Topological methods in model theory

Ludomir Newelski

Instytut Matematyczny
Uniwersytet Wrocławski

June 2013

Set-up

M is a model,

$M = (\mathbb{R}, +, \cdot, <, \dots)$

$M = (\mathbb{Z}, +)$

$T = Th(M)$ in language $L = L(M)$

$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.

$\varphi(x) = \exists y \; x \cdot y = 1$

$U = \varphi(M)$.

$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.

Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.

Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.

Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is **definable** if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y\ x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y\ x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{\text{definable subsets of } M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y\ x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{\text{definable subsets of } M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Set-up

M is a model,
$M = (\mathbb{R}, +, \cdot, <, \dots)$
$M = (\mathbb{Z}, +)$
$T = Th(M)$ in language $L = L(M)$
$U \subseteq M$ is definable if U is a solution set of an equation (with parameters from M) or more generally a formula $\varphi(x)$ with quantifiers.
$\varphi(x) = \exists y \; x \cdot y = 1$
$U = \varphi(M)$.
$Def(M) = \{$definable subsets of $M\}$ this is a Boolean algebra.
Assume $M \prec N$ and $U = \varphi(M) \in Def(M)$.
Let $U^N = \varphi(N)$. So $U^N \in Def(N)$.
Let $a \in N$.

$$tp(a/M) = \{U \in Def(M) : a \in U^N\}$$

$$= \{\varphi(x) \in L(M) : a \in \varphi(N)\}$$

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.

Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.

$S(M)$ is called the space of complete types over M.

$tp(a/M) \in S(M)$.

Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.

$S(M)$ is a compact topological space:

$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.

More generally, a type over M is a filter in $Def(M)$.

Similarly, for $A \subseteq M$, (complete) types over A:

$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

$tp(a/M)$ is an ultrafilter in $Def(M)$.
Let $S(M) = S(Def(M))$ be the Stone space of ultrafilters in $Def(M)$.
$S(M)$ is called the space of complete types over M.
$tp(a/M) \in S(M)$.
Every $\mathcal{U} \in S(M)$ equals $tp(a/M)$ for some $N \succ M$ and $a \in N$.
$S(M)$ is a compact topological space:
$U \in Def(M) \rightsquigarrow [U] = \{p \in S(M) : U \in p\}$ a basic clopen set in $S(M)$.
More generally, a type over M is a filter in $Def(M)$.
Similarly, for $A \subseteq M$, (complete) types over A:
$S(A) = \{\text{complete types over } A\} = S(Def_A(M))$.

Types and automorphisms

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Types and automorphisms

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Types and automorphisms

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Types and automorphisms

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C}),\ p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Types and automorphisms

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Types and automorphisms

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

Types and automorphisms

Let $\mathfrak{C} \succ M$ be large, saturated (a monster model).
For a small $A \subseteq \mathfrak{C}$ let $Aut(\mathfrak{C}/A) = \{f \in Aut(\mathfrak{C}) : f|_A = id_A\}$.
$Aut(\mathfrak{C}/A)$ acts on:

- $\mathfrak{C}$
- $Def(\mathfrak{C})$ (by automorphisms)
- $S(\mathfrak{C}) = S(Def(\mathfrak{C}))$, by homeomorphisms.

The orbits of this action = sets of the form $p(\mathfrak{C})$, $p \in S(A)$.
$Def_A(\mathfrak{C}) \subseteq Def(\mathfrak{C})$ subalgebra
$r : S(\mathfrak{C}) \to S(A)$ restriction function
$S(\mathfrak{C}) \ni p \mapsto r(p) = p \cap Def_A(\mathfrak{C})$

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories

In model theory we

- count types (stability hierarchy
  stable theories, models = theories, models with few types)
- measure types and definable sets (with various ranks):

$S(A)$ is a compact topological space.
The Cantor-Bendixson rank on $S(A)$, $S(M)$ (coming from CB-derivative) is called the Morley rank:
$RM : S(M) \to Ord \cup \{\infty\}$

- the main tool in Morley categoricity theorem (1964)
- large types = types with large RM.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Stable theories and forking

Let $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then $RM(q) \leq RM(p)$.
q is a large extension of p if $RM(q) = RM(p)$.
This leads to:

- the notion of non-forking extension of a type (Shelah).
- forking independence
- geometric stability theory (Zilber, Hrushovski, Pillay, ...)

The definition of forking given in combinatorial terms.
Works well for stable theories.
Extensions of the method to some unstable theories:

- theories with NIP (including o=minimality, $\mathbb{R}$)
- simple theories (random graph, pseudo-finite fields)

Here forking loses its explaining power.

Orbits

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Orbits

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Orbits

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Orbits

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Orbits

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Orbits

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Orbits

Fact

Assume T is stable, $A \subset \mathfrak{C}$, $p \in S(A)$, $q \in S(\mathfrak{C})$ and $p \subseteq q$. Then TFAE:

1. q is a non-forking extension of p.
2. The orbit of q under $Aut(\mathfrak{C}/A)$ has bounded size (actually, $\leq 2^{|T|+|A|}$).

Assume T is unstable.

- 1. and 2. are no longer equivalent.
- Instead of considering 1. we may consider 2.

Idea

q is a large type extending p iff
the orbit of q under $Aut(\mathfrak{C}/A)$ is small.

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Topological dynamics

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Definition continued

Assume X is a G-flow and $p \in X$.

(3) p is periodic if the orbit Gp is finite.

(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.

(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.

(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.

(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.

$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$

$Gen(X) = \{p \in X : p \text{ is generic}\}$

$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.

(3) p is periodic if the orbit Gp is finite.

(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.

(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.

(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.

(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.

$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$

$Gen(X) = \{p \in X : p \text{ is generic}\}$

$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.

(3) p is periodic if the orbit Gp is finite.

(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.

(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.

(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.

(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.

$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$

$Gen(X) = \{p \in X : p \text{ is generic}\}$

$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.

(3) p is periodic if the orbit Gp is finite.

(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.

(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.

(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.

(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.

$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$

$Gen(X) = \{p \in X : p \text{ is generic}\}$

$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(3) p is periodic if the orbit Gp is finite.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X) U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(3) p is periodic if the orbit Gp is finite.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G) AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X) U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(3) p is periodic if the orbit Gp is finite.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(3) p is periodic if the orbit Gp is finite.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(3) p is periodic if the orbit Gp is finite.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G) AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X) U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Topological dynamics

Definition continued

Assume X is a G-flow and $p \in X$.
(3) p is periodic if the orbit Gp is finite.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Let X be a G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X, \ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r.t. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Let X be a G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r.t. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Let X be a G-flow.
$G \ni g \ \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Let X be a G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X$, $\pi_g(x) = g \cdot x$,

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r.t. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Let X be a G-flow.
$G \ni g \rightsquigarrow \pi_g : X \stackrel{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Let X be a G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an **ideal** if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an **idempotent** if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Model theory

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.

$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.

$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Model theory

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Model theory

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Model theory

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Model theory

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Let $A \subseteq \mathfrak{C}$, $p \in S(A)$, $S_p(\mathfrak{C}) = \{q \in S(\mathfrak{C}) : p \subseteq q\}$.
$S_p(\mathfrak{C})$ is a closed subspace of $S(\mathfrak{C})$.
$G := Aut(\mathfrak{C}/A)$ acts on $S(\mathfrak{C})$ by homeomorphisms.

- $S(\mathfrak{C})$ is a G-flow.
- $S_p(\mathfrak{C})$ is a G-subflow.

Almost periodic/[weakly] generic types $q \in S_p(\mathfrak{C})$ good candidates for "large" extensions of p.

Specialized notions

- $U \in Def(\mathfrak{C})$ is p-generic if $p(\mathfrak{C})$ is covered by finitely many A-conjugates of U.
- $U \in Def(\mathfrak{C})$ is weakly p-generic if $U \cup V$ is p-generic for some non-p-generic $V \in Def(\mathfrak{C})$.
- $q \in S_p(\mathfrak{C})$ is [weakly] p-generic if every $U \in q$ is.

Fact

1. $WGen(S_p(\mathfrak{C})) = cl(APer(S_p(\mathfrak{C})))$.
2. If $Gen(S_p(\mathfrak{C})) \neq \emptyset$, then $Gen(S_p(\mathfrak{C})) = WGen(S_p(\mathfrak{C})) = APer(S_p(\mathfrak{C}))$.
3. If T is stable, then $Gen(S_p(\mathfrak{C})) \neq \emptyset$ and it consists exactly of the non-forking extensions of p.

This was used recently by Kaplan, Miller, Simon to prove a conjecture of Pillay and others on Lascar strong types.

Fact

1. $WGen(S_p(\mathfrak{C})) = cl(APer(S_p(\mathfrak{C})))$.
2. If $Gen(S_p(\mathfrak{C})) \neq \emptyset$, then $Gen(S_p(\mathfrak{C})) = WGen(S_p(\mathfrak{C})) = APer(S_p(\mathfrak{C}))$.
3. If T is stable, then $Gen(S_p(\mathfrak{C})) \neq \emptyset$ and it consists exactly of the non-forking extensions of p.

This was used recently by Kaplan, Miller, Simon to prove a conjecture of Pillay and others on Lascar strong types.

Fact

1. $WGen(S_p(\mathfrak{C})) = cl(APer(S_p(\mathfrak{C})))$.
2. If $Gen(S_p(\mathfrak{C})) \neq \emptyset$, then $Gen(S_p(\mathfrak{C})) = WGen(S_p(\mathfrak{C})) = APer(S_p(\mathfrak{C}))$.
3. If T is stable, then $Gen(S_p(\mathfrak{C})) \neq \emptyset$ and it consists exactly of the non-forking extensions of p.

This was used recently by Kaplan, Miller, Simon to prove a conjecture of Pillay and others on Lascar strong types.

Fact

1. $WGen(S_p(\mathfrak{C})) = cl(APer(S_p(\mathfrak{C})))$.
2. If $Gen(S_p(\mathfrak{C})) \neq \emptyset$, then $Gen(S_p(\mathfrak{C})) = WGen(S_p(\mathfrak{C})) = APer(S_p(\mathfrak{C}))$.
3. If T is stable, then $Gen(S_p(\mathfrak{C})) \neq \emptyset$ and it consists exactly of the non-forking extensions of p.

This was used recently by Kaplan, Miller, Simon to prove a conjecture of Pillay and others on Lascar strong types.

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M)$ has nice model-theoretic properties.
The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.
Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.

Group and semi-group connection

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M))$ has nice model-theoretic properties.
The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.
Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M))$ has nice model-theoretic properties.
The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.
Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M))$ has nice model-theoretic properties. The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.
Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.

Group and semi-group connection

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M)$ has nice model-theoretic properties.
The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.
Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.

Group and semi-group connection

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M))$ has nice model-theoretic properties.
The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.
Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.

Group and semi-group connection

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M)$ has nice model-theoretic properties. The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.

Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.

Group and semi-group connection

Assume $G \subseteq M$ is a definable group. Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is a Boolean algebra od sets, closed under left translation in G.
- $S_G(M) = S(Def_G(M))$ is the space of G-types over M.
- G acts on $S_G(M)$ by left translation.
- $S_G(M)$ is a G-flow.

The Ellis semigroup $E(S_G(M)$ has nice model-theoretic properties. The ideal subgroups of $E(S_G(M))$ are closely related to some model-theoretic connected components of G.
Questions on the model-theoretic absoluteness of the topological-dynamic notions in model theory.