group theory is also still useful in our approach: we show an application of the Švarc-Milnor lemma, for certain families of semigroups, to obtain some results about finite presentability.

REFERENCES

[1] R. Gray and M. Kambites. A Švarc-Milnor lemma for monoids acting by isometric embeddings. Internat. J. Algebra Comput., 21:1135–1147, 2011.

[2] R. Gray and M. Kambites. Groups acting on semimetric spaces and quasi-isometries of monoids. Trans. Amer. Math. Soc, 365:555–578, 2013.

[3] R. Gray and M. Kambites. Quasi-isometry and finite presentations for left cancellative monoids. Internat. J. Algebra Comput., to appear.

On semifields of order q^4 with center F_q, admitting a Klein four-group of automorphisms

MASHHOUR BANI ATA

Department of Mathematics

PAAET, KUWAIT CITY (KUW)

mashhour_ibrahim@yahoo.com

In this talk we investigate the semifields of order q^4 over a finite field of order q, q an odd prime-power, admitting a Klein four-group of automorphisms. Here a semifield over a field K is understood as a central division algebra over a field or as a semifield containing K in its center. The obtained results obtained will be used in a subsequent paper to show that there are no such semifields in higher dimension i.e. for odd prime powers q and even prime powers $n > 4$ there are no semifields over F_q of order q^n admitting an elementary abelian group of automorphisms of order n.

On multipalindromic sequences

BOJAN BAŠIĆ
Department of Mathematics and Informatics
University of NOVI SAD (SRB)
bojan.basic@dmi.uns.ac.rs

We call a number a *palindrome in base b* if for its expansion in base b, say

$$\langle c_{d-1}, c_{d-2}, \ldots, c_0 \rangle_b$$

($c_{d-1} \neq 0$), the equality $c_j = c_{d-1-j}$ holds for every $0 \leqslant j \leqslant d-1$. This talk focuses on numbers that are simultaneously palindromes in more different bases.

We first answer the question recently posed in the literature: what is the longest list of bases for which there exists a number that is simultaneously a d-digit palindrome in all the bases from the list. It turns out that there is no upper bound on the length of such a list, that is, a number can be a d-digit palindrome in arbitrarily many bases, even if d is given in advance. We then concentrate on palindromic sequences $\langle c_{d-1}, c_{d-2}, \ldots, c_0 \rangle$, $c_{d-1} \neq 0$, such that for any $K \in \mathbb{N}$ there exists a base b for which the number $n = \langle c_{d-1}, c_{d-2}, \ldots, c_0 \rangle_b$ is simultaneously a palindrome, respectively a d-digit palindrome, in K different bases. Regarding the first question (palindromes with a variable number of digits) it is easy to show that all the palindromic sequences have the described property. Regarding the second question, the cases $d = 1$ and $d = 2$ are easy, while for $d = 3$ we present two different constructions that show that again all the palindromic sequences have the described property. Finally, we give some further comments on the case $d \geqslant 4$.

Relational Structure Theory – localising algebras, and more

MIKE BEHRISCH
Institute of Algebra
University of Technology, DRESDEN (GER)
mike.behrisch@mailbox.tu-dresden.de

Relational Structure Theory is a localisation theory for algebras, originating from ideas by K. Kearnes and Á. Szendrei ([Kea01]) on how to interpret key concepts of Tame Congruence Theory for term operations instead of polynomials and arbitrary finitary compatible relations instead of congruences. In this context, algebras are localised to special subsets, called *neighbourhoods*, which are images of idempotent unary operations in the least locally closed clone containing all fundamental operations. Every algebra is associated with a *relational counterpart* carrying all its compatible relations, and localisation is achieved by taking the induced relational substructure and converting it back to an algebra. From a collection of neighbourhoods satisfying a certain separation property w.r.t. compatible relations, one can reconstruct an algebra up to local term equivalence, i.e. up to equality of the associated clone of compatible relations. Such collections are called *covers*, and the local structures in a cover can be used to describe the variety generated by an algebra up to categorical equivalence (at least for algebras in locally finite varieties).

In this sense covers can be seen as a means of decomposition of algebras belonging to locally finite varieties. It is known that finite algebras always possess covers satisfying a special optimality condition, called *non-refinability*, and that non-refinable covers are unique in a natural way. This result constitutes the basis for characterisations of categorical equivalence of finite algebras, see e.g. [Iza12].

In the talk, new concepts and constructions are presented that allow to considerably extend the scope of the existence and uniqueness theorem for non-refinable covers from finite algebras to so-called poly-Artinian algebras in 1-locally finite varieties. The abstract machinery developed to obtain this result suggests that also more general structures than algebras might be localised in a similar way, and hints at which tasks have to be solved to get decompositions that are unique in a certain sense.

References

[Iza12] Shohei Izawa, *Composition of matrix products and categorical equivalence*, to appear in Algebra Universalis, 2012.

[Kea01] Keith A. Kearnes, *Tame Congruence Theory is a localization theory*, Lecture Notes from "A Course in Tame Congruence Theory" Workshop, Paul Erdős Summer Research Center of Mathematics, Budapest, 2001, available at `http://www.math.u-szeged.hu/confer/algebra/2001/lec+ex1.ps`, July 2001.

Groups that are almost synchronizing

WOLFRAM BENTZ

Centre of Algebra
University of LISBON (POR)
wfbentz@fc.ul.pt

Synchronization can be understood as a method of error recovery. From time to time software enters a faulty state. To recover from this state, systems can use a backward error recovery approach, such as restarting a database from a checkpoint. When this is not possible, an alternative approach consists in doing forward error recovery: something that can bring the process to a known state, irrespective of its current state. For an automaton, a forward recovery option consists of a sequence of instructions, which ends up in the same state irrespective of the state in which it starts; this is called a *reset* (or *synchronizing*) word. Not all automata have a synchronizing word, hence we would like to classify those that do.

Algebraically we can address this question by asking if the transition semigroup associated to the automaton contains a constant map. An important case of this approach is the situation where this semigroup is generated by a primitive group G together with a singular transformation t, both acting on the state set X. By recent results proved by P. M. Neumann, for some G and singular transformations t of uniform kernel, the semigroup $\langle G, t \rangle$ does not generate a constant map. We say that a primitive group G on X is *synchronizing* if G together with any non-invertible map on X generates a constant.

In this talk, we generalize this notion, saying that a primitive group G on X is *almost synchronizing* if G together with any map of non-uniform kernel generates a constant. We introduce two methods to find groups that are not synchronizing, but are almost synchronizing, and provide several infinite families of such groups. We close by raising various questions on synchronization motivated by our results.

This is a joint work with JOÃO ARAÚJO (Centro de Álgebra, Universidade de Lisboa), and PETER J. CAMERON (Queen Mary, University of London).

Automaton semigroup constructions

TARA BROUGH
School of Mathematics and Statistics
University of ST ANDREWS (GBR)
tara@mcs.st-andrews.ac.uk

Automaton semigroups are a natural generalisation of the self-similar automata groups introduced by Grigorchuk and others in the 1980s as a source of exotic examples in group theory. In this talk I will introduce some of the basic theory of automaton semigroups and present joint work with ALAN CAIN (Porto) on the extent to which the class of automaton semigroups is closed under various standard semigroup constructions.

Algebraic approach to coloring by oriented trees

JAKUB BULÍN
Department of Algebra
Charles University, PRAGUE (CZE)
jakub.bulin@gmail.com

Given a directed graph H, the H-coloring problem is the following decision problem: input another digraph G and ask whether there exists a homomorphism from G to H. The CSP dichotomy conjecture of Feder and Vardi says that the H-coloring problem is always either in P or NP-complete. The so-called algebraic approach to CSP, a link of this combinatorial problem to universal algebra, has led to a significant progress towards solving this conjecture and has brought a new insight into some structural properties of digraphs. We will present a few results and tools from this area and then apply them on the case when H is an oriented tree. We will show that the CSP dichotomy holds for a certain class of oriented trees, and discuss possible directions for future research as well as related open problems.

Autocommutators and special automorphisms in certain 2-groups

MIHAI CHIŞ
West University of TIMIŞOARA (ROM)
chis@math.uvt.ro

We determine the autocommutator subgroup $K(G)$, generated by the autocommutators $[g,\alpha] = g^{-1}g^{\alpha}$, with $g \in G$ and $\alpha \in \mathrm{Aut}(G)$, for certain families of groups, including dihedral groups, quasidihedral groups, generalized quaternion groups, and also for all 2-groups of order ≤ 32. We also investigate for particular characteristic subgroups N of a group G the N-al automorphisms of the group G, which are the automorphisms whose autocommutators belong to N. In particular, we determine the orbits of the natural actions of the central automorphisms (for $N = Z(G)$) and of the derival automorphisms (for $N = G'$) on certain 2-groups.

This is joint work with CODRUŢA CHIŞ (Timişoara).

Coordinatization of join-distributive lattices

GÁBOR CZÉDLI
Bolyai Institute
University of SZEGED (HUN)
czedli@math.u-szeged.hu

Join-distributive lattices are finite, meet-semidistributive, and semimodular lattices. They are the same as Dilworth's lattices in 1940, and many alternative definitions and equivalent concepts have been discovered or rediscovered since then. Let L be a join-distributive lattice of length n, and let k denote the width of the set of join-irreducible elements of L. A result of P. H. Edelman and R. E. Jamison, translated from Combinatorics to Lattice Theory, says that L can be described by $k-1$ permutations acting on the set $\{1,\dots,n\}$. We prove a similar result, see arXiv:1208.3517 of 17 August 2012 (latest revision 12 October 2012), *within Lattice Theory*: there exist $k-1$ permutations acting on $\{1,\dots,n\}$ such that the elements of L are coordinatized by k-tuples over $\{0,\dots,n\}$, and the permutations determine which k-tuples are allowed. Since the concept of join-distributive lattices is equivalent to that of antimatroids and convex geometries, our result offers a coordinatization for these combinatorial structures.

Later, a joint work with KIRA ADARICHEVA (Yeshiva Univ.), see arXiv:1210.3376 of 11 October 2012, pointed out that both our lattice theoretic approach and the Edelman–Jamison combinatorial approach can be derived from each other. A new characterization of join-distributive lattices was also given.

A generalization of the Kaloujnine–Krasner Theorem

TAMÁS DÉKÁNY
Bolyai Institute
University of SZEGED (HUN)
`tdekany@math.u-szeged.hu`

In this talk we will generalize a well-known theorem of group theory for the class of completely simple semigroups. The extensions of groups play a fundamental role both in the structure theory and in the theory of varieties of groups. Kaloujnine and Krasner (1950) proved that for any groups N and H, every extension of N by H is embeddable in the wreath product of N by H, i.e. in a semidirect product of a direct power of N by H determined by N and H.

Any group congruence ρ of a completely simple semigroup S determines a normal subgroup N in every maximal subgroup G of S, and their union is the identity ρ-class, which is a completely simple subsemigroup in S, and S/ρ is isomorphic to G/N. A completely simple semigroup S is called an extension of a completely simple semigroup K by a group H if there exists a group congruence ρ on S such that the identity ρ-class, as a completely simple subsemigroup of S, is isomorphic to K, and S/ρ is isomorphic to H.

In the talk, we give a completely simple semigroup which is an extension of a completely simple semigroup K by a group H, and which is not embeddable in the wreath product of K by H. On the other hand we show that any extension of a completely simple semigroup K by a group H is embeddable in a semidirect product of a completely simple semigroup T by H, where the maximal subgroups of T are direct powers of the maximal subgroups of K.

This is a joint work with MÁRIA B. SZENDREI (University of Szeged).

Expressibility of graph homomorphism obstructions in the logic $LFP + Rank$

DEJAN DELIĆ
Department of Mathematics
Ryerson University, TORONTO (CAN)
ddelic@ryerson.ca

One of the fundamental problems in finite model theory is the quest for the logic which captures polynomial time on finite (di)graphs. From the algebraic point of view, an interesting restriction of this problem asks whether there is a logic L strong enough to capture, given a finite digraph G, the class $\neg HOM(G)$ of all finite digraphs not homomorphic to G and such that the truth of L-sentences on finite digraphs can be decided in polynomial time. In 2007, Atserias, Bulatov, and Dawar showed that the $LFP + C$ cannot capture the homomorphism problem on digraphs, where C is the counting operator. Recently, with Bulín, Jackson, and Niven, we refined the original method of Feder and Vardi of translating the constraint satisfaction problem for general relational structures to digraphs in such a way that it preserves the algebraic reasons for polynomial time solvability. In this talk, we present a very recent result, obtained with F. McInerney and C. Heggerud, which shows that, under the aforementioned transformation, if $\neg HOM(\mathbb{A})$ is definable by a $LFP + Rank$ sentence for a finite relational template $\mathbb{A}$, then $\neg HOM(D_{\mathbb{A}})$ is definable in the same logic, where $D_{\mathbb{A}}$ is the digraph obtained from the relational template $\mathbb{A}$. In conclusion, we discuss some related conjectures.

Infinite partition monoids

JAMES EAST
School of Computing, Engineering and Mathematics
University of Western Sydney, SYDNEY (AUS)
J.East@uws.edu.au

We investigate the partition monoids $\mathcal{P}_X$ on an infinite set X, taking as our inspiration various classical results in the theory of infinite transformation semigroups. We prove that $\mathcal{P}_X$ may be generated by the symmetric group $\mathcal{S}_X$ or the set of idempotent partitions $E(\mathcal{P}_X)$ together with two (but no fewer) additional partitions, and we classify the pairs $\alpha, \beta \in \mathcal{P}_X$ such that $\mathcal{P}_X$ is generated by $\mathcal{S}_X \cup \{\alpha, \beta\}$ or $E(\mathcal{P}_X) \cup \{\alpha, \beta\}$. In the case of $\mathcal{S}_X$, the classification depends crucially on whether X is countable, uncountable but regular, or singular. Among other things, we show that any countable subset of $\mathcal{P}_X$ is contained in a 4-generator subsemigroup of $\mathcal{P}_X$, and that the length function on $\mathcal{P}_X$ with respect to any generating set is bounded.

On enumerating transformation semigroups

ATTILA EGRI-NAGY
School of Computing, Engineering and Mathematics
University of Western Sydney, SYDNEY (AUS)
a.egri-nagy@uws.edu.au

Motivated by other algorithmic problems, we are aiming to enumerate transformation semigroups on n points by finding all subsemigroups of the full transformation semigroup T_n.

Pen and paper calculation shows that there are 10 subsemigroups of T_2 in 8 conjugacy classes. Brute force computer calculation (checking all subsets) gives the answer for $n = 3$: there are 1299 subsemigroups in 283 conjugacy classes.

Due to the huge search space, for $n = 4$, we have to use a different method. With some heuristics applied we recursively reduce the multiplication table of the semigroup. Computations are under way, but we know that there are (at least) 3788252 subsemigroups in 162332 conjugacy classes of $K_{4,2}$, the semigroup of all transformations on 4 points with image size of maximum 2. Due to the generality of the method, we can enumerate the subsemigroups of an arbitrary transformation semigroup and we can ask special questions like "What are the semigroups that contain no constant maps?". This can be used for enumerating automata with no synchronizing words.

In this talk we describe the reduction algorithm and the obtained data sets in more detail. This is a joint work with JAMES D. MITCHELL (University of St Andrews) and JAMES EAST (University of Western Sydney).

Ordered sets in information theory

STEPHAN FOLDES
TAMPERE University of Technology (FIN)
stephan.foldes@tut.fi

Some connections between uniquely decipherable codes and partially ordered sets will be commented on. These will relate in particular to the ordering of codes by refinement and the Kraft sum, and to executions of Huffman's algorithm viewed as chains in a partition lattice.

On automorphisms, derivations and elementary operators

ILJA GOGIĆ
Department of Mathematics
University of ZAGREB (CRO)
ilja@math.hr

Let R be a unital semiprime ring. An interesting class of additive maps $d : R \to R$ including both automorphisms and (generalized) derivations is the class of *generalized skew derivations*, that is, those satisfying

$$d(xy) = \delta(x)y + \sigma(x)d(y) \qquad (x, y \in R),$$

for some map $\delta : R \to R$ and automorphism $\sigma \in \mathrm{Aut}(R)$.

On the other hand, an attractive and fairly large class of additive maps $\phi : R \to R$ is the class of *elementary operators*, that is, those which can be expressed as a finite sum

$$\phi(x) = \sum_i a_i x b_i \qquad (x \in R),$$

where $a_i, b_i \in R$.

Motivated by the fact that elementary operators comprise both inner automorphisms $x \mapsto axa^{-1}$ and generalized inner derivations $x \mapsto ax - xb$, we consider the following problem:

Problem. *Describe the form of generalized skew derivations which are also elementary operators.*

In this talk we shall present some results, examples, and open questions on the above problem.

This is a joint work with DANIEL EREMITA (University of Maribor) and DIJANA ILIŠEVIĆ (University of Zagreb).

Islands and proximity domains

ESZTER K. HORVÁTH
Bolyai Institute
University of SZEGED (HUN)
horeszt@math.u-szeged.hu

An *island domain* is a pair $(\mathcal{C}, \mathcal{K})$, where $\mathcal{C} \subseteq \mathcal{K} \subseteq \mathcal{P}(U)$ for some nonempty finite set U such that $U \in \mathcal{C}$. By a *height function* we mean a map $h\colon U \to \mathbb{R}$. We denote the cover relation of the poset $(\mathcal{K}, \subseteq)$ by $\prec$, and we write $K_1 \preceq K_2$ if $K_1 \prec K_2$ or

$K_1 = K_2$. We say that S is an *island* with respect to the triple $(\mathcal{C}, \mathcal{K}, h)$, if every $K \in \mathcal{K}$ with $S \prec K$ satisfies

$$h(u) < \min h(S) \text{ for all } u \in K \setminus S.$$

An island domain $(\mathcal{C}, \mathcal{K})$ is called a *connective island domain* if

$$\forall A, B \in \mathcal{C} : \ (A \cap B \neq \emptyset \text{ and } B \nsubseteq A) \implies \exists K \in \mathcal{K} : A \subset K \subseteq A \cup B.$$

We define a binary relation $\delta \subseteq \mathcal{C} \times \mathcal{C}$ that expresses the fact that a set $B \in \mathcal{C}$ is in some sense close to a set $A \in \mathcal{C}$:

$$A\delta B \Leftrightarrow \exists K \in \mathcal{K} : \ \ A \preceq K \text{ and } K \cap B \neq \emptyset.$$

The island domain $(\mathcal{C}, \mathcal{K})$ is called a *proximity domain*, if it is a connective island domain and the relation δ is symmetric for nonempty sets, that is

$$\forall A, B \in \mathcal{C} \setminus \{\emptyset\} : \ A\delta B \Leftrightarrow B\delta A.$$

We characterize systems of islands in proximity domains. We investigate the following condition on $(\mathcal{C}, \mathcal{K})$, which is stronger than that of being a connective island domain:

$$\forall K_1, K_2 \in \mathcal{K} : \ K_1 \cap K_2 \neq \emptyset \implies K_1 \cup K_2 \in \mathcal{K}.$$

If we have a graph structure on U, and $(\mathcal{C}, \mathcal{K})$ is a corresponding island domain, then this above condition holds.

This is a joint work with STEPHAN FOLDES (Tampere University of Technology), SÁNDOR RADELECZKI (University of Miskolc) and TAMÁS WALDHAUSER (University of Szeged).

Sierpiński rank and universal sets

JAMES HYDE
School of Mathematics and Statistics
University of ST ANDREWS (GBR)
jth4@st-andrews.ac.uk

The Sierpiński rank of a semigroup S is the least n such that any countable subset of S is contained in some n-generated subsemigroup of S.

A subset X of F_n the free semigroup on n points is universal for S if every function from X to S is the restriction of some homomorphism from F_n to S. The universal set rank of a semigroup S is the least n (possibly infinite) such that F_n has an infinite subset which is universal for S.

We will see some ranks and some examples of universal sets for various semigroups and give some properties of universal sets.

This is a joint work with JULIUS JONUŠAS, JAMES MITCHELL and YANN PÉRESSE.

Small expansions of $(\omega, <)$ and $(\omega + \omega^*, <)$

DEJAN ILIĆ
Faculty of Transport and Traffic Engineering
University of BELGRADE (SRB)
d.ilic@sf.bg.ac.rs

An *expansion* of a first-order structure is any structure obtained from it by adding additional finitary relations and functions. By a *unary expansion* we mean an expansion which is obtained by adding only unary relations. Two first-order structures are *definitionally equivalent* iff they have the same domain and the same definable sets. An expansion is *essentially unary* if it is definitionally equivalent to a unary expansion. An expansion is *definitional* if it is definitionally equivalent to original structure.

We consider expansions of either $(\omega, <)$ or $(\omega + \omega^*, <)$ (in which ω is not definable), requiring that the complete first-order theory is small: there are only countably many complete types without parameters. Examples are unary expansions $(\omega, <, P_d)$ where $P_d(x)$ is "d divides x", and $(\omega + \omega^*, <, P_d)$ where $P_d(x)$ is "d divides x" for $x \in \omega \cup \omega^*$. The main question is whether any considered expansion has to be essentially unary. We prove a partial result in this direction:

Theorem: Let $\mathcal{M}$ be $(\omega, <, ...)$ or $(\omega + \omega^*, <, ...)$ and such that $\mathrm{Th}(\mathcal{M})$ is small and $\mathrm{CB}(x = x) = 1$. Then $\mathcal{M}$ is an essentially unary expansion. Particularly,

1) If $\mathrm{CBdeg}(x = x) = 1$, then $\mathcal{M}$ is definitional expansion;

2) If $\mathrm{CBdeg}(x = x) = d > 1$, then $\mathcal{M}$ is definitionally equivalent to a variation of the above examples. □

Pillay and Steinhorn proved that $(\omega, <)$ is definitionally equivalent to any of its o-minimal expansions (Theorem 3.1 in *Discrete O-Minimal Structures*, Annals of Pure and Applied Logic 34, 1987). This result easily implies case 1) above. Our proof here is new even in this case, and it works simultaneously for both $(\omega, <)$ and $(\omega + \omega^*, <)$.

Monoid varieties with continuum many subvarieties

MARCEL JACKSON
Department of Mathematics and Statistics
La Trobe University, MELBOURNE (AUS)
m.g.jackson@latrobe.edu.au

There is extensive literature concerning finitely generated varieties of semigroups with uncountable subvariety lattices, but it seems substantially harder to construct examples in the language of monoids. For example, there are broad classes K for which a semigroup from K generates a variety with continuum many subvarieties if and only if it generates a semigroup variety containing the seven element monoid $S(xyx)$, consisting of the Rees quotient of the free monoid $\{x,y\}^*$ by the ideal of all words not containing xyx as a subword. But in contrast, the *monoid* variety generated by $S(xyx)$ has a subvariety lattice isomorphic to the five element chain.

We identify an infinite independent system of monoid identities, and use this system to show that a number of finite monoids generate monoid varieties whose subvariety lattice embeds the powerset lattice $\wp(\mathbb{N})$. We show that this property is held by the monoid variety of the 9 element monoid $S(xyxy)$, as well as by any finitely generated inherently nonfinitely based variety of monoids. Other results that follow include an example of two Cross varieties of monoids whose join has a subvariety lattice failing both the ACC and the DCC.

This is a joint work with EDMOND W. H. LEE (Nova Southeastern University).

On homomorphism-homogeneous point-line geometries

ÉVA JUNGÁBEL
Department of Mathematics and Informatics
University of Novi Sad (SRB)
eva.jungabel@dmi.uns.ac.rs

In this talk we discuss one class of homomorphism-homogenous point-line geometries. A structure S is homomorphism-homogeneous if every homomorphism from S' to S'', where S' and S'' are two finitely induced substructures, can be extended to an endomorphism of S. A point-line geometry is a non-empty finite set of points, together with a collection of subsets called lines such that every line contains at least two points and any pair of points is contained in at most one line. A line which contains more than two points is referred to as a regular line. A line which contains exactly two points is called singular. Homomorphism-homogenous point-line geometries containing two regular intersecting lines have already been described. In order to complete the characterization of homomorphism-homogeneous point-line geometries it is necessary to describe homomorphism-homogenous point-line geometries where no two regular lines intersect. In this talk we first show that the problem of deciding homomorphism-homogeneity of a point-line geometry where no two regular lines intersect and there exist points that do not lie on regular lines is a *coNP*-complete problem. Therefore, we focus on point-line geometries where every point lies on a regular line. We fully classify point-line geometries with only two regular nonintersecting lines, and in the case of more than two regular pairwise nonintersecting lines we provide a partial classification.

Reaching the minimum ideal in a finite semigroup

NASIM KARIMI
Department of Mathematics
University of PORTO (POR)
nasim.karimi@fc.up.pt

Consider a finite semigroup S with a generating set A. By the length of an element $s \in S$, with respect to A, we mean the minimum length of a sequence which represents s in terms of generators in A. Define the parameter $N(S, A)$ to be the minimum length of elements contained in the minimum ideal of S. Let the parameters $N(S)$, $M(S)$ be the minimum and the maximum of $N(S, A)$ over all generating sets of minimum size, respectively; and denote by $M'(S)$ the maximum of $N(S, A)$ over all generating sets of S. In the first part of this talk, we shall present some classes of semigroups for which the above-mentioned parameters have been estimated. Furthermore, we will present an upper bound for $N(S)$, provided that S is a wreath product of two finite semigroups. If the factors of the product do not have trivial groups of units, the diameter of a semidirect product of groups will appear in the obtained upper bound; and in the special case that one of factors has trivial group of units, the diameter of a direct power of a finite group will appear. In the second part of this talk, we will discuss the diameter of a direct power of a finite group.

How to decide absorption

ALEXANDR KAZDA
Department of Algebra
Charles University, PRAGUE (CZE)
alex.kazda@gmail.com

Let A be a finite algebra and B a subalgebra of A. If B is an absorbing subalgebra of A, then many kinds of connectivity properties of A are also true for B. This makes absorption very useful for proofs by induction.

Given A and B, it is not obvious if B is an absorbing subalgebra of A. A result by LIBOR BARTO and JAKUB BULÍN tells us how to decide absorption in finitely related algebras by looking for a chain of Jónsson-like terms.

In our talk we will show a one attempt to obtain a similar absorption-deciding algorithm for algebras that are perhaps not finitely related, but have a finite number of basic operations.

This is a joint work with LIBOR BARTO (Charles University in Prague).

Minimal matrix centralizers over the field $\mathbb{Z}_2$

DAMJANA KOKOL BUKOVŠEK
Faculty of Economics
University of LJUBLJANA (SVN)
damjana.kokol.bukovsek@ef.uni-lj.si

We partially order the set of all n-by-n matrices over a field as follows: $A \leq B$ if and only if the centralizer of A is a subset of the centralizer of B. Minimal matrices over infinite fields with respect to this partial order are well characterized, while in the case of finite fields the problem which matrices are minimal appears to be difficult. In the talk we study the minimal matrices over the field with two elements. We characterize the minimal matrices with their minimal polynomial of the form $p(x) = m(x)^k$, where $m(x)$ is an irreducible polynomial and $k \in \mathbb{N}$. We also characterize all minimal matrices with spectrum in $\mathbb{Z}_2$.

This is a joint work with DAVID DOLŽAN (University of Ljubljana).

Semigroup varieties determined by $zx^2 = zx$ and $zkxyw = zkyxw$

SAMUEL J. L. KOPAMU
Division of Mathematics and Computing
University of GOROKA (PNG)
samkopamu@gmail.com

A.V. Tischenko (2007) determined completely the lattice of all semigroup varieties determined by the pair of identities $zx^2 = zx$ and $zxy = zyx$. The finite sublattice so formed, which resembles somewhat like a box with its lid opened upwards, is comprised of 10 elements. Moreover, the variety of semigroups determined by this pair of identities may also be considered as the class of all semigroups S whose quotient $S/\theta(1,0)$ forms a semilattice, where the congruence $\theta(1,0)$ is defined by: a and b are $\theta(1,0)$-related if and only if $za = zb$ for all $z \in S$. In this talk, we describe completely the larger sublattice formed by all subvarieties of the semigroup variety determined by the pair of identities $zx^2 = zx$ and $zkxyw = zkyxw$. This variety is precisely the class of all semigroups S whose quotient $S/\theta(1,0)$ forms a normal band. We show that the lattice is finite with cardinality 26. Our proof makes use of certain earlier results proved by Melnik (1971) concerning the lattice of all 2-nilpotent extensions of rectangular bands, and Petrich (1974) which describes the lattice formed by joins of the variety of semilattices with varieties of 2-nilpotent extensions of rectangular bands.

Additively divisible commutative semirings: The 1-generated case

MIROSLAV KORBELÁŘ
Department of Mathematics and Statistics
Masaryk University, BRNO (CZE)
miroslav.korbelar@gmail.com

A commutative semiring is an algebraic structure with two commutative and associative binary operations (an addition and a multiplication) such that the multiplication distributes over the addition. It is well known that no finitely generated commutative ring (whether unitary or not) contains a copy of the field of rational numbers. More generally, the additive group of a non-trivial finitely generated commutative ring is not divisible. In the case of commutative semirings, various examples together with the classification of the simple ones indicate, that the every finitely generated additively divisible commutative semirings should be additively idempotent. We confirm validity of this conjecture for the one-generated case.

Fractional universal algebra

ANDREI KROKHIN
School of Engineering and Computer Sciences
DURHAM University (GBR)
andrei.krokhin@durham.ac.uk

I will discuss fractional universal algebras where operations are replaced by probability distributions over operations. I will explain how fractional algebraic notions appeared in the study of constraint satisfaction and give a short overview of some complexity classification results using these notions.

On multisemigroups

GANNA KUDRYAVTSEVA
Department of Information and Computer Science
University of LJUBLJANA (SVN)
ganna.kudryavtseva@fri.uni-lj.si

The talk is based on a joint work [1] with VOLODYMYR MAZORCHUK (Uppsala University).

A *multisemigroup* is a set S equipped with a map $m : S \times S \to \mathcal{P}(S)$, called the *multivalued multiplication*, which is *associative*, which means that for any $a, b, c \in S$

we have

$$\bigcup_{x \in m(a,b)} m(x,c) = \bigcup_{y \in m(b,c)} m(a,y).$$

Multisemigroups arise naturally in different mathematical settings. We outline some of them, particularly the appearance of topological multisemigroups as duals to some ordered structures with additional operations. We present several interesting and quite surprising counterexamples that show that many basic properties of semigroups can not be immediately extended to multisemigroups. As a positive result, we describe the structure of multisemigroup analogues of finite simple semigroups.

References

[1] G. Kudryavtseva, V. Mazorchuk, On multisemigroups, arXiv:1203.6224v1.

Reconstructing functions from identification minors

Erkko Lehtonen

Computer Science and Communications Research Unit
University of LUXEMBOURG (LUX)
erkko.lehtonen@uni.lu

Let A and B be arbitrary nonempty sets, and let $f\colon A^n \to B$. For any two-element subset I of $\{1, \dots, n\}$, the function $f_I\colon A^{n-1} \to B$ given by

$$f_I(a_1, \dots, a_{n-1}) = f(a_1, \dots, a_{\max I - 1}, a_{\min I}, a_{\max I}, \dots, a_{n-1}),$$

for all $a_1, \dots, a_{n-1} \in A$, is called an *identification minor* of f. Functions $f\colon A^n \to B$ and $g\colon A^n \to B$ are *equivalent* if there exists a permutation σ of $\{1, \dots n\}$ such that $f(a_1, \dots, a_n) = f(a_{\sigma(1)}, \dots, a_{\sigma(n)})$ for all $a_1, \dots, a_n \in A$.

We consider the following problem: Is a function $f\colon A^n \to B$ uniquely determined, up to equivalence, by the collection of its identification minors? Not every function is reconstructible in this sense. Consider, for example, the ternary functions induced by the polynomials $x_1 + x_2 + x_3$ and $x_1x_2 + x_1x_3 + x_2x_3$ over the two-element field. All identification minors of these two functions are projections.

In this talk, we report on recent discoveries – both positive and negative results – about the reconstructibility of functions.

The research reported here is in part joint work with Miguel Couceiro (Université Paris-Dauphine) and Karsten Schölzel (University of Luxembourg).

Algebraic models of computation

NICHOLAS J. LOUGHLIN
School of Mathematics and Statistics
NEWCASTLE University (GBR)
n.j.loughlin@ncl.ac.uk

In classical decision and recognition theory, machine models are widely disparate in form and function; there is no well-studied "natural" simultaneous generalisation of the notions of deterministic finite automaton and the Turing machine for example.

In this short talk, which is mainly expository in nature, we take some ideas and results of Kambites and Kambites-Render which develop a possible candidate for a unifying framework, namely automata with a multiply-only register consisting of a monoid, and discuss them in the context of some specific examples. In particular, given a decision problem of arbitrarily hard complexity, we can build a machine which will recognise it.

Commutative orders in semigroups

LÁSZLÓ MÁRKI
Alfréd Rényi Institute of Mathematics
Hungarian Academy of Sciences, BUDAPEST (HUN)
marki.laszlo@renyi.mta.hu

We consider commutative orders, that is, commutative semigroups having a semigroup of quotients in a local sense defined as follows. An element $a \in S$ is *square-cancellable* if for all $x, y \in S^1$ we have that $xa^2 = ya^2$ implies $xa = ya$ and also $a^2x = a^2y$ implies $ax = ay$. It is clear that being square-cancellable is a necessary condition for an element to lie in a subgroup of an oversemigroup. In a commutative semigroup S, the square-cancellable elements constitute a subsemigroup $\mathcal{S}(S)$. Let S be a subsemigroup of a semigroup Q. Then S is a *left order* in Q and Q is a *semigroup of left quotients* of S if every $q \in Q$ can be written as $q = a^\sharp b$ where $a \in \mathcal{S}(S)$, $b \in S$ and $a^\sharp$ is the inverse of a in a subgroup of Q and if, in addition, every square-cancellable element of S lies in a subgroup of Q. *Right orders* and *semigroups of right quotients* are defined dually. If S is both a left order and a right order in Q, then S is an *order* in Q and Q is a *semigroup of quotients* of S. We remark that if a commutative semigroup is a left order in Q, then Q is commutative so that S is an order in Q. A given commutative order S may have more than one semigroup of quotients. The semigroups of quotients of S are pre-ordered by the relation $Q \geq P$ if and only if there exists an onto homomorphism $\phi : Q \to P$ which restricts to the identity on S. Such a ϕ is referred to as an *S-homomorphism*; the classes of the associated equivalence relation are the S-isomorphism classes of orders, giving us a partially ordered set $\mathcal{Q}(S)$. In the best case, $\mathcal{Q}(S)$ contains maximum and minimum elements. In a commutative order S, $\mathcal{S}(S)$ is also an order and has a maximum semigroup of quotients R, which is a Clifford semigroup. We investigate how much of the relation between $\mathcal{S}(S)$ and its semigroups of quotients can be lifted to S and its semigroups of quotients.

This is a joint work with P. N. ÁNH, P. A. GRILLET, AND V. GOULD.

Quasivarietes of idempotent, entropic and symmetric groupoids

KATARZYNA MATCZAK
Faculty of Civil Engineering, Mechanics and Petrochemistry
Warsaw University of Technology, PŁOCK (POL)
matczak.katarzyna@gmail.com

Symmetric, idempotent and entropic groupoids were investigated by A.B. Romanowska and B. Roszkowska. B. Roszkowska described the lattice of subvarieties of the variety of such groupoids. We provide a description of the lattice of some quasivarieties of symmetric, idempotent and entropic groupoids. We will show that the lattice of quasivarieties of cancellative symetric, idempotent and entropic groupoids is isomorphic to the lattice of quasivarieties of commutative idempotent and entropic quasigroups. A universal-algebraic variety $\underline{\underline{V}}$ is called deductive if every subquasivariety is a variety. We prove that there exist deductive varieties of symmetric, idempotent and entropic groupoids.

Supernilpotence prevents dualizability

PETER MAYR
Department of Algebra
Johannes Kepler Universit, LINZ (AUT)
peter.mayr@jku.at

An algebraic structure is dualizable in the sense of Clark and Davey if there exists a certain natural duality between the quasi-variety it generates and some category of topological-relational structures. As a classical example, Boolean algebras are dualizable by Boolean spaces via Stone's representation theorem.

We consider the question which Mal'cev algebras are dualizable. In the work of Quackenbush and Szabó on groups (2002) and Szabó on rings (1999) nilpotence appears as an obstacle to dualizability. We extend these results and show that actually a stronger version of nilpotence, called supernilpotence by Aichinger and Mudrinski (2010), is the real culprit. In particular, we will present a non-abelian nilpotent expansion of a group that is dualizable.

This is a joint work with WOLFRAM BENTZ (Lisbon).

Cayley automaton semigroups

ALEX MCLEMAN
School of Mathematics and Statistics
University of St Andrews
alexm@mcs.st-andrews.ac.uk

Automaton semigroups are semigroups of endomorphisms of labelled rooted trees generated by actions of automata. After a brief overview of automaton semigroups, we will move on to a particular class - Cayley Automaton Semigroups. We obtain the automata in these cases by viewing the Cayley Table of a semigroup as an automaton. The aim then is to try to establish connections between a semigroup and the Cayley Automaton Semigroup arising from it.

After looking at some of the basic properties of these semigroups (established in [1], [2] and [3]), we will look at the Cayley Automaton semigroups of some classes of semigroups (such as nilpotent and cancellative semigroups) before addressing the question of self-automaton semigroups - those which are isomorphic to their Cayley Automaton semigroup.

REFERENCES

[1] A.J. Cain, *Automaton Semigroups*, Theoretical Computer Science 410 (2009) 5022-5038
[2] A. Mintz, *On the Cayley Semigroup of a Finite Aperiodic Semigroup*, International Journal of Algebra and Computation 19 (2009) 723-746
[3] P.V. Silva and B. Steinberg, *On a Class of Automata Groups Generalizing Lamplighter Groups*, International Journal of Algebra and Computation 15 (2005) 1213-1234

Semigroup near-rings

JOHN MELDRUM
School of Mathematics
University of EDINBURGH (GBR)
meldrum.john@gmail.com

A satisfactory definition of group near-rings was given by Le Riche et al. We extend the definition to semigroup near-rings. The ideal structure is examined, in particular the structure of the augmentation ideal. In the group near-ring case this ideal has interesting properties linking the group structure with the near-ring structure and we look for how this translates in the case of semigroup near-rings.

Asymmetric regular types

SLAVKO MOCONJA
Faculty of Mathematics
University of BELGRADE (SRB)
slavko@matf.bg.ac.rs

Asymmetric regular types in a complete first-order theory carry a natural closure operation on its locus. We show that it is induced by a certain degenerated pregeometry operation combined with a linear ordering relation. Any Morley sequence in a fixed asymmetric regular type is linearly ordered and the order type of a maximal Morley sequence in a fixed model M does not depend on a particular choice of the sequence.

As an example we show that any $\emptyset$-invariant, global type in a small, o-minimal theory is regular. Under some additional assumptions we show that any model of the theory is, up to isomorphism, determined by the order-types of maximal Morley sequences of a fixed family of 1-types. These assumptions hold if the theory has fewer than continuum countable models, giving a new description of Laura Mayer's proof of Vaught's Conjecture for o-minimal theories.

This is a joint work with PREDRAG TANOVIĆ (MI SANU, Belgrade).

Duality via truth for distributive interlaced bilattices

ANNA MUĆKA
Faculty of Mathematics and Information Science
Warsaw University of Technology (POL)
a.mucka@mini.pw.edu.pl

The Priestley style duality is well known for many classes of algebras with lattice reduct. This type of duality includes a representation of algebras in terms of a topological structure. Duality via truth is a duality between classes of algebras and the classes of their associated relational systems (frames) without any topology. Our intention is to view algebras and frames as being semantic structures for formal languages. Having a semantics of a formal language under consideration, we can define the notion of truth of its formulas α (or sequents $\alpha \vdash \beta$). A duality principle underlying the duality via truth stated that a given class of algebras and a class of frames provide equivalent semantics in the sense that a formula α (resp. a sequent $\alpha \vdash \beta$) is true with respect to one semantics iff it is true with respect to the other semantics. As a consequence, the algebras and the frames express equivalent notion of truth.
An algebra $(L, \wedge, \vee, \cdot, +, \neg)$ is a bilattice, if both the reducts, $L_1 = (L, \wedge, \vee)$ and $L_2 = (L, \cdot, +)$ are lattices and $\neg$ is an involution order reserving in L_1 and order

preserving in L_2. We present representation theorems and duality via truth for some classes of bilattices.

This is a joint work with ANNA RADZIKOWSKA (Warsaw University of Technology).

A characterization of 2-supernilpotent Mal'cev algebras

NEBOJŠA MUDRINSKI
Department of Mathematics and Informatics
University of NOVI SAD (SRB)
nmudrinski@dmi.uns.ac.rs

It is well-known that every abelian Mal'cev algebra is polynomially equivalent to a module over a ring. We obtain a similar characterization for Mal'cev algebras that satisfy $[1, 1, 1] = 0$.

On generalizations of the Cantor and Aleksandrov cube

ALEKSANDAR PAVLOVIĆ
Department of Mathematics and Informatics
University of NOVI SAD (Serbia)
apavlovic@dmi.uns.ac.rs

We investigate topologies on complete Boolean algebras generated by convergence of sequences. One example is the sequential topology, related to the problem of von Neumann, which is generated by the generalization of the convergence on the Cantor cube. Another example is a topology generated by a generalization of the convergence on the Aleksandrov cube. It is known that the union of the topology on Aleksandrov cube and its algebraic dual is a subbase for the Cantor cube, and a sequence converges in the Cantor cube to a point a iff it converges to a in the Aleksandrov cube and its dual. Examining these two properties on the generalizations of the Cantor and Aleksandrov cube, we prove that both of these properties hold in the class of Maharam algebras (ccc and weakly distributive), the second one holds in the class of algebras satisfying condition ($\hbar$) (which follows from $\mathfrak{t}$-cc and implies $\mathfrak{s}$-cc), and, using a notion of base matrix tree, we define a Boolean algebra and a sequence in it which witness that those properties do not hold in general.

This is a joint work with MILOŠ KURILIĆ (University of Novi Sad).

On the Bergman property for clones

CHRISTIAN PECH
Institute of Algebra
University of Technology, DRESDEN (GER)
Christian.Pech@tu-dresden.de

An infinite group has the Bergman property if each of its connected Cayley graphs has a finite diameter. In this talk we define the Bergman property for clones and study its connection with cofinality questions. For a large class of countable homogeneous structures we show that their clones of polymorphisms have uncountable cofinality and the Bergman property.

This is a joint work with MAJA PECH (University of Novi Sad).

On generating sets of polymorphism clones of homogeneous structures

MAJA PECH
Department of Mathematics and Informatics
University of NOVI SAD (SRB)
and
Institute of Algebra
University of Technology, DRESDEN (GER)
maja@dmi.uns.ac.rs, Maja.Pech@tu-dresden.de

We show for a large class of countable homogeneous structures that their polymorphism clones are generated by the monoid of homomorphic self-embeddings together with one further endomorphism and one further binary polymorphism. Our results generalize a classical theorem by Sierpiński, that the clone of all function on an arbitrary set is generated by its binary part.

This is a joint work with CHRISTIAN PECH (TU Dresden).

Semigroups with low difficulty word problem

MARKUS PFEIFFER
School of Mathematics and Statistics
University of ST ANDREWS (GBR)
markus.pfeiffer@morphism.de

The word problem for groups is a well-studied notion in computational group theory. Results of Anisimov, Muller and Schupp, Lehnert and Schweitzer, Holt, Roever, Thomas and many more relate the word problem and the coword problem of (classes of) groups to (classes of) formal languages, for example regular languages and context-free languages. In my research I considered a natural definition of the word problem and the coword problem of semigroups. Using the notions of recognisable, rational, and extended rational subsets of monoids, I extended some of the results about groups to semigroups. I then defined a hierarchy of semigroups by difficulty of their word problem. In my talk I will give an overview of the results. I will also give a few open questions which I hope to answer in the near future.

The persistence lattice

João Pita Costa
Department of Artificial Inteligence
Jozef Štefan Institut, LJUBLJANA (SVN)
joao.pitacosta@ijs.si

Persistent homology is a recent addition to topology, where it has been applied to a variety of problems including to data analysis. It has been in the center of the interest of computational topology for the past twenty years. In this talk we will introduce a generalized version of persistence based on lattice theory, unveiling universal rules and reaching deeper levels of understanding. Its algorithmic construction leads to two operations on homology groups which describe a diagram of spaces that can be described as a complete Heyting algebra. Unlike the lattice of subspaces of a vector space, these lattice operations are constructed using equalizers and coequalizers that guarantee distributivity. We will discuss the further study of the structure properties of these objects of great interest, and their interpretation within the framework of persistence.

This is a joint work with Primož Škraba (Jozef Štefan Institut, Ljubljana).

Congruence lattices and Compact Intersection Property

Miroslav Ploščica
Mathematical Institute
Slovak Academy of Sciences, KOŠICE (SVK)
miroslav.ploscica@upjs.sk

The problem of characterizing the class Con$\mathcal{V}$ of all congruence lattices of algebras in a given variety $\mathcal{V}$ has proved to be very difficult and probably intractable, even for the most common kinds of algebras like groups or lattices. A large part of the difficulties is caused by the fact that the compact elements of the congruence lattices $\mathrm{Con}(A)$ need not form a lattice. In general, it is only a join-semilattice and various refinement properties come into play. This is the motivation to study varieties $\mathcal{V}$, where this complication does not arise.

We say that a variety $\mathcal{V}$ has the Compact Intersection Property (CIP) if the intersection of two compact congruences on every $A \in \mathcal{V}$ is compact.

We present a systematical investigation of congruence lattices of algebras in finitely generated congruence distributive varieties with CIP. We develop two approaches: by means of direct limits, and by means of Priestley duality. We prove some general results, study several illustrative concrete cases, and formulate some conjectures.

Some classes of Archimedean power semigroups

Žarko Popović
Faculty of Economics
University of NIŠ (SRB)
zpopovic@eknfak.ni.ac.rs

Power semigroups of various classes of semigroups were studied by a number of authors: S. Bogdanović, M. Ćirić, M. Putcha, T. Tamura, J. Shafer, Ž. Popović and other. In this paper we give a structural characterization for semigroups whose power semigroups are k-Archimedean semigroups.

By $\mathbf{Z}^+$ we denote the set of all positive integers. If X is a non-empty set, then with $P(X)$ we denote the partitive set of the set X, i.e. the set of all subsets of X. Let S be a semigroup. On the partitive set of a semigroup S we define a multiplication with

$$A \cdot B = \{x \in S | (\exists a \in A)(\exists b \in B)\ x = ab\}, \qquad A, B \in P(S).$$

Then under this operation the set $P(S)$ is a semigroup which we call a power semigroup of a semigroup S.

Let $k \in \mathbf{Z}^+$ be a fixed integer. S. Bogdanović and Ž. Popović introduced notations for some new classes of semigroups like as k-Archimedean, left k-Archimedean, right k-Archimedean and t-k-Archimedean. A semigroup S is k-Archimedean if $a^k \in S^1bS^1$, for all $a, b \in S$. A semigroup S is a k-nil semigroup if $a^k = 0$ for every $a \in S$. This notion was introduced by T. Tamura. An ideal extension S of a semigroup I is a k-nil-extension of I if S/I is a k-nil semigroup.

In the present paper we are going to prove that the power semigroup of a semigroup S is k-Archimedean if and only if S is a k-nil-extension of a rectangular band.

On the genus of the intersection graph of ideals of a commutative ring

MARKO RADOVANOVIĆ
Faculty of Mathematics
University of BELGRADE (SRB)
markor@matf.bg.ac.rs

To each commutative ring R we can associate a graph $G(R)$, called *the intersection graph of ideals*, whose vertices are nontrivial ideals of R. In this talk, we try to establish some connections between commutative ring theory and graph theory, by study of the genus of the intersection graph of ideals. We classify all graphs of genus at most two that are intersection graphs of ideals and obtain some lower bounds for the genus of the intersection graphs of ideals of nonlocal rings.

This is a joint work with ALEKSANDRA ERIĆ and ZORAN PUCANOVIĆ (Faculty of Civil Engineering, University of Belgrade).

Topological index of some carbon nanotubes and the symmetry group for nanotubes and unit cells in solids

HAMID SAATI
Department of Mathematics
Georgian Technical University, TBILISI (GEO)
hamidfrir@gmail.com

The Padmakar-Ivan (PI) index of a graph G is defined as $PI(G) = \sum[n_{eu}(e|G) + n_{ev}(e|G)]$, where $n_{eu}(e|G)$ is the number of edges of G lying closer to u than to v, $n - ev(e|G)$ is the number of edges of G lying closer to v than to u and summation goes over all edges of G. Let G be a connected graph. $M_{eu}(e|G)$ is the number of vertices of G lying closer to u and $M_{ev}(e|G)$ is the number of vertices of G lying closer to v. Then the Szeged index of G is defined as the sum of $M_{eu}(e|G)M_{ev}(e|G)$, over edges of G. The PI index of G is the Szeged-like topological index defined as the sum of $[n_{eu}(e|G) + n_{ev}(e|G)]$, where $n_{eu}(e|G)$ is the number of edges of G lying closer to u than to v, $n_{ev}(e|G)$ is the number of edges of G lying closer to v than to u and summation goes over all edges of G. Types of symmetry groups are commonly used in chemistry. Point groups are used for molecules, whereas, for solids, the 230 space groups are used. Neither of these types of symmetry groups are suitable for representing unit cells in solids the symmetry of which is intermediate between that of point groups and space groups. To represent the symmetry of unit cells in an infinite lattice, a third type of symmetry group must be used. An algorithmic method of generating these symmetry groups is described. It can be demonstrated that these groups are valid by use of conventional symmetry group theory; this technique has been applied to the two-dimensional graphite lattice. Because the new method generates symmetry tables using only the topology of the system, the symmetry properties of graphs can also readily be derived. Last, the relationship between these groups and the other two types of groups is identified.

On Zariski topologies of Abelian groups with operations

DENIS I. SAVELIEV
Steklov Mathematical Institute
Russian Academy of Sciences, MOSCOW (RUS)
d.i.saveliev@gmail.com

We consider universal algebras consisting of an Abelian group endowed with operations (of arbitrary arity) satisfying the generalized distributivity law, i. e. such that the unary operations obtaining from them by fixing all but one arguments are endomorphisms of the group. Instances of such algebras include rings, modules, linear algebras, differential rings, etc. Given such an algebra K, a closed basis of the Zariski topology on its Cartesian product K^n consists of finite unions of sets of solutions of equations $t(x_1, \ldots, x_n) = 0$ for all terms t of n variables over K; it is the least T_1 topology in which all operations are continuous. We prove that for every such infinite K and any n, the space K^n is closed nowhere dense in the space K^{n+1}. A fortiori, all such K are nondiscrete (this fact was previously established for commutative associative rings by Arnautov [1]). Our proof uses a multidimensional generalization of Hindman's Finite Sums Theorem, a strong Ramsey-theoretic result obtained via algebra of ultrafilters [2].

REFERENCES

[1] V. I. Arnautov, Nondiscrete topologizability of countable groups, Doklady Akademii Nauk SSSR, vol. 191 (1970), pp. 747–750.
[2] N. Hindman, D. Strauss, Algebra in the Stone-Čech compactification, de Gruyter Expositions in Mathematics 27, Walter de Gruyter, 1998.

Structure of weak suborders of a poset

BRANIMIR ŠEŠELJA
Department of Mathematics and Informatics
University of NOVI SAD (SRB)
seselja@dmi.uns.ac.rs

For a given poset (P, ρ), we deal with weakly reflexive, antisymmetric and transitive subrelations of ρ.

The lattice of all these subrelations is algebraic and we investigate its structure.

We also connect these with more general lattices arising from algebras and posets and we show that they have several common properties.

In the second part, we deal with analogue notions and properties in the framework of lattice valued orderings.

As an application, we present an introduction to lattice valued ordered groupoids and groups.

This is a joint work with ANDREJA TEPAVČEVIĆ (University of Novi Sad) and MIRNA UDOVIČIĆ (University of Tuzla, Bosnia and Herzegovina).

Finitely generated varieties which are finitely decidable are residually finite

MATTHEW SMEDBERG
Department of Mathematics
Vanderbilt University, NASHVILLE (USA)
matthew.smedberg@vanderbilt.edu

It is known since the 1990s that a locally finite congruence-modular variety $\mathcal{V}$ such that the first-order theory of $\mathcal{V}_{\text{fin}}$ is decidable must satisfy stringent structural conditions, such as that the solvable radical of a finite subdirectly irreducible algebra in $\mathcal{V}$ is abelian, and the congruences above this radical must be linearly ordered. We generalize a number of these properties to varieties admitting the unary tame-congruence type, and conclude that each finitely generated, finitely decidable $\mathcal{V}$ has a finite residual bound.

This is joint work with RALPH MCKENZIE (Vanderbilt University).

Random bipartite graphs

BORIS ŠOBOT
Department of Mathematics and Informatics
University of NOVI SAD (SRB)
sobot@dmi.uns.ac.rs

We explore various properties of random bipartite graphs (short: bigraphs). These structures naturally correspond to independent families, which are very important in various set-theoretic constructions. We investigate their robustness, universality, possibility of factorization and maximality. The main result is the classification of all countable bipartite graphs G having a naturally defined partition property:

$\mathcal{P}'$: for every partition of the set of vertices of G into finitely many pieces that each induce $(\aleph_0, \aleph_0)$-bigraphs at least one of the induced sub-bigraphs is isomorphic to G.

($(\aleph_0, \aleph_0)$-bigraphs are bipartite graphs having $\aleph_0$ vertices on each side.) Let S be a bigraph such that one of the vertices on its right side is connected to all of the vertices on the left, and others are isolated. Let S' be obtained by exchanging the left and the right side of S. Random bipartite graphs play an important role in the proof of the following theorem:

Theorem. The only $(\aleph_0, \aleph_0)$-bigraphs with the property $\mathcal{P}'$ up to isomorphism are the empty $(\aleph_0, \aleph_0)$-bigraph, the complete $(\aleph_0, \aleph_0)$-bigraph, the bigraphs S and S' and their complements.

Epimorphisms of partially ordered semigroups

NASIR SOHAIL
Faculty of Mathematics and Computer Science
University of TARTU (EST)
snasir@ut.ee

A partially ordered semigroup (briefly posemigroup) is a semigroup S endowed with a partial order $\leq$ that is compatible with the binary operation (i.e. for all $s_1, s_2, t_1, t_2 \in S$, $s_1 \leq t_1$, $s_2 \leq t_2$ implies $s_1 s_2 \leq t_1 t_2$). A posemigroup homomorphism $f : S \longrightarrow T$ is a monotone semigroup homomorphism (i.e. for all $s_1, s_2 \in S$, $f(s_1 s_2) = f(s_1) f(s_2)$ and $s_1 \leq s_2$ in S implies $f(s_1) \leq f(s_2)$ in T). In my talk, I shall discuss a criterion (namely zigzag theorem for posemigroups) to check if a posemigroup homomorphism f is an epimorphism (in the sense of category theory). As one may easily observe, f is necessarily an epimorphism in the category of posemigroups if it is such in the category of semigroups. I shall also show that the converse of this statement, which may not be true in general, holds in certain varieties of semigroups (equivalently posemigroups).

On morphisms of lattice-valued formal contexts

SERGEJS SOLOVJOVS
Department of Mathematics and Statistics
Masaryk University, BRNO (CZE)
solovjovs@math.muni.cz

A *formal context* is a triple (G, M, I), which comprises a set of objects G, a set of attributes M, and a binary incidence relation I between G and M, where $g\, I\, m$ stands for "object g has attribute m" [3]. There exist at least three different ways of defining a morphism between formal contexts (G_1, M_1, I_1) and (G_2, M_2, I_2).

(1) The theory of *Formal Concept Analysis (FCA)* [3] employs pairs of maps $G_1 \xrightarrow{\alpha} G_2$, $M_1 \xrightarrow{\beta} M_2$ such that $g\, I_1\, m$ iff $\alpha(g)\, I_2\, \beta(m)$ for every $g \in G_1$, $m \in M_1$.
(2) The theory of *Chu spaces* [5] uses pairs of maps $G_1 \xrightarrow{\alpha} G_2$, $M_2 \xrightarrow{\beta} M_1$ such that $g\, I_1\, \beta(m)$ iff $\alpha(g)\, I_2\, m$ for every $g \in G_1$, $m \in M_2$.
(3) The theory of *Galois connections* [4] relies on the pairs of maps $\mathcal{P}(G_1) \xrightarrow{\alpha} \mathcal{P}(G_2)$, $\mathcal{P}(M_2) \xrightarrow{\beta} \mathcal{P}(M_1)$, where $\mathcal{P}(X)$ stands for the powerset of X, such that the diagrams

$$\begin{tikzcd} \mathcal{P}(G_1) \arrow[r, "\alpha"] \arrow[d, "H_1"'] & \mathcal{P}(G_2) \arrow[d, "H_2"] \\ \mathcal{P}(M_1) & \mathcal{P}(M_2) \arrow[l, "\beta"] \end{tikzcd} \quad \text{and} \quad \begin{tikzcd} \mathcal{P}(M_1) \arrow[d, "K_1"'] & \mathcal{P}(M_2) \arrow[l, "\beta"'] \arrow[d, "K_2"] \\ \mathcal{P}(G_1) \arrow[r, "\alpha"'] & \mathcal{P}(G_2) \end{tikzcd}$$

commute, where $H_j(S) = \{m \in M_j \mid s\, I_j\, m \text{ for every } s \in S\}$ and $K_j(T) = \{g \in G_j \mid g\, I_j\, t \text{ for every } t \in T\}$ (called *Birkhoff operators*).

Recently, [2] compared the approaches of items (2) and (3) by considering their respective categories of *lattice-valued formal contexts* (in the sense of [1]) over a commutative quantale Q, and constructing an embedding of each category into its counterpart. The conclusion was: the two viewpoints on formal context morphisms are not categorically isomorphic.

This talk (a shortened version of [6]) compares all three approaches in the setting of lattice-valued formal contexts over a category of (not necessarily commutative) quantales (constructing a number of embeddings between their respective categories of formal contexts), and shows that the approach of item (3) falls out of the FCA framework in the lattice-valued case. More precisely, while in the crisp case, there is a one-to-one correspondence between binary relations $I \subseteq G \times M$ and order-reversing Galois connections on $(\mathcal{P}(G), \mathcal{P}(M))$, in the lattice-valued case, every map $G \times M \xrightarrow{I} Q$ gives an order-reversing Galois connection on (Q^G, Q^M), but the converse way is possible under additional requirements only.

Acknowledgements. This research was supported by ESF Project Nr. CZ.1.07/2.3.00/20.0051 "Algebraic methods in Quantum Logic" of the Masaryk University in Brno, Czech Republic.

References

[1] R. Bělohlávek, *Lattices generated by binary fuzzy relations*, Tatra Mt. Math. Publ. **16** (1999), no. 1, 11–19.

[2] J. T. Denniston, A. Melton, and S. E. Rodabaugh, *Formal concept analysis and lattice-valued Chu systems*, Fuzzy Sets Syst. **216** (2013), 52–90.
[3] B. Ganter and R. Wille, *Formale Begriffsanalyse. Mathematische Grundlagen*, Berlin: Springer, 1996.
[4] J. M. McDill, A. C. Melton, and G. E. Strecker, *A category of Galois connections*, Category Theory and Computer Science, Proc. Conf., Edinburgh/U.K. 1987, Lect. Notes Comput. Sci. 283, 290-300 (1987).
[5] V. Pratt, *Chu spaces*, School on category theory and applications. Lecture notes of courses, Coimbra, Portugal, July 13-17, 1999. Coimbra: Universidade de Coimbra, Departamento de Matemática. Textos Mat., Sér. B. 21, 39–100 (1999).
[6] S. Solovyov, *Lattice-valued topological systems as a framework for lattice-valued Formal Concept Analysis*, Journal of Mathematics **2013** (2013), 33 pages, Article ID 506275, http://dx.doi.org/10.1155/2013/506275.

Commutator theory for loops

DAVID STANOVSKÝ
Department of Algebra
Charles University, PRAGUE (CZE)
`stanovsk@karlin.mff.cuni.cz`

Using the Freese-McKenzie commutator theory for congruence modular varieties, we develop commutator theory for the variety of loops. The main result is a relation between generators of the congruence commutator, and generators of the total inner mapping group of a loop.

We argue that some standard definitions of loop theory, drawn upon direct analogy to group theory, should be revised. In particular, we show that Bruck's notion of solvability is strictly weaker than solvability in the sense of commutator theory, and question certain results, such as Glaubermann's extension of the Feit-Thompson odd order theorem to Moufang loops.

This is a joint work with PETR VOJTĚCHOVSKÝ (University of Denver).

Combinatorial semigroups and induced/deduced operators

G. STACEY STAPLES
Department of Mathematics and Statistics
Southern Illinois University, EDWARDSVILLE (USA(
`sstaple@siue.edu`

A number of combinatorially interesting semigroups can be generated by commuting and anti commuting unit- and null-squares. The corresponding semigroup algebras have proven useful in addressing problems in graph theory and computer science, from random walks on hypercubes to multi-constrained routing in wireless sensor networks. Moreover, as subalgebras of Clifford algebras, they have natural connections with quantum probability and quantum computing.

Considering linear operators on vector spaces over the generators of such semigroups, one can induce corresponding operators on the associated semigroup algebra. Conversely, starting with a linear operator on a semigroup algebra, one can define various "deduced" operators on the vector space spanned by generators.

For example, let A denote the adjacency matrix of a simple graph with vertex set V, viewed as a linear transformation on the vector space generated by V. Let $A^{\vee k}$ denote the corresponding multiplication-induced operator on the grade-k subspace of the semigroup algebra $\mathcal{C}\ell_V{}^{\text{nil}}$, generated by commuting null-squares. For fixed subset $I \subseteq V$, let X_I denote the number of disjoint cycle covers of the subgraph induced by I. Similarly, let M_J denote the number of perfect matchings on the subgraph induced by $J \subseteq V$ (nonzero only for J of even cardinality). Then,

$$\operatorname{tr}(A^{\vee k}) = \sum_{\substack{I \subset V \\ |I|=k}} \sum_{J \subseteq I} X_{I \setminus J} M_J.$$

Other notions of induced and deduced operators will be discussed, along with some of their interesting combinatorial properties and applications.

Automorphism groups of free Steiner triple systems

IZABELLA STUHL

Institute of Mathematics and Statistics
University of SÃO PAULO (BRA)
izabella@ime.usp.br

We consider Steiner triple systems from the algebraic point of view. That is, we study Steiner loops that are in a one-to-one correspondence with Steiner triple systems.

Because Steiner loops form a variety, we can operate with free objects on this variety and use the term free Steiner triple system for the combinatorial object corresponding to the free Steiner loop.

The main result is that all automorphisms of the free Steiner loops are tame. Furthermore, the group of automorphisms of a free Steiner loop cannot be finitely generated when the loop is generated by more than 3 elements. Finally, in the case of a 3-generated free Steiner loop we specify the (triples of) generators of the automorphism group.

This is a joint work with A. GRISHKOV (University of São Paulo) and M. AND D. RASSKAZOVA (Omsk University).

Generically stable regular types

PREDRAG TANOVIĆ
Mathematical Institute
Serbian Academy of Sciences and Arts, BELGRADE (SRB)
tane@mi.sanu.ac.rs

Regular types produce an invariant of a model (of a complete first-order theory). There are two kinds of regular types: symmetric, their invariants are cardinal numbers (for example dimension of vector spaces), and asymmetric, whose invariants are linear-order types. We study the relation of non-orthogonality on symmetric regular types and prove that generically stable strongly regular ones behave very much like in the stable context.

Automorphic equivalence of many-sorted algebras

ARKADY TSURKOV
Institute of Mathematics and Statistics
University of SÃO PAULO (BRA)
arkady.tsurkov@gmail.com

Universal algebras H_1, H_2 of the variety Θ are geometrically equivalent if they have same structure of the algebraic closed sets. Automorphic equivalence of algebras is a generalization of this notion. We can say that universal algebras H_1, H_2 of the variety Θ are geometrically equivalent if the structures of the algebraic closed sets of these algebras coincides up to changing of coordinates defined by some automorphism of the category Θ^0. Θ^0 is a category of the free finitely generated algebras of the variety Θ. The quotient group $\mathfrak{A}/\mathfrak{Y}$ determines the difference between geometric and automorphic equivalence of algebras of the variety Θ, where $\mathfrak{A}$ is a group of the all automorphisms of the category Θ^0, $\mathfrak{Y}$ is a group of the all inner automorphisms of this category.

The method of the verbal operations was worked out in: B. Plotkin, G. Zhitomirski, *On automorphisms of categories of free algebras of some varieties*, 2006 - for the calculation of the group $\mathfrak{A}/\mathfrak{Y}$. By this method the automorphic equivalence was reduced to the geometric equivalence in: A. Tsurkov, *Automorphic equivalence of algebras*, 2007. All these results were true for the one-sorted algebras: groups, semigroups, linear algebras...

Now we reprove these results for the many-sorted algebras: representations of groups, actions of semigroups over sets and so on.

Direction cones for the representation of tomonoids

THOMAS VETTERLEIN
Department of Knowledge-Based Mathematical Systems
Johannes Kepler University, LINZ (AUT)
Thomas.Vetterlein@jku.at

Tomonoids are monoids endowed with a translation-invariant total order. We focus on tomonoids that are commutative, positive, and finitely generated, or c.p.f. for short. In the finite case, these tomonoids correspond exactly to MTL-algebras, which are of relevance in fuzzy logic as well as for the theory of triangular norms.

C.p.f. tomonoids arise from finitely generated free monoids by the formation of a quotient and the subsequent extension of the natural partial order to a total order. Both steps can be combined into one, namely, by considering the preorder that a tomonoid induces on the free monoid. In fact, c.p.f. tomonoids are describable by translation-invariant, positive total preorders on an $\mathbb{N}^n$, or monomial preorders for short.

If a monomial preorder is a total order, it can be represented by a positive cone of the associated group $\mathbb{Z}^n$. In general, we can associate with a monomial preorder what we call a direction cone. Direction cones are subsets of the $\mathbb{Z}^n$ characterised similarly to the positive cone of partially ordered groups. It turns out that each monomial preorder can be restricted to a preorder that is fully described by a direction cone. As a result, all c.p.f. tomonoids are quotients of tomonoids representable by direction cones.

On this basis, we develop a representation theory for nilpotent c.p.f. tomonoids and finally for finite c.p.f. tomonoids in general.

The relation between pentagonal and GS-quasigroups

STIPE VIDAK
Department of Mathematics
University of ZAGREB (CRO)
stipe.vidak@gmail.com

Pentagonal quasigroups are idempotent medial quasigroups in which identity $(ab \cdot a)b \cdot a = b$ holds. GS-quasigroups are idempotent medial quasigroups in which one of the mutually equivalent identities $a(ab \cdot c) \cdot c = b$, $a \cdot (a \cdot bc)c = b$ hold. We show that in every pentagonal quasigroup we can define GS-quasigroup. Using that we define geometric concepts of GS-trapezium and affine regular pentagon in pentagonal quasigroups, concepts already defined and studied in GS-quasigroups. Consequently, pentagonal quasigroups inherit the entire geometry of GS-quasigroups. Geometric representations of some theorems regarding mentioned concepts are given in the quasigroup $C(q)$, where q is a solution of the equation $q^4 - 3q^3 + 4q^2 - 2q + 1 = 0$.

Invariance groups of finite functions and orbit equivalence of permutation groups

TAMÁS WALDHAUSER
Bolyai Institute
University of SZEGED (HUN)
twaldha@math.u-szeged.hu

Which subgroups of the symmetric group S_n arise as invariance groups of n-variable functions defined on a k-element domain? It appears that the higher the difference $n - k$, the more difficult it is to answer this question. For $k \geq n$, the answer is easy: all subgroups of S_n are invariance groups. We give a complete answer in the cases $k = n - 1$ and $k = n - 2$, and we also give a partial answer in the general case: we describe invariance groups when n is much larger than $n - k$. The proof utilizes Galois connections and the corresponding closure operators on S_n, which turn out to provide a generalization of orbit equivalence of permutation groups. We also present some computational results, which show that all primitive groups except for the alternating groups arise as invariance groups of functions defined on a three-element domain.

This is a joint work with ESZTER K. HORVÁTH (University of Szeged), GÉZA MAKAY (University of Szeged) and REINHARD PÖSCHEL (Technische Universität Dresden).

Lattices of regular closed sets in closure spaces: semidistributivity and Dedekind-MacNeille completions

FRIEDRICH WEHRUNG
Department of Mathematics
University of CAEN (FRA)
friedrich.wehrung01@unicaen.fr

For a closure space (P, φ) with $\varphi(\varnothing) = \varnothing$, the closures of open subsets of P, called the *regular closed* subsets, form an ortholattice $\mathrm{Reg}(P, \varphi)$, extending the poset $\mathrm{Clop}(P, \varphi)$ of all clopen subsets. Every ortholattice arises in this fashion (Mayet 1982, Katrnoška 1982), but if (P, φ) is a finite convex geometry, then the lattice $\mathrm{Reg}(P, \varphi)$ is pseudocomplemented. This construction extends the following known ones:

(1) The permutohedron on a given finite number of letters (Guilbaud and Rosenstiehl 1963).

(2) The permutohedron on an arbitrary poset (Pouzet *et al.* 1995).
(3) The lattice of all bipartitions on an arbitrary set (Foata and Zeilberger 1996, Han 1996, Hetyei and Krattenthaler 2011).
(4) The poset of regions of any central hyperplane arrangement (Edelman 1984).

Analogy with contexts related to (1)–(4) above suggests applying the construction to the following contexts:

(5) P is an arbitrary join-semilattice, and $\varphi(X)$ is the join-subsemilattice of P generated by X, whenever $X \subseteq P$.
(6) P is the collection of all connected subsets of a graph G, and $\varphi(X)$ is the closure of X under connected disjoint unions. This construction is related to the one of the weak Bruhat ordering on a finite Coxeter group, thus to (4) above.

We shall give a short introduction to the following results:

(DM) In all contexts above except (6), $\mathrm{Reg}(P, \varphi)$ is always the Dedekind-MacNeille completion of $\mathrm{Clop}(P, \varphi)$.
(B) In all contexts above except (3), and if P is finite, then $\mathrm{Reg}(P, \varphi)$ is a bounded homomorphic image of a free lattice; in particular, it is semidistributive.

This is a joint work with LUIGI SANTOCANALE (Université de Provence, Marseille).

Free idempotent generated semigroups over bands

DANDAN YANG
Department of Mathematics
University of YORK (GBR)
ddy501@york.ac.uk

There have been significant recent advances in the study of maximal subgroups of *free idempotent generated semigroups* $\mathrm{IG}(E)$ over *biordered sets* E. In my talk, we will investigate $\mathrm{IG}(E)$ in a different direction, i.e. for which kind of biordered sets E, is $\mathrm{IG}(E)$ *(weakly) abundant*? It is proved that for any semilattice E, $\mathrm{IG}(E)$ is adequate – i.e. it belongs to a quasivariety of algebras introduced in York by Fountain over 30 years ago, for which the free objects have recently been described. Furthermore, we can show that if E is a semilattice of left/right zero bands, then $\mathrm{IG}(E)$ is weakly abundant with the congruence condition. More remarkably, $\mathrm{IG}(E)$ is abundant for any strong semilattice of left/right zero bands E. Finally, we conjecture that for any band E, $\mathrm{IG}(E)$ is weakly abundant.

This is a joint work with VICTORIA GOULD (University of York).

The similarities of positive Jónsson theories in admissible enrichments of signatures

AIBAT R. YESHKEYEV
Insitute of Applied Mathematics
KARAGANDA State University (KAZ)
modth1705@mail.ru

In this talk I would like to report some results concerning the syntactic and semantic similarity of positive Jónsson theories in admissible enrichments of signatures. The main result shows that for any such theory there is exists syntactically similar for it some Jónsson theory of a polygon. A polygon is an algebra of unars over monoid with some conditions.

Restricted semidirect products via inductive categories

RIDA-E ZENAB
Department of Mathematics
University of YORK (GBR)
rzz500@york.ac.uk

The semidirect product of two inverse semigroups need not be inverse in general. Billhardt showed how to get round this difficulty by modifying the definition of semidirect product of two inverse semigroups to obtain what he termed a *λ-semidirect product* [1] and that we call here a *restricted semidirect product*. Billhardt later extended his construction, in the case where one component was a semilattice, to left ample semigroups [2]. Again in this special case, this was extended further to the restricted semidirect product of a semilattice and a left restriction semigroup [3]. Wazzan found a new approach in the inverse case by first building an inductive groupoid [5].

We extend the above in two ways. First, we consider the restricted semidirect product of arbitrary left restriction semigroups. Using the notion of double actions taken from [4] we then introduce the restricted semidirect product of (two-sided) restriction semigroups. Following Wazzan's technique we first construct an inductive category and then obtain the corresponding restriction semigroup.

REFERENCES

[1] B. Billhardt, 'On a wreath product embedding and idempotent pure congruences on inverse semigroups' *Semigroup Forum* **45** (1992), 45-54.
[2] B. Billhardt, 'Extensions of semilattices by left type-A semigroups' *Glasgow Math. J.* **39** (1997), 7-16.
[3] M. Branco, G. Gomes and V. Gould, 'Extensions and covers for semigroups whose idempotents form a left regular band' *Semigroup Forum* **81** (2010), 51-70.

[4] J. Fountain, G. M. S. Gomes and V. Gould, 'The free ample monoid' *I. J. A.C* **19** (2009), 527-554.
[5] S. Wazzan, 'The Zappa-Szép product of semigroups' *PhD Thesis, Heriot-Watt University* 2008.

On free $(\ell r, rr)$-dibands

ANATOLIY V. ZHUCHOK
Department of Mathematical Analysis and Algebra
LUHANSK Taras Shevchenko National University (UKR)
zhuchok_a@mail.ru

Recall that a nonempty set D equipped with two binary associative operations $\dashv$ and $\vdash$ satisfying the following axioms:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z), \ (x \vdash y) \dashv z = x \vdash (y \dashv z),$$
$$(x \dashv y) \vdash z = x \vdash (y \vdash z) \text{ for all } x, y, z \in D,$$

is called a dimonoid [1]. A dimonoid $(D, \dashv, \vdash)$ will be called a $(\ell r, rr)$-diband, if $(D, \dashv)$ is a left regular band and $(D, \vdash)$ is a right regular band. A dimonoid $(D, \dashv, \vdash)$ is called a $(\ell n, rn)$-diband [2], if $(D, \dashv)$ is a left normal band and $(D, \vdash)$ is a right normal band.

Note that every left (right) normal band is left (right) regular. The converse statement is not true. It is natural to consider the similar question for $(\ell r, rr)$-dibands and $(\ell n, rn)$-dibands.

Theorem. A dimonoid $(D, \dashv, \vdash)$ is a $(\ell r, rr)$-diband if and only if $(D, \dashv, \vdash)$ is a $(\ell n, rn)$-diband.

Corollary. The variety of $(\ell r, rr)$-dibands coincides with the variety of $(\ell n, rn)$-dibands.

In terms of dibands of subdimonoids [3] we also describe the structure of free $(\ell r, rr)$-dibands. It turns out that operations of an idempotent dimonoid with left (right) regular bands coincide and it is a left (right) regular band.

REFERENCES

[1] J.-L. Loday, *Dialgebras, In: Dialgebras and related operads, Lect. Notes Math.* **1763**, Springer-Verlag, Berlin, 2001, 7–66.
[2] A.V. Zhuchok, *Free normal dibands*, Algebra and Discrete Math. **12** (2011), no. 2, 112–127.
[3] A.V. Zhuchok, *Dimonoids*, Algebra and Logic **50** (2011), no. 4, 323–340.

Monoids of endomorphisms of relational structures

YURIY V. ZHUCHOK
Department of Mathematical Analysis and Algebra
LUHANSK Taras Shevchenko National University (UKR)
zhuchok_y@mail.ru

Let $\mathscr{R}$ be an arbitrary set of relations on a set X. A pair $(X, \mathscr{R})$ is called a relational structure over X. Examples of relational structures are arbitrary relational clones, ordered sets, quasi-ordered sets, graphs, hypergraphs, different algebras of relations etc. The most important relational structures are those in which each relation from $\mathscr{R}$ is a binary relation. For instance, such structures are so-called coherent configurations and, in particular, associative schemes, Heming's schemes, Johnson's schemes (see, e.g., [1]).

Assume $(X, \mathscr{R})$ be a relational structure over X. A transformation φ of the set X is called an endomorphism of $(X, \mathscr{R})$ if φ is an endomorphism of each relation from $\mathscr{R}$. The set of all endomorphisms of $(X, \mathscr{R})$ is a semigroup with respect to the ordinary operation of the composition of transformations. This semigroup is called the monoid of endomorphisms of the relational structure $(X, \mathscr{R})$ and is denoted by $End\,(X, \mathscr{R})$. The monoid of strong endomorphisms [2] and the group of automorphisms of $(X, \mathscr{R})$ are defined by the similar way.

We define the concept of a connectivity in the relational structure (X, ρ) with a single relation $\rho \in \mathscr{R}$ and consider the reduction's problem of the description of the semigroup of endomorphisms (resp. the monoid of strong endomorphisms, the group of automorphisms) of any disconnected relational structure (X, ρ) to the description of its connected components. Thus, knowing how to construct the monoid $End\,(X, \rho)$ (or some its submonoid) for all relations $\rho \in \mathscr{R}$ we obtain the description of the monoid $End\,(X, \mathscr{R})$ (or some its submonoid).

REFERENCES

[1] Sushchansky V. I., Sikora V. S. *Operations on groups of permutations*, Chernivtsi: Ruta, 2003, 255 p. (in Ukrainian).
[2] Knauer U., Nieporte M. *Endomorphisms of graphs I. The monoid of strong endomorphisms*, Arch. Math., 52 (1989), 607–614.

POSTER PRESENTATIONS

On Baer triple $(V, G, \mathfrak{R})_{F_2}$

ABDULLAH ALAZEMI
Department of Mathematics
PAAET, KUWAIT CITY (KUW)
alazmi95@gmail.com

We classify the Baer triples $(V, G, \mathfrak{R})_{F_2}$, where G is isomorphic to an elementary abelian 2-group $\mathbb{Z}_2^2$, and F_2 denotes the Galois field $GF(2)$.

Ideas for improving notation for finite total transformations

ATTILA EGRI-NAGY
School of Computing, Engineering and Mathematics
University of Western Sydney, SYDNEY (AUS)
a.egri-nagy@uws.edu.au

Given a total transformation, for example

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 3 \end{pmatrix},$$

we can simply write it with the one-line notation as

`[2,1,2,3,3],`

or following [2] we can use

`[4,3,2,1|2][5,3|3],`

or with recent improvements in [1]

`([[4,5;3];2],1),`

or we can even come up with something new and write

`([{4,5},3,2],1).`

We compare the properties (length, readability, etc.) of these different notations, aiming to single out an optimal variant that can be used in computer algebra systems.

REFERENCES

[1] O. Ganyushkin and V. Mazorchuk, *Classical Transformation Semigroups*, Springer, 2009.

[2] G. Ayik, H. Ayik and J. M. Howie, On factorisations and generators in transformation semigroups, *Semigroup Forum* **70** (2005), 225–237.