Invited Talks

On the applications of counting independent sets in hypergraphs

József Balogh
University of Illinois

Recently, Balogh-Morris-Samotij and Saxton-Thomason developed a method of counting independent sets in hypergraphs. I will survey the field, and explain several applications. This includes counting maximal triangle-free graphs, maximal sum-free sets, intersecting families of permutations, intersecting families set-systems, estimate volume of metric spaces, etc.

These results are partly joint with Das, Delcourt, Liu, Morris, Mycroft, Petrickova, Samotij, Sharifzadeh, Treglown and Wagner.

Degree anti-Ramsey numbers of graphs

Dan Hefetz
University of Birmingham

The degree anti-Ramsey number $AR_d(H)$ of a graph H is the smallest integer k for which there exists a graph G with maximum degree at most k such that any proper edge colouring of G yields a rainbow copy of H. In this talk I will present a general upper bound on degree anti-Ramsey numbers, determine the precise value of the degree anti-Ramsey number of any forest, and prove an upper bound on the degree anti-Ramsey number of cycles, which is best possible up to a multiplicative factor of 2.

Based on joint work with Shoni Gilboa.

Intelligence vs Randomness: the power of choices

Michael Krivelevich
Tel Aviv University

Consider the following very standard experiment: n balls are thrown independently at random each into n bins (if you are practically inclined, think about distributing n jobs at random between n machines). It is quite easy to see that the maximum load over all bins will typically be about $\ln \frac{n}{\ln \ln n}$. If however each ball is allowed to choose between two randomly drawn bins, the typical maximum bin load drops dramatically to about $\ln \ln n$, as shown in a seminal paper of Azar, Broder, Karlin and Upfal from 1994 – an exponential improvement!

The above described result is just one manifestation of a recently widely studied phenomenon: a limited manipulation of the otherwise truly random input is frequently capable to advance various goals significantly. In this talk I will discuss results of this type, mainly focusing on the so called controlled random graph processes, where at each stage an algorithm is presented with a collection of randomly drawn edges and is allowed to manipulate this collection in a certain predefined, and rather limited, way.

Half-random Maker-Breaker games

Tibor Szabó
Freie Universität Berlin

We study Maker-Breaker positional games between two players, one of whom is playing randomly against an opponent with an optimal strategy. In both such scenarios, that is when Maker plays randomly and when Breaker plays randomly, we determine the sharp threshold bias of classical graph games, such as connectivity, Hamiltonicity, and minimum degree-k. The traditional, deterministic version of these games, with two optimal players playing, are known to obey the so-called probabilistic intuition. That is, the threshold bias of these games is asymptotically equal to the threshold bias of their random counterpart, where players just take edges uniformly at random. We find, that despite this remarkable agreement of the results of the deterministic and the random games, playing randomly against an optimal opponent is not a good idea: the threshold bias becomes significantly higher in favor of the "clever" player. An important qualitative aspect of the probabilistic intuition nevertheless carries through: for Maker to occupy a connected graph or a Hamilton cycle, the bottleneck is still the ability to achieve that there is no isolated vertex in his graph.

The talk represents joint work with Jonas Groschwitz.

Contributed Talks

Making spanning graphs

Peter Allen
London School of Economics and Political Science

I will explain how to combine a Maker-Breaker result of Ferber, Krivelevich and Naves with a random graph embedding result of the speaker with Boettcher, Han, Kohayakawa and Person to give new results on the critical bias for the Maker-Breaker H-game on K_n, where H is a large (possibly spanning) bounded degree graph.

Time permitting, I will also explain the limitations of this approach and how to overcome some of them. The latter is joint work with Boettcher, Kohayakawa, Naves and Person.

Manipulative Waiter does not always win

Sylwia Antoniuk
Polish Academy of Sciences

We consider a variation of the connectivity Waiter-Client game $WC(n, q, \mathcal{C})$. This is a two player, perfect information game played on a n-vertex graph G consisting of $q + 1$ edge-disjoint spanning trees. In each round Waiter presents $q + 1$ edges from G from which Client chooses one and the remaining edges are discarded. The aim of Waiter is to force Client to build a connected graph. If this happens Waiter wins. Otherwise Client is the winner. It is known that for $q + 1 = 2$ and $q + 1 = n/2$ (even n), Waiter has a winning strategy. In this talk we show that in most of the remaining cases there are graphs for which Client is the winner.

From red to violet – fragments of the rainbow matching theme

János Barát
MTA-ELTE GAC Research Group

If $M_1, M_2, \ldots, M_n$ are matchings and $e_1 \in M_1$, $e_2 \in M_2, \ldots, e_n \in M_n$ such that $e_1, \ldots, e_n$ form a matching, then $\{M_1, M_2, \ldots, M_n\}$ possesses a *rainbow matching.* Note that we do *not* require the matchings M_i to be disjoint. A $k \times n$ matrix R, containing symbols from an alphabet S, can be viewed as listing edges in a bipartite graph G_R. The two parts of G_R correspond respectively to the columns of R, and to S. Each cell in R corresponds to an edge in G_R that links the column and the symbol in the cell. A *partial transversal of length* ℓ in R is a selection of ℓ cells of R that contain ℓ different symbols and come from different rows and columns. A matrix is *generalised latin* if no symbol is repeated in any row or column. The $n \times n$ matrix L is a *latin square* if it is generalised latin and $n = |S|$. The following conjecture, attributed variously to Brualdi and Stein.

Conjecture: Every $n \times n$ latin square contains a partial transversal of length $(n - 1)$.

One might think that relaxing the latin constraint should lead to a full transversal. This observation has led to various generalizations. We will mention results, conjectures and counterexamples in these directions.

For instance, Aharoni and Berger conjectured that if a bipartite multigraph is the union of n matchings of size $n + 1$, then it contains a rainbow matching.

We showed that there is a set of $\lfloor n/2 \rfloor - 1$ matchings of size $n + 1$ and $n - \lfloor n/2 \rfloor + 1$ matchings of size n, which together have no rainbow matching.

Number theory problems in combinatorial games

Małgorzata Bednarska-Bzdęga
Adam Mickiewicz University

Let X be a finite set, $\mathcal{A}$ be a family of subsets of X and let a, b be positive integers. In the Avoider-Enforcer game in each round Avoider selects exactly a free elements of X and Enforces answers by selecting exactly b free elements of X. The game ends when there is no free element left. Enforcer tries to force Avoider to select all elements of at least one set from $\mathcal{A}$. It appears that the result of the game depends often on the reminder of the division of X by $a + b$. We will talk on two number theory problems arising in the context of such games.

Local resilience of spanning subgraphs in sparse random graphs

Julia Böttcher
London School of Economics and Political Science

Dellamonica, Kohayakawa, Rödl and Ruciński showed that for $p = C(\log n/n)^{1/d}$ the random graph $G(n,p)$ contains asymptotically almost surely all spanning graphs H with maximum degree d as subgraphs. In this talk I will discuss a resilience version of this result, which shows that for the same edge density, even if a $(1/k - \epsilon)$-fraction of the edges at every vertex is deleted adversarially from $G(n,p)$, the resulting graph continues to contain asymptotically almost surely all spanning H with maximum degree d, with sublinear bandwidth and with at least $C \max(p^{-2}, p^{-1} \log n)$ vertices not in triangles. Neither the restriction on the bandwidth, nor the condition that not all vertices are allowed to be in triangles can be removed. The proof uses a sparse version of the Blow-Up Lemma.

Joint work with Peter Allen, Julia Ehrenmüller, Anusch Taraz.

Rainbow matchings in multigraphs

Dennis Clemens
Technische Universität Hamburg-Harburg

A conjecture of Brualdi and Stein states that every Latin square of order n should contain a transversal of size $n-1$. Related to this problem, Aharoni and Berger conjectured that every family of n matchings of size $n+1$ in a bipartite multigraph contains a rainbow matching of size n. We prove that matching sizes of $3n/2 + o(n)$ suffice to guarantee such rainbow matching, improving previous results by Aharoni, Charbit and Howard, and Kotlar and Ziv. We also discuss a similar problem when the colour classes of the given multigraph are disjoint unions of cliques, thus addressing a question of Grinblat.

Joint work with Julia Ehrenmüller and Alexey Pokrovskiy.

Regularity inheritance in 3-uniform hypergraphs

Ewan Davies
London School of Economics and Political Science

We discuss a theory of regularity inheritance in 3-uniform hypergraphs, and as an application, a strengthening of the counting lemma of Frankl and Rödl. The method is ripe for extensions to higher uniformities and to more powerful embedding results such as a hypergraph Blow-Up lemma.

Hanani-Tutte for Radial Drawings

Radoslav Fulek
IST Austria

A drawing of a graph G is *radial* if the vertices of G are placed on concentric circles $C_1, \ldots, C_k$ with common center c and edges are drawn *radially*: every edge crosses every circle centered at c at most once. G is *radial planar* if it has a radial embedding, that is, a crossing-free radial drawing. If the vertices of G are ordered or partitioned into ordered levels (as they are for leveled graphs), we require that the assignment of vertices to circles corresponds to the given ordering or leveling.

We show that a graph G is radial planar if G has a radial drawing in which every two edges cross an even number of times; the radial embedding has the same leveling as the radial drawing. In other words, we establish the weak variant of the Hanani-Tutte theorem for radial planarity. This generalizes a result by Pach and Tóth.

This is a joint work with Marcus Schaefer and Michael Pelsmajer.

Intersecting P-free families

Dániel Gerbner
Renyi Institute

We study the problem of determining the size of the largest intersecting P-free family for a given partially ordered set P. In particular we find the exact size of the largest intersecting B-free family where B is the butterfly poset and classify the cases of equality.

Joint work with Abhishek Methuku and Casey Tompkins.

The Multicolour Ramsey Number of a Long Odd Cycle

Matthew Jenssen
London School of Economics and Political Science

For a graph G, the k-colour Ramsey number $R_k(G)$ is the least integer N such that every k-colouring of the edges of the complete graph K_N contains a monochromatic copy of G. Let C_n denote the cycle on n vertices. We show that for fixed $k \geq 3$ and n odd and sufficiently large,

$$R_k(C_n) = 2^{k-1}(n-1) + 1.$$

This generalises a result of Kohayakawa, Simonovits and Skokan and resolves a conjecture of Bondy and Erdős for large n. We also establish a surprising correspondence between extremal k-colourings for this problem and perfect matchings in the hypercube Q_k. This allows us to prove a stability-type strengthening of the above. The proof is analytic in nature, the first step of which is to use the Regularity Lemma to relate this problem in Ramsey Theory to one in Convex Optimisation.

Infinite dimensional finitely forcible graphon

Tereza Klimošová
University of Warwick

Graphons are analytic objects associated with convergent sequences of graphs. Problems from extremal combinatorics and theoretical computer science led to a study of graphons determined by finitely many subgraph densities, which are referred to as finitely forcible. We show that there exists a finitely forcible graphon such that the topological space of its typical vertices has infinite Lebesgue covering dimension, disproving the conjecture by Lovasz and Szegedy.

The talk is based on joint work with Roman Glebov and Dan Kral.

New developments on the Erdős-Hajnal conjecture

Anita Liebenau
University of Warwick

The Erdős-Hajnal conjecture asserts that for every fixed graph H there is a constant $c > 0$ such that any H-free graph G on n vertices contains a clique or an independent set of size n^c. This is in stark contrast to the random graph which contains no clique or independent of size $O(\log n)$, yet it contains every subgraph of fixed size. The conjecture is still widely open. In this talk I will survey what is known and I will present a new result related to the Erdős-Hajnal conjecture which provides new tools to tackle the problem.

Sparsity and dimension

Piotr Micek
Technische Universität Berlin & Jagiellonian University Krakow

We prove that posets of bounded height whose cover graphs belong to a fixed class with bounded expansion have bounded dimension. Bounded expansion, introduced by Nešetřil and Ossona de Mendez as a model for sparsity in graphs, is a property that is naturally satisfied by a wide range of graph classes, from graph structure theory (graphs excluding a minor or a topological minor) to graph drawing (e.g. graphs with constant book thickness). Therefore, our theorem generalizes a number of results including the most recent one for posets of bounded height with cover graphs excluding a fixed graph as a topological minor (Walczak, SODA 2015). We also show that the result is in a sense best possible, as it does not extend to nowhere dense classes; in fact, it already fails for cover graphs with locally bounded treewidth.

On-line list coloring of random graphs

Dieter Mitsche
Université Nice

In this paper, the on-line list colouring of binomial random graphs $G(n,p)$ is studied. We show that the on-line choice number of $G(n,p)$ is asymptotically almost surely asymptotic to the chromatic number of $G(n,p)$, provided that the average degree $d = p(n-1)$ tends to infinity faster than $(\log \log n)^{1/3}(\log n)^2 n^{2/3}$. For sparser graphs, we are slightly less successful; we show that if $d > (\log n)^{2+\epsilon}$ for some $\epsilon > 0$, then the on-line choice number is larger than the chromatic number by at most a multiplicative factor of C, where $C \in [2,4]$, depending on the range of d. Also, for $d = O(1)$, the on-line choice number is by at most a multiplicative constant factor larger than the chromatic number.

Joint work with Alan Frieze, Xavier Pérez-Giménez and Pawel Prałat.

On the subgraphs of minimum degree 3

Rajko Nenadov
ETH Zürich

Given a graph G on n vertices and $2n-1$ edges, what can we say about the smallest subset $S \subseteq V(G)$ such that $\delta(G[S]) = 3$? It is easy to see that G contains an induced subgraph with minimum degree 3, and Erdős, Faudree, Rousseau and Schelp showed that $|S| \leq n - \Theta(\sqrt{n})$. Furthermore, Erdős conjecture that there exists an absolute constant $\varepsilon > 0$ such that $|S| \leq (1-\varepsilon)n$. Note that $2n-1$ is the smallest number of edges for which this question is sensible, since a wheel on n vertices has $2n-2$ edges and no proper induced subgraph with minimum degree 3.

We make a small progress towards this conjecture. Implicitly, Erdős et al. showed that if $\Delta(G) = 4$ then such ε exists. Here we show that if $\Delta(G) = 5$ then $|S| \leq n - \Theta(n^{0.6})$.

This is joint work with Ralph Keusch, Frank Mousset, Andreas Noever, Yury Person and Nemanja Škorić.

Counting subgraphs in F-free graphs

Cory Palmer
University of Montana

For a fixed graph F we say that a graph is F-free if it contains no copy of F as a subgraph. The standard question in extremal graph theory is to determine the maximum number of edges possible in an n-vertex F-free graph. In this talk we will discuss a generalization of this problem. Given fixed graphs F and H, determine the maximum number of copies of a subgraph H in an n-vertex F-free graph. We will survey what is known about this problem and present a new result that generalizes the Erdős-Stone-Simonovits theorem.

Joint work with Dániel Gerbner.

Decomposition of Multiple Coverings

Dömötör Pálvölgyi
ELTE Budapest

I survey old and new results related to decomposition of multiple coverings. For example, there is an m such that any m-fold covering of the plane with axis parallel unit squares decomposes into two coverings of the plane, i.e., we can color the squares red and blue such that both families form a covering of the whole plane, but the same is not true if we replace squares with disks.

Finding a majority element

Balázs Patkós
MTA-ELTE GAC Research Group & MTA Renyi Institute

Suppose we are given n (indexed) balls in an urn each colored by some color. A *majority ball* is a ball the color class of which is larger than $n/2$. A *k-majority ball* is a ball the color class of which has size at least k. A *plurality ball* is a ball the color class of which is strictly larger than any other color class. Our aim is to find (the index of) a majority / k-majority / plurality ball (or to show that there exists none) with as few queries as possible. A query is always a subset S of the set of balls and we investigate different models according to whether a query asks (i) if all balls in S have the same color (ii) for a majority ball in S (iii) etc. We mostly study cases when queries must be of size 2 or 3.

The Ramsey Numbers of Squares of Paths and Cycles

Barnaby Roberts
London School of Economics and Political Science

The square of a graph G is the graph on the same vertex set in which vertices are joined if they are at distance at most two in G. We determine the Ramsey numbers of squares of paths with a view to getting a general upper bound on the Ramsey numbers of bounded degree tri-partite graphs with sublinear bandwidth.

Joint work with Peter Allen and Jozef Skokan.

Embedding the binomial random graph in the regular random graph

Matas Šileikis
University of Oxford

Let $G(n,p)$ be the binomial random graph and $R(n,d)$ the random d-regular graph. In 2004 Kim and Vu proved that one can define a joint distribution of these two graphs (with $p \sim d/n$) in such a way that $G(n,p)$ is a subgraph of $R(n,d)$ with probability tending to 1, provided $\log n \ll d \ll n^{1/3}/\log^2 n$.

Such an embedding allows deduction of monotone increasing properties (like Hamiltonicity) of $R(n,d)$ from the simpler model $G(n,p)$. We relax the restriction on d to $\log n \ll d \ll n$ and provide an extension to random hypergraphs.

Joint work with A. Dudek, A. Frieze, and A. Ruciński.

A tight Erdős-Pósa function for long cycles

Nemanja Škorić
ETH Zürich

A family H of graphs is said to have the Erdös-Pósa property if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that every graph G contains either k vertex-disjoint members of H or a set X of at most $f(k)$ vertices such that $G - X$ has no subgraph isomorphic to a graph in H. We show that, for every l, the family of cycles of length at least l satisfies the Erdös-Pósa property with $f(k) = O(kl + k \log k)$. This is optimal up to a constant factor.

Joint work with Frank Mousset, Andreas Noever and Felix Weissenberger.

On Kinetic Range Spaces and their Applications

Shakhar Smorodinsky
Ben-Gurion University of the NEGEV & EPFL

We study geometric hypergrahs in a kinetic setting. That is, the set of vertices of the hypergraph is a set of moving points in $\Re^d$ with coordinates that are polynomials in time. The hyperedges are all subsets that can be realized by intersecting the set of points at some fixed time with some "simple" geometric shape, such as, say, a halfspace. We show that for many of the static cases where the VC-dimension of the hypergraph is bounded, the kinetic counterpart also has bounded VC-dimension. This allows us to prove our main result: for any set of n moving points in $\Re^d$ and any parameter $1 < k < n$, one can select a non-empty subset of the points of size $O(k \log k)$ such that the Voronoi diagram of this subset is "balanced" at any given time. By that, we mean that at any time, each Voronoi cell contains at most $O(n/k)$ of the points. We also show that the bound $O(k \log k)$ is near optimal already for the one dimensional case (i.e., $d = 1$) and points moving linearly (i.e., with constant speed). As an application, we show that we can assign a communication radius to a collection of n moving sensors so that at any given time, their interference is $O(\sqrt{n \log n})$. This is optimal up to an $O(\sqrt{\log n})$ factor.

The guarding game is E-complete

Tomáš Valla
Czech Technical University in Prague

The guarding game is a game in which several cops try to guard a region in a (directed or undirected) graph against Robber. Robber and the cops are placed on the vertices of the graph; they take turns in moving to adjacent vertices (or staying), cops inside the guarded region, Robber on the remaining vertices (the robber-region). The goal of Robber is to enter the guarded region at a vertex with no cop on it. The problem is to determine whether for a given graph and given number of cops the cops are able to prevent Robber from entering the guarded region.

Fomin et al. [Fomin, Golovach, Hall, Mihalák, Vicari, Widmayer: How to Guard a Graph? Algorithmica 61(4), 839–856 (2011)] proved that the problem is NP-complete when the robber-region is restricted to a tree. Further they prove that is it PSPACE-complete when the robber-region is restricted to a directed acyclic graph, and they ask about the problem complexity for arbitrary graphs. In this paper we prove that the problem is E-complete for arbitrary directed graphs.