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CONNECTIONS BETWEEN THE DOUBLE ALTERNATED ABSOLUTE DIFFERENTIAL OF CURVATURE TENSORS OF THE FINSLER SPACE AND INDUCED CURVATURE TENSORS OF ITS SUBSPACE

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1. INTRODUCTION

The subspace F_m of a Finsler space F_n is given by the equations:

$$x^i = x^i(u^1, u^2, \dots, u^m) \qquad i,j,k,l,\dots = 1,2,\dots,n,$$

if the rank of the matrix

$$\|B^i_\alpha\| = \left\|\frac{\partial x_i}{\partial u^\alpha}\right\| \qquad \alpha,\beta,\gamma,\delta,\varepsilon,\iota, \ ,\eta,\dots,=1,2,\dots,m$$

is assumed to be m. To every vector $\dot{u}^\alpha$, which is tangent to F_m, may be associated a vector $\dot{x}^i$ in the following way:

$$\dot{x}^i = B^i_\alpha \dot{u}^\alpha .$$

At every point P of F_m there are n-m linearly independent vectors

$$\overset{\nu}{N}_i \qquad \mu,\nu,\zeta,\rho,\Psi = n-m+1,\dots,n$$

which satisfy the relations of |1|

$$\overset{\nu}{N}_i B^i_\alpha = 0 \qquad \underset{\mu}{N}^i \underline{\underline{\mathrm{def}}}\ g^{ij}(x,\dot{x})\overset{\mu}{N}_j$$

$$g_{ij}(x,\dot{x})\underset{\nu}{N}^i\underset{\mu}{N}^j = \delta_{\nu\mu}$$

If D and Δ are absolute differentials which correspond to the motion from $(u^\beta, \dot{u}^\beta)$ to $(u^\beta + du^\beta, \dot{u} + d\dot{u}^\beta)$ and $(u^\beta + \delta u^\beta, \dot{u}^\beta + \delta\dot{u}^\beta)$

in the subspace F_m of a Finsler space F_n then as in |2|* we have

$$(1.1)\quad [\Delta D] B^i_\alpha = \bar{\Omega}_\alpha{}^\delta(d,\delta) B^i_\delta + \bar{\Omega}_\alpha{}^\mu(d,\delta) \underset{\mu}{N}^i + \tilde{D} B^i_\alpha$$

$$(1.2)\quad [\Delta D] \underset{\mu}{N}^i = \bar{\Omega}_\mu{}^\delta(d,\delta) B^i_\delta + \bar{\Omega}_\mu{}^\nu(d,\delta) \underset{\nu}{N}^i + \tilde{D}\underset{\mu}{N}^i$$

$$(1.3)\quad \bar{\Omega}_\alpha{}^\delta(d,\delta) = \frac{1}{2} \overset{2}{R}_\alpha{}^\delta{}_{\beta\gamma}[du^\beta \delta u^\gamma] + \overset{2}{P}_\alpha{}^\delta{}_{\beta\gamma}[du^\beta \bar{\Delta}\ell^\gamma] + \frac{1}{2} \overset{2}{S}_\alpha{}^\delta{}_{\beta\gamma}[\bar{D}\ell^\beta \bar{\Delta}\ell^\gamma]$$

$$(1.4)\quad \bar{\Omega}_\alpha{}^\mu(d,\delta) = \frac{1}{2} \overset{2}{R}_\alpha{}^\mu{}_{\beta\gamma}[du^\beta \delta u^\gamma] + \overset{2}{P}_\alpha{}^\mu{}_{\beta\gamma}[du^\beta \bar{\Delta}\ell^\gamma] + \frac{1}{2} \overset{2}{S}_\alpha{}^\mu{}_{\beta\gamma}[\bar{D}\ell^\beta \bar{\Delta}\ell^\gamma]$$

$$(1.5)\quad \bar{\Omega}_{\mu\alpha} = -\bar{\Omega}_{\alpha\mu}$$

$$(1.6)\quad \bar{\Omega}_\mu{}^\nu(d,\delta) = \frac{1}{2} \overset{2}{R}_\mu{}^\nu{}_{\beta\gamma}[du^\beta \delta u^\gamma] + \overset{2}{P}_\mu{}^\nu{}_{\beta\gamma}[du^\beta \bar{\Delta}\ell^\gamma] + \frac{1}{2} \overset{2}{S}_\mu{}^\nu{}_{\beta\gamma}[\bar{D}\ell^\beta \bar{\Delta}\ell^\gamma]$$

For the arbitrary vector field ξ defined on the subspace F_m of the Funsler space F_n we have

$$(1.7)\quad \xi^i = B^i_\alpha \xi^\alpha + \underset{\mu}{N}^i \xi^\mu$$

It is known that

$$(1.8)\quad [\Delta D]\xi^i = \frac{1}{2} R_j{}^i{}_{hk}\xi^j[dx^h \delta x^k] + P_j{}^i{}_{hk}\xi^j[dx^h \Delta\ell^k] + \frac{1}{2} S_j{}^i{}_{hk}\xi^j[D\ell^h \Delta\ell^k] + \tilde{D}\xi^i$$

where

$$(1.9)\quad dx^h = B^h_\alpha du^\alpha$$

$$(1.10)\quad D\ell^k = B^k_\beta \bar{D}\ell^\beta + \bar{H}^k_\beta du^\beta$$

On the other hand

$$(1.11)\quad [\Delta D]\xi^i = \xi^\alpha[\Delta D]B^i_\alpha + \xi^\mu[\Delta D]\underset{\mu}{N}^i + (\delta d - d\delta)\xi^\alpha B^i_\alpha\ (\delta d - d\delta)\xi^\mu \underset{\mu}{N}^i$$

*) One can easelyconclude what the tensors $\overset{2}{R}, \overset{2}{P}, \overset{2}{S}$ are from (2.1), (29'),(2.18) and (2.19),(2,27),(2.33) in |2| . They are explicitly defined in |3| .

Substituting (1.9), (1.10) into (1.8) and (1.1)-(1.6) into (1.11) we get two relations for $[\Delta D]\xi^i$. Equating the coefficients of bivectors

$[du^\beta\ \delta u^\gamma], [du^\beta \bar{\Delta}\ell^\gamma]$ and $[\bar{D}\ell^\beta \bar{\Delta}\ell^\gamma]$, neglecting inifinitesimals of a higher order, we obtain the relations:

$$(1.12)\quad R_j{}^i{}_{hk}\xi^j B^{hk}_{\beta\gamma} + P_j{}^i{}_{hk}\xi^j B^h_{[\beta}\ \bar{H}^k_{\gamma]} + S_j{}^i{}_{hk}\xi^j \bar{H}^k_\beta\ \bar{H}^k_\gamma =$$
$$= \overset{2}{R}_\alpha{}^\varepsilon{}_{\beta\gamma}\xi^\alpha B^i_\varepsilon + \overset{2}{R}_\alpha{}^\mu{}_{\beta\gamma}\xi^\alpha N^i_\mu + \overset{2}{R}_\mu{}^\varepsilon{}_{\beta\gamma}\xi^\mu B^i_\varepsilon + \overset{2}{R}_\mu{}^\nu{}_{\beta\gamma}\xi^\mu N^i_\nu,$$

$$(1.13)\quad P_j{}^i{}_{hk}\xi^j B^{hk}_{\beta\gamma} + S_j{}^i{}_{hk}\xi^j\ \bar{H}^h_\beta B^k_\gamma =$$
$$\overset{2}{P}_\alpha{}^\varepsilon{}_{\beta\gamma}\xi^\alpha B^i_\varepsilon + \overset{2}{P}_\alpha{}^\mu{}_{\beta\gamma}\xi^\alpha N^i_\mu + \overset{2}{P}_\mu{}^\varepsilon{}_{\beta\gamma}\xi^\mu B^i_\varepsilon + \overset{2}{P}_\mu{}^\nu{}_{\beta\gamma}\xi^\mu\ N^i_\nu,$$

$$(1.14.)\quad S_j{}^i{}_{kh}\ \xi^j\ B^{hk}_{\beta\gamma} =$$
$$\overset{2}{S}_\alpha{}^\varepsilon{}_{\beta\gamma}\xi^\alpha B^i_\varepsilon + \overset{2}{S}_\alpha{}^\mu{}_{\beta\gamma}\xi^\alpha N^i_\mu + \overset{2}{S}_\mu{}^\varepsilon{}_{\beta\gamma}\xi^\mu B^i_\varepsilon + \overset{2}{S}_\mu{}^\nu{}_{\beta\gamma}\xi^\mu N^i_\mu.$$

Using (1,7) and putting $\xi^\mu=0$, then $\xi^\alpha=0$ in (1.12), (1.13) and (1.14) we get

$$(1.15)\quad R_j{}^i{}_{hk}B^{jhk}_{\alpha\beta\gamma} + P_j{}^i{}_{hk}B^j_\alpha B^h_{[\beta}\bar{H}^k_{\gamma]} + S_j{}^i{}_{hk}B^j_\alpha \bar{H}^h_\beta \bar{H}^k_\gamma =$$
$$= \overset{2}{R}_\alpha{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon + \overset{2}{R}_\alpha{}^\mu{}_{\beta\gamma}N^i_\mu$$

$$(1.16)\quad R_j{}^i{}_{hk}N^j B^{hk}_{\beta\gamma} + P_j{}^i{}_{hk}N^j_\mu B^k_{[\beta}\ \bar{H}^k_{\gamma]} + S_j{}^i{}_{hk}N^j_\mu \bar{H}^j_\beta\ \bar{H}^k_\gamma =$$
$$= \overset{2}{R}_\mu{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon + \overset{2}{R}_\mu{}^\nu{}_{\beta\gamma}N^i_\nu$$

$$(1.17)\quad P_j{}^i{}_{hk}\ B^{jhk}_{\alpha\beta\gamma} + S_j{}^i{}_{hk}\ B^j_\alpha\ \bar{H}^h_\beta\ B^k_\gamma = \overset{2}{P}_\alpha{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon + \overset{2}{P}_\alpha{}^\mu{}_{\beta\gamma}N^i_\mu$$

$$(1.18)\quad P_j{}^i{}_{hk}N^j_\mu B^{hk}_{\beta\gamma} + S_j{}^i{}_{hk}N^j_\mu \bar{H}^h_\beta B^k_\gamma = \overset{2}{P}_\mu{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon + \overset{2}{P}_\mu{}^\nu{}_{\beta\gamma}N^i_\nu$$

$$(1.19)\quad S_j{}^i{}_{hk}\ B^{jhk}_{\alpha\beta\gamma} = \overset{2}{S}_\alpha{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon + \overset{2}{S}_\alpha{}^\mu{}_{\beta\gamma}N^i_\mu$$

(1.20) $$S_j{}^i{}_{hk}N^j_\mu B^{hk}_{\beta\gamma} = \overset{2}{S}_\mu{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon + \overset{2}{S}_\mu{}^\nu{}_{\beta\gamma}N^i_\nu$$

The curvature tensors R, P, S of the Finsler space F_n and induced curvature tensors $\overset{2}{R}$, $\overset{2}{P}$, $\overset{2}{S}$ of its subspace F_m are connected by the relations (1.15)-(1.20).

2. DOUBLE ALTERNATED ABSOLUTE DIFFERENTIALS OF CURVATURE TENSORS IN F_n

If D_1 and D_2 are the absolute differentials which correspond to the motion from $(u^\beta, \dot u^\beta)$ to $(u^\beta d_1 u^\beta, u^\beta + d_1 u^\beta)$ and $(u^\beta + d_2 u^\beta, \dot u^\beta + d_2\dot u^\beta)$ in the subspace F_m of the Finsler space F_n, then we have from (1.15)-(1.20), using (1.1), (1.2) and

(2.1) $$\bar H^k_\gamma = \bar\Theta_\iota{}^\mu{}_\gamma \ell^\iota N^k_\mu ,$$

(2.2) $$[D_1 D_2]R_j{}^i{}_{hk}B^{jhk}_{\alpha\beta\gamma} + R_j{}^i{}_{hk}(\bar\Omega_\alpha{}^\delta B^j_\delta + \bar\Omega_\alpha{}^\nu N^j_\nu)B^{hk}_{\beta\gamma} +$$
$$+ R_j{}^i{}_{hk}B^j_\alpha(\bar\Omega_\beta{}^\delta B^h_\delta + \bar\Omega_\beta{}^\nu N^h_\nu)B^k_\gamma + R_j{}^i{}_{hk}B^{jh}_{\alpha\beta}(\bar\Omega_\gamma{}^\delta B^k_\delta + \bar\Omega_\gamma{}^\nu N^k_\nu) +$$
$$+ [D_1 D_2]P_j{}^i{}_{hk}B^{jh}_{\alpha[\beta}\bar H^k_{\gamma]} + P_j{}^i{}_{hk}(\bar\Omega_\alpha{}^\delta B^j_\delta + \Omega_\alpha{}^\nu N^j_\nu)B^h_{[\beta}\bar H^k_{\gamma]} +$$
$$+ P_j{}^i{}_{hk}B^j_\alpha(\bar\Omega_{[\beta}{}^\delta B^k_{|\delta|} + \bar\Omega_{[\beta}{}^\nu N^j_\nu)\bar H^k_{\gamma]} + P_j{}^i{}_{hk}B^j_\alpha B^h_{[\beta}\ \bar\Theta_{\iota\gamma}{}^\mu \ell^\iota\ (\bar\Omega_\mu{}^\delta B^k_\delta +$$
$$+ \bar\Omega_\mu{}^\nu N^k_\nu) + [D_1 D_2]S_j{}^i{}_{hk}B^j_\alpha \bar H^h_\beta \bar H^k_\gamma + S^i_{jhk}(\bar\Omega_\alpha{}^\delta B^j_\delta +$$
$$+ \bar\Omega_\alpha{}^\nu N^j_\nu)\bar H^h_\beta \bar H^k_\gamma + S_j{}^i{}_{hk}B^j_\beta\bar\Theta_\iota{}^\mu{}_\beta \ell^\iota(\bar\Omega_\mu{}^\delta B^h_\delta + \bar\Omega_\mu{}^\nu N^h_\nu)\bar H^k_\gamma +$$
$$+ S_j{}^i{}_{hk}B^j_\alpha \bar H^h_\beta\ \Theta_\iota{}^\nu{}_\gamma \ell^\iota(\bar\Omega_\nu{}^\delta B^k_\delta + \bar\Omega_\nu{}^\mu N^k_\mu) =$$
$$= \overset{2}{R}_\alpha{}^\varepsilon{}_{\beta\gamma}(\bar\Omega_\varepsilon{}^\delta B^i_\delta + \bar\Omega_\varepsilon{}^\nu N^i_\nu) + \overset{2}{R}_\alpha{}^\mu{}_{\beta\gamma}(\bar\Omega_\mu{}^\delta B^i_\delta + \bar\Omega_\mu{}^\nu N^i_\nu),$$

(2.3) $$[D_1 D_2]R_j{}^i{}_{hk}N^j_\mu B^{hk}_{\beta\gamma} + R_j{}^i{}_{hk}(\bar\Omega_\mu{}^\delta B^j_\delta + \bar\Omega_\mu{}^\nu N^j_\nu)B^{hk}_{\beta\gamma} +$$
$$+ R_j{}^i{}_{hk}N^j_\mu(\bar\Omega_\beta{}^\delta B^h_\delta + \bar\Omega_\beta{}^\nu N^h_\nu)B^k_\gamma + R_j{}^i{}_{hk}N^j_\mu B^h_\beta(\bar\Omega_\gamma{}^\delta B^k_\delta + \bar\Omega_\gamma{}^\nu N^k_\nu) +$$

$$+ [D_1D_2]P_j{}^i{}_{hk}N^j_\mu B^k_{[\beta}\bar{H}^k_{\gamma]} + P_j{}^i{}_{hk}(\bar{\Omega}_\mu{}^\delta B^j_\delta + \bar{\Omega}_\mu{}^\nu N^j_\nu)B^h_{[\beta}\bar{H}^k_{\gamma]} +$$

$$+ P_j{}^i{}_{hk}N^j_\mu(\bar{\Omega}_{[\beta}{}^\delta B^k_{|\delta|} + \bar{\Omega}_{[\beta}{}^\nu N^k_\nu)\bar{H}^k_{\gamma]} + P_j{}^i{}_{hk}N^j_\mu B^h_\beta\bar{\Theta}_\iota{}^\mu{}_\gamma \ell^\iota(\bar{\Omega}_\mu{}^\delta B^k_\delta +$$

$$+ \bar{\Omega}_\mu{}^\nu N^k_\nu) + [D_1D_2]S_j{}^i{}_{hk}N^j_\mu\bar{H}^h_\beta\bar{H}^k_\gamma + S_j{}^i{}_{hk}(\bar{\Omega}_\mu{}^\delta B^j_\delta + \bar{\Omega}_\mu{}^\nu N^j_\nu)\bar{H}^h_\beta\bar{H}^k_\gamma +$$

$$+ S_j{}^i{}_{hk}N^j_\mu\Theta_\iota{}^\Psi{}_\beta \ell^\iota(\bar{\Omega}_\Psi{}^\delta B^k_\delta + \bar{\Omega}_\Psi{}^\nu N^k_\nu)\bar{H}^k_\gamma + S_j{}^i{}_{hk}N^j_\mu\bar{H}^k_\beta\bar{\Theta}_\iota{}^\Psi{}_\gamma \ell^\iota(\bar{\Omega}_\Psi{}^\delta B^k_\delta +$$

$$+ \bar{\Omega}_\Psi{}^\nu N^k_\nu) = \overset{2}{R}_\mu{}^\varepsilon{}_{\beta\gamma}(\bar{\Omega}_\varepsilon{}^\delta B^i_\delta + \bar{\Omega}_\varepsilon{}^\nu N^i_\nu) + \overset{2}{R}_\mu{}^\nu{}_{\beta\gamma}(\bar{\Omega}_\nu{}^\delta B^i_\delta + \bar{\Omega}_\nu{}^\Psi N^i_\Psi)$$

(2.4)
$$[D_1D_2]P_j{}^i{}_{hk}B^{jhk}_{\alpha\beta\gamma} + P_j{}^i{}_{hk}(\bar{\Omega}_\alpha{}^\delta B^j_\delta + \bar{\Omega}_\alpha{}^\nu N^j_\nu)B^{hk}_{\beta\gamma} +$$

$$+ P_j{}^i{}_{hk}B^j_\alpha(\bar{\Omega}_\beta{}^\delta B^h_\delta + \bar{\Omega}_\beta{}^\nu N^h_\nu)B^k_\gamma + P_j{}^i{}_{hk}B^{jh}_{\alpha\beta}(\bar{\Omega}_\gamma{}^\delta B^k_\delta + \bar{\Omega}_\gamma{}^\nu N^k_\nu) +$$

$$+ [D_1D_2]S_j{}^i{}_{hk}B^j_\alpha\bar{H}^k_\beta B^k_\gamma + S_j{}^i{}_{hk}(\bar{\Omega}_\alpha{}^\delta B^j_\delta + \bar{\Omega}_\alpha{}^\nu N^j_\nu)\bar{H}^h_\beta B^k_\gamma +$$

$$+ S_j{}^i{}_{hk}B^j_\alpha\bar{\Theta}_\iota{}^\Psi{}_\beta \ell^\iota(\bar{\Omega}_\Psi{}^\delta B^h_\delta + \bar{\Omega}_\Psi{}^\nu N^h_\nu)B^k_\gamma + S_{jhk}^i B^j_\alpha\bar{H}^h_\beta(\bar{\Omega}_\gamma{}^\delta B^k_\delta + \bar{\Omega}_\gamma{}^\nu N^k_\nu)$$

$$= \overset{2}{P}_\alpha{}^\varepsilon{}_{\beta\gamma}(\bar{\Omega}_\varepsilon{}^\delta B^i_\delta + \bar{\Omega}_\varepsilon{}^\nu N^i_\nu) + \overset{2}{P}_\alpha{}^\mu{}_{\beta\gamma}(\bar{\Omega}_\mu{}^\delta B^i_\delta + \bar{\Omega}_\mu{}^\nu N^i_\nu)$$

(2.5)
$$[D_1D_2]P_j{}^i{}_{hk}N^j_\mu B^{hk}_{\beta\gamma} + P_j{}^i{}_{hk}(\bar{\Omega}_\mu{}^\delta B^j_\delta + \bar{\Omega}_\mu{}^\nu N^j_\nu)B^{hk}_{\beta\gamma} +$$

$$+ P_j{}^i{}_{hk}N^j_\mu(\bar{\Omega}_\beta{}^\delta B^h_\delta + \bar{\Omega}_\beta{}^\nu N^h_\nu)B^k_\gamma + P_j{}^i{}_{hk}N^j_\mu B^h_\beta(\bar{\Omega}_\gamma{}^\delta B^k_\delta + \bar{\Omega}_\gamma{}^\nu N^k_\nu) +$$

$$+ [D_1D_2]S_j{}^i{}_{hk}N^j_\mu\bar{H}^h_\beta B^k_\gamma + S_j{}^i{}_{hk}(\bar{\Omega}_\mu{}^\delta B^j_\delta + \bar{\Omega}_\mu{}^\nu N^j_\nu)\bar{H}^h_\beta B^k_\gamma +$$

$$+ S_j{}^i{}_{hk}N^j_\mu\bar{\Theta}_\iota{}^\Psi{}_\beta \ell^\iota(\bar{\Omega}_\Psi{}^\delta B^h_\delta + \bar{\Omega}_\Psi{}^\nu N^h_\nu)B^k_\gamma + S_j{}^i{}_{hk}N^j_\mu\bar{H}^h_\beta(\bar{\Omega}_\gamma{}^\delta B^k_\delta +$$

$$+ \bar{\Omega}_\gamma{}^\nu N^k_\nu) = \overset{2}{P}_\mu{}^\varepsilon{}_{\beta\gamma}(\bar{\Omega}_\varepsilon{}^\delta B^i_\delta + \bar{\Omega}_\varepsilon{}^\nu N^i_\nu) + \overset{2}{P}_\mu{}^\Psi{}_{\beta\gamma}(\bar{\Omega}_\Psi{}^\delta B^i_\delta + \bar{\Omega}_\Psi{}^\nu N^i_\nu)$$

(2.6)
$$[D_1D_2]S_j{}^i{}_{hk}B^{jhk}_{\alpha\beta\gamma} + S_j{}^i{}_{hk}(\bar{\Omega}_\alpha{}^\delta B^j_\delta + \bar{\Omega}_\alpha{}^\nu N^j_\nu)B^{hk}_{\beta\gamma} + S_j{}^i{}_{hk}B^j_\alpha(\bar{\Omega}_\beta{}^\delta B^h_\delta +$$

$$+ \bar{\Omega}_\alpha{}^\nu N^h_\nu)B^k_\gamma + S_j{}^i{}_{hk}B^{jh}_{\alpha\beta}(\bar{\Omega}_\gamma{}^\delta B^k_\delta + \bar{\Omega}_\gamma{}^\nu N^k_\nu) = \overset{2}{S}_\alpha{}^\varepsilon{}_{\beta\gamma}(\bar{\Omega}_\varepsilon{}^\delta B^i_\delta + \bar{\Omega}_\varepsilon{}^\nu N^i_\nu) +$$

$$+ \overset{2}{S}_\alpha{}^\mu{}_{\beta\gamma}(\bar{\Omega}_\mu{}^\delta B^i_\delta + \bar{\Omega}_\mu{}^\nu N^i_\nu)$$

$$(2.7)\quad [D_1\ D_2] S_j{}^i{}_{hk} \underset{\mu}{N}{}^j B^{hk}_{\beta\gamma} + S_j{}^i{}_{hk} (\bar{\Omega}_\mu{}^\delta B^j_\delta + \bar{\Omega}_\mu{}^\nu \underset{\nu}{N}{}^j) B^{hk}_{\beta\gamma} + $$
$$+ S_j{}^i{}_{hk} \underset{\mu}{N}{}^j (\bar{\Omega}_\beta{}^\delta B^h_\delta + \bar{\Omega}_\beta{}^\nu \underset{\nu}{N}{}^h) B^k_\gamma + S_j{}^i{}_{hk} \underset{\mu}{N}{}^j B^h_\beta (\bar{\Omega}_\gamma{}^\delta B^k_\delta + \Omega_\gamma{}^\nu \underset{\nu}{N}{}^k) =$$
$$= \overset{2}{S}{}_\mu{}^\varepsilon{}_{\beta\gamma} (\bar{\Omega}_\varepsilon{}^\delta B^i_\delta + \bar{\Omega}_\varepsilon{}^\nu \underset{\nu}{N}{}^i) + \overset{2}{S}{}_\mu{}^\psi{}_{\beta\gamma} (\bar{\Omega}_\psi{}^\delta B^i_\delta + \bar{\Omega}_\psi{}^\nu \underset{\nu}{N}{}^i)$$

In formulas (2.2)-(7.2)

$$\bar{\Omega} = \bar{\Omega}(d_2, d_1)$$

for all indices of $\bar{\Omega}$.

3. A SPECIAL CASE

If the space and its subspace are Riemannian, then from (2.2), (2.3) we obtain (in case the Riemannian space tensors P and S are zero)

$$(3.1)\quad [D_1 D_2] R_j{}^i{}_{hk} B^{jhk}_{\alpha\beta\gamma} + R_j{}^i{}_{hk} (\bar{\Omega}_\alpha{}^\delta B^j_\delta + \bar{\Omega}_\alpha{}^\nu \underset{\nu}{N}{}^j) B^{hk}_{\beta\gamma} +$$
$$+ R_j{}^i{}_{hk} B^j_\alpha (\bar{\Omega}_\beta{}^\delta B^h_\delta + \bar{\Omega}_\beta{}^\nu \underset{\nu}{N}{}^h) B^k_\gamma + R_j{}^i{}_{hk} B^{jh}_{\alpha\beta} (\bar{\Omega}_\gamma{}^\delta B^k_\delta + \bar{\Omega}_\gamma{}^\nu \underset{\nu}{N}{}^k) =$$
$$= \overset{2}{R}{}_\alpha{}^\varepsilon{}_{\beta\gamma} (\bar{\Omega}_\varepsilon{}^\delta B^i_\delta + \bar{\Omega}_\varepsilon{}^\nu \underset{\nu}{N}{}^i) + \overset{2}{R}{}_\alpha{}^\mu{}_{\beta\gamma} (\bar{\Omega}_\mu{}^\delta B^i_\delta + \bar{\Omega}_\mu{}^\nu \underset{\nu}{N}{}^i)$$

$$(3.2)\quad [D_1 D_2] R_j{}^i{}_{hk} \underset{\mu}{N}{}^j B^{hk}_{\beta\gamma} + R_j{}^i{}_{hk} (\bar{\Omega}_\mu{}^\delta B^j_\delta + \bar{\Omega}_\mu{}^\nu \underset{\nu}{N}{}^j) B^{hk}_{\beta\gamma} +$$
$$+ R_j{}^i{}_{hk} \underset{\mu}{N}{}^j (\bar{\Omega}_\beta{}^\delta B^k_\delta + \bar{\Omega}_\beta{}^\nu \underset{\nu}{N}{}^k) B^k_\gamma + R_j{}^i{}_{hk} \underset{\mu}{N}{}^j B^k_\beta (\bar{\Omega}_\gamma{}^\delta B^k_\delta + \bar{\Omega}_\gamma{}^\nu \underset{\nu}{N}{}^k) =$$
$$= \overset{2}{R}{}_\mu{}^\varepsilon{}_{\beta\gamma} (\bar{\Omega}_\varepsilon{}^\delta B^i_\delta + \bar{\Omega}_\varepsilon{}^\nu \underset{\nu}{N}{}^i) + \overset{2}{R}{}_\mu{}^\nu{}_{\beta\gamma} (\bar{\Omega}_\nu{}^\delta B^i_\delta + \bar{\Omega}_\nu{}^\psi \underset{\psi}{N}{}^i)$$

In the two above formulas

$$\bar{\Omega}_\beta{}^\delta = \frac{1}{2} \overset{2}{R}{}_\beta{}^\delta{}_{\gamma\delta} [d_2 u^\gamma, d_1 u^\delta]$$
$$\bar{\Omega}_\alpha{}^\mu = \frac{1}{2} \overset{2}{R}{}_\alpha{}^\mu{}_{\beta\gamma} [d_2 u^\beta, d_1 u^\gamma]$$
$$\bar{\Omega}_\mu{}^\nu = \frac{1}{2} \overset{2}{R}{}_\mu{}^\nu{}_{\beta\gamma} [d_2 u^\beta, d_1 u^\gamma]$$
$$\overset{2}{R}{}_\alpha{}^\mu{}_{\beta\gamma} = \bar{R}_\alpha{}^\mu{}_{\beta\gamma} + \bar{\Theta}_\alpha{}^\mu{}_{[\beta} \bar{\Theta}_{|\mu|}{}^\varepsilon{}_{\gamma]}$$

$$\overset{2}{R}_{\alpha}{}^{\mu}{}_{\beta\gamma} = \bar{R}_{\alpha}{}^{\mu}{}_{\beta\gamma} + \bar{\Theta}_{\alpha[\beta}{}^{\nu}\,\bar{\lambda}_{|\nu|}{}^{\mu}{}_{\gamma]}$$

$$\overset{2}{R}_{\mu}{}^{\nu}{}_{\beta\gamma} = \bar{R}_{\mu}{}^{\nu}{}_{\beta\gamma} + \bar{\lambda}_{\mu[\beta}{}^{\xi}\,\bar{\lambda}_{|\xi|\gamma]}{}^{\nu}$$

$$\bar{R}_{\alpha}{}^{\varepsilon}{}_{\beta\gamma} = \partial_{[\gamma}T_{|\alpha|\beta]}{}^{\varepsilon} + \bar{T}_{\alpha[\beta}{}^{\kappa}\,\bar{T}_{|\kappa|}{}^{\varepsilon}{}_{\gamma]}$$

$$\bar{R}_{\alpha}{}^{\mu}{}_{\beta\gamma} = \partial_{[\gamma}\,\bar{\Theta}_{|\alpha|\beta]}{}^{\mu} + \bar{\Gamma}_{\alpha[\beta}{}^{\kappa}\,\bar{\Theta}_{|\kappa|\gamma]}{}^{\mu}$$

$$\bar{R}_{\mu}{}^{\nu}{}_{\beta\gamma} = \partial_{[\gamma}\,\bar{\lambda}_{|\mu|\beta]}{}^{\nu} + \bar{\Theta}_{\mu[\beta}{}^{\kappa}\,\bar{\Theta}_{|\kappa|}{}^{\nu}{}_{\gamma]}$$

$$\bar{\Gamma}_{\alpha\gamma\beta} = g_{ir}\,B^{r}_{\gamma}(B^{i}_{\alpha\beta} + \Gamma_{j}{}^{i}{}_{k}\,B^{jk}_{\alpha\beta})$$

$$\bar{\Theta}_{\alpha\nu\beta} = g_{ir}N^{r}_{\nu}(B^{i}_{\alpha\beta} + \Gamma_{j}{}^{i}{}_{k}B^{jk}_{\alpha\beta})$$

$$\bar{\lambda}_{\mu}{}^{\psi}{}_{\gamma} = \overset{\psi}{N}_{i}\,(\partial_{\gamma}N^{i}_{\mu} + \Gamma_{j}{}^{i}{}_{k}N^{j}_{\mu}B^{k}_{\gamma})$$

Multiplying (3.1) and (3.2) with B^{κ}_{i} and $\overset{\rho}{N}_{i}$ we obtain

$$(3.3)\quad ([D_1D_2]R_{j}{}^{i}{}_{hk})B^{j\kappa}_{\alpha i}B^{hk}_{\beta\gamma} + R_{j}{}^{i}{}_{hk}(\bar{\Omega}_{\alpha}{}^{\delta}B^{j}_{\delta} + \bar{\Omega}_{\alpha}{}^{\nu}N^{j}_{\nu})B^{hk}_{\beta\gamma}\,B^{\kappa}_{i} +$$
$$+ R_{j}{}^{i}{}_{hk}B^{j\kappa}_{\alpha i}\,(\bar{\Omega}_{\beta}{}^{\delta}B^{h}_{\delta}+\bar{\Omega}_{\beta}{}^{\nu}N^{h}_{\nu})B^{k}_{\gamma} + R_{j}{}^{i}{}_{hk}B^{j\kappa}_{\alpha i}B^{h}_{\beta}(\bar{\Omega}_{\gamma}{}^{\delta}B^{k}_{\delta} + \bar{\Omega}_{\gamma}{}^{\nu}\,N^{k}_{\nu}) =$$
$$= \overset{2}{R}_{\alpha}{}^{\varepsilon}{}_{\beta\gamma}\,\bar{\Omega}_{\varepsilon}{}^{\kappa} + \overset{2}{R}_{\alpha}{}^{\mu}{}_{\beta\gamma}\,\bar{\Omega}_{\mu}{}^{\kappa}$$

$$(3.4)\quad ([D_1D_2]R_{j}{}^{i}{}_{hk})B^{j}_{\alpha}\,\overset{\rho}{N}_{i}\,B^{hk}_{\beta\gamma} + R_{j}{}^{i}{}_{hk}(\bar{\Omega}_{\alpha}{}^{\delta}B^{j}_{\delta} + \bar{\Omega}_{\alpha}{}^{\nu}N^{j}_{\nu})\overset{\rho}{N}_{i}B^{hk}_{\beta\gamma} +$$
$$+ R_{j}{}^{i}{}_{hk}\,B^{j}_{\alpha}\overset{\rho}{N}_{i}(\bar{\Omega}_{\alpha}{}^{\delta}B^{h}_{\delta} + \bar{\Omega}_{\beta}{}^{\nu}N^{h}_{\nu})B^{k}_{\gamma} + R_{j}{}^{i}{}_{hk}B^{j}_{\alpha}\overset{\rho}{N}_{i}B^{h}_{\beta}(\bar{\Omega}_{\gamma}{}^{\delta}B^{k}_{\delta} + \bar{\Omega}_{\gamma}{}^{\nu}N^{k}_{\nu}) =$$
$$= \overset{2}{R}_{\alpha}{}^{\varepsilon}{}_{\beta\gamma}\,\bar{\Omega}_{\varepsilon}{}^{\rho} + \overset{2}{R}_{\alpha}{}^{\mu}{}_{\beta\gamma}\,\bar{\Omega}_{\mu}{}^{\rho}.$$

$$(3.5)\quad ([D_1D_2]R_{j}{}^{i}{}_{hk})N^{j}_{\mu}B^{\kappa}_{i}B^{hk}_{\beta\gamma} + R_{j}{}^{i}{}_{hk}(\bar{\Omega}_{\mu}{}^{\delta}B^{j}_{\delta} + \bar{\Omega}_{\mu}{}^{\nu}N^{j}_{\nu})B^{\kappa}_{i}B^{hk}_{\beta\gamma} +$$
$$+ R_{j}{}^{i}{}_{hk}N^{i}_{\mu}B^{\kappa}_{i}(\bar{\Omega}_{\beta}{}^{\delta}B^{h}_{\delta} + \bar{\Omega}_{\beta}{}^{\nu}N^{h}_{\nu})B^{k}_{\gamma} + R_{j}{}^{i}{}_{hk}N^{j}_{\mu}B^{\kappa}_{i}B^{h}_{\beta}(\bar{\Omega}_{\gamma}{}^{\delta}B^{k}_{\delta} + \bar{\Omega}_{\gamma}{}^{\nu}N^{k}_{\nu}) =$$
$$= \overset{2}{R}_{\mu}{}^{\varepsilon}{}_{\beta\gamma}\,\bar{\Omega}_{\varepsilon}{}^{\kappa} + \overset{2}{R}_{\mu}{}^{\nu}{}_{\beta\gamma}\,\bar{\Omega}_{\nu}{}^{\kappa}\ .$$

$$(3.6)\quad ([D_1D_2]R_{j}{}^{i}{}_{hk})N^{j}_{\mu}\overset{\rho}{N}_{i}B^{hk}_{\beta\gamma} + R_{j}{}^{i}{}_{hk}(\bar{\Omega}_{\mu}{}^{\delta}B^{j}_{\delta} + \bar{\Omega}_{\mu}{}^{\nu}N^{j}_{\nu})\overset{\rho}{N}_{i}B^{hk}_{\beta\gamma} +$$
$$+ R_{j}{}^{i}{}_{hk}N^{j}_{\mu}\overset{\rho}{N}_{i}(\bar{\Omega}_{\beta}{}^{\delta}B^{k}_{\delta} + \bar{\Omega}_{\beta}{}^{\nu}N^{h}_{\nu})B^{k}_{\gamma} + R_{j}{}^{i}{}_{hk}N^{j}_{\mu}\overset{\rho}{N}_{i}B^{h}_{\beta}(\bar{\Omega}_{\gamma}{}^{\delta}B^{k}_{\delta} + \bar{\Omega}_{\gamma}{}^{\nu}N^{k}_{\nu}) =$$
$$= \overset{2}{R}_{\mu}{}^{\varepsilon}{}_{\beta\gamma}\,\bar{\Omega}_{\varepsilon}{}^{\rho} + \overset{2}{R}_{\mu}{}^{\nu}{}_{\beta\gamma}\,\bar{\Omega}_{\nu}{}^{\rho}$$

Let us define the tensors

$$\overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\beta\gamma} = R_j{}^i{}_{hk} B^j_\alpha B^\kappa_i B^{hk}_{\beta\gamma} \qquad \overset{2}{\bar{R}}_{\alpha}{}^{\mu}{}_{\beta\gamma} = R_j{}^i{}_{hk} B^j_\alpha \overset{\mu}{N}_i B^{hk}_{\beta\gamma}$$

$$\overset{2}{\bar{R}}_{\mu}{}^{\kappa}{}_{\beta\gamma} = R_j{}^i{}_{hk} \underset{\mu}{N}{}^j B^\kappa_i B^{hk}_{\beta\gamma} \qquad \overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\mu\gamma} = R_j{}^i{}_{hk} B^j_\alpha B^\kappa_i \underset{\mu}{N}{}^k B^k_\gamma$$

(3.7)
$$\overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\beta\mu} = R_j{}^i{}_{hk} B^j_\alpha B^\kappa_i B^h_\beta \underset{\mu}{N}{}^k \qquad \overset{2}{\bar{R}}_{\nu}{}^{\mu}{}_{\beta\gamma} = R_j{}^i{}_{hk} \underset{\nu}{N}{}^j \overset{\mu}{N}_i B^{hk}_{\beta\gamma}$$

$$\overset{2}{\bar{R}}_{\alpha}{}^{\mu}{}_{\nu\gamma} = R_j{}^i{}_{hk} B^j_\alpha \overset{\mu}{N}_i \underset{\nu}{N}{}^h B^k_\gamma \qquad \overset{2}{\bar{R}}_{\alpha}{}^{\mu}{}_{\beta\nu} = R_j{}^i{}_{hk} B^j_\alpha \overset{\mu}{N}_i B^h_\beta \underset{\nu}{N}{}^k$$

$$\overset{2}{\bar{R}}_{\mu}{}^{\nu}{}_{\psi\gamma} = R_j{}^i{}_{hk} \underset{\mu}{N}{}^j \overset{\nu}{N}_i \underset{\psi}{N}{}^h B^k_\gamma \qquad \overset{2}{\bar{R}}_{\mu}{}^{\nu}{}_{\beta\psi} = R_j{}^i{}_{hk} \underset{\mu}{N}{}^j \overset{\nu}{N}_i B^h_\beta \underset{\psi}{N}{}^k$$

Some of these tensors appear in (1.5) and (1.6) and for the Riemannian space and subspace are the same as those (above) defined.

Now (3.3) - (3.6) have the from

(3.8)
$$([D_1 D_2] R_j{}^i{}_{hk}) B^{j\kappa}_{\alpha i} B^{hk}_{\beta\gamma} =$$
$$= \overset{2}{\bar{R}}_{\alpha}{}^{\delta}{}_{\beta\gamma} \bar{\Omega}_\delta{}^\kappa - \overset{2}{\bar{R}}_{\delta}{}^{\kappa}{}_{\beta\gamma} \bar{\Omega}_\alpha{}^\gamma - \overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\delta\gamma} \bar{\Omega}_\beta{}^\delta - \overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\beta\delta} \bar{\Omega}_\gamma{}^\delta +$$
$$+ \overset{2}{\bar{R}}_{\alpha}{}^{\mu}{}_{\beta\gamma} \bar{\Omega}_\mu{}^\kappa - \overset{2}{\bar{R}}_{\mu}{}^{\kappa}{}_{\beta\gamma} \bar{\Omega}_\alpha{}^\mu - \overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\mu\gamma} \bar{\Omega}_\beta{}^\mu - \overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\beta\mu} \bar{\Omega}_\gamma{}^\mu \stackrel{\text{def}}{=} [\bar{D}_1 \bar{D}_2] \overset{2}{\bar{R}}_{\alpha}{}^{\kappa}{}_{\beta\gamma}$$

(3.9)
$$([D_1 D_2] R_j{}^i{}_{hk}) B^j_\alpha \overset{\nu}{N}_i B^{hk}_{\beta\gamma} =$$
$$= \overset{2}{\bar{R}}_{\alpha}{}^{\delta}{}_{\beta\gamma} \bar{\Omega}_\delta{}^\nu - \overset{2}{\bar{R}}_{\delta}{}^{\nu}{}_{\beta\gamma} \bar{\Omega}_\alpha{}^\delta - \overset{2}{\bar{R}}_{\alpha}{}^{\beta}{}_{\delta\gamma} \bar{\Omega}_\beta{}^\delta - \overset{2}{\bar{R}}_{\alpha}{}^{\nu}{}_{\beta\delta} \bar{\Omega}_\gamma{}^\delta +$$
$$+ \overset{2}{\bar{R}}_{\alpha}{}^{\mu}{}_{\beta\gamma} \bar{\Omega}_\mu{}^\nu - \overset{2}{\bar{R}}_{\mu}{}^{\rho}{}_{\beta\gamma} \bar{\Omega}_\alpha{}^\mu - \overset{2}{\bar{R}}_{\alpha}{}^{\nu}{}_{\beta\mu} \bar{\Omega}_\gamma{}^\mu - \overset{2}{\bar{R}}_{\alpha}{}^{\nu}{}_{\beta\mu} \bar{\Omega}_\gamma{}^\mu \stackrel{\text{def}}{=} [\bar{D}_1 \bar{D}_2] \overset{2}{\bar{R}}_{\alpha}{}^{\nu}{}_{\beta\gamma}$$

(3.10)
$$([D_1 D_2] R_j{}^i{}_{hk}) \underset{\mu}{N}{}^j B^\kappa_i B^{hk}_{\beta\gamma} =$$
$$= \overset{2}{\bar{R}}_{\mu}{}^{\delta}{}_{\beta\gamma} \bar{\Omega}_\delta{}^\kappa - \overset{2}{\bar{R}}_{\delta}{}^{\kappa}{}_{\beta\gamma} \bar{\Omega}_\mu{}^\delta - \overset{2}{\bar{R}}_{\mu}{}^{\kappa}{}_{\delta\gamma} \bar{\Omega}_\beta{}^\delta - \overset{2}{\bar{R}}_{\mu}{}^{\kappa}{}_{\beta\delta} \bar{\Omega}_\gamma{}^\delta +$$
$$+ \overset{2}{\bar{R}}_{\mu}{}^{\nu}{}_{\beta\gamma} \bar{\Omega}_\nu{}^\kappa - \overset{2}{\bar{R}}_{\nu}{}^{\kappa}{}_{\beta\gamma} \bar{\Omega}_\mu{}^\nu - \overset{2}{\bar{R}}_{\mu}{}^{\kappa}{}_{\nu\gamma} \bar{\Omega}_\beta{}^\nu - \overset{2}{\bar{R}}_{\mu}{}^{\kappa}{}_{\beta\nu} \bar{\Omega}_\gamma{}^\nu \stackrel{\text{def}}{=} [\bar{D}_1 \bar{D}_2] \overset{2}{\bar{R}}_{\mu}{}^{\kappa}{}_{\beta\gamma} .$$

$$(3.11)\quad ([D_1 D_2] R_j{}^i{}_{hk} N^j_\mu \overset{\nu}{N}_i B^{hk}_{\beta\gamma} = $$
$$= \overset{2}{\bar{R}}_\mu{}^\delta{}_{\beta\gamma} \bar{\Omega}_\delta{}^\nu - \overset{2}{\bar{R}}_\delta{}^\nu{}_{\beta\gamma} \bar{\Omega}_\mu{}^\delta - \overset{2}{\bar{R}}_\mu{}^\nu{}_{\delta\gamma} \bar{\Omega}_\beta{}^\delta - \overset{2}{\bar{R}}_\mu{}^\nu{}_{\beta\delta} \bar{\Omega}_\gamma{}^\delta +$$
$$+ \overset{2}{\bar{R}}_\mu{}^\Psi{}_{\beta\gamma} \bar{\Omega}_\Psi{}^\nu - \overset{2}{\bar{R}}_\Psi{}^\nu{}_{\beta\gamma} \bar{\Omega}_\mu{}^\Psi - \overset{2}{\bar{R}}_\mu{}^\nu{}_{\Psi\gamma} \bar{\Omega}_\beta{}^\Psi - \overset{2}{\bar{R}}_\mu{}^\nu{}_{\beta\Psi} \bar{\Omega}_\gamma{}^\Psi \underset{=}{\underline{\mathrm{def}}}$$
$$[\bar{D}_1 \bar{D}_2] \overset{2}{\bar{R}}_\mu{}^\nu{}_{\beta\gamma} .$$

In the Riemannian space the double alternated differentials of the curvature tensor of the space and its subspace are connected by (3.8)-(3.11).

4. RECURRENT RIEMANNIAN SPACE OF THE SECOND ORDER

If the surrounding Riemannian space has the property

$$(4.1)\quad R_j{}^i{}_{hk|p|q} = a_{pq} R_j{}^i{}_{hk}$$

then from

$$[D_1 D_2] R_j{}^i{}_{hk} = R_j{}^i{}_{hk[|p|q]} [d_2 x^p, d_1 x^q]$$

it follows that

$$(4.2)\quad [D_1 D_2] R_j{}^i{}_{hk} = \tfrac{1}{2}(a_{pq} - a_{qp}) R_j{}^i{}_{hk} [d_2 x^p, d_1 x^q] =$$
$$= b_{pq} R_j{}^i{}_{hk} [d_2 x^p, d_1 x^q]$$

where b_{pq} is the second order antisymmetric covariant tensor

$$b_{pq} = \tfrac{1}{2} (a_{pq} - a_{qp})$$

In this case (3.8) becomes

$$(4.3)\quad [\bar{D}_1 \bar{D}_2] \overset{2}{\bar{R}}_\alpha{}^\kappa{}_{\beta\gamma} = b_{pq} [d_2 x^p, d_1 x^q] R_j{}^i{}_{hk} B^{j}_{\alpha}{}^{\kappa}_{\iota}{}^{hk}_{\beta\gamma}$$

Denoting

$$K = b_{pq} [d_2 x^p, d_1 x^q]$$

we have for (4.3)

$$(4.4)\qquad [\bar{D}_1\bar{D}_2]\ \overset{2}{\bar{R}}{}_{\alpha}{}^{\kappa}{}_{\beta\gamma} = K R_j{}^i{}_{hk}\ B_{\alpha}^{j}{}^{\kappa}{}_{\beta}^{h}{}_{\gamma}^{k} = K \overset{2}{\bar{R}}{}_{\alpha}{}^{\kappa}{}_{\beta\gamma}$$

In a similar manner (3.9), (3.10) and (3.11) become in this case

$$(4.5)\qquad [\bar{D}_1\bar{D}_2]\ \overset{2}{\bar{R}}{}_{\alpha}{}^{\nu}{}_{\beta\gamma} = K \overset{2}{\bar{R}}{}_{\alpha}{}^{\nu}{}_{\beta\gamma}$$

$$(4.6)\qquad [\bar{D}_1\bar{D}_2]\ \overset{2}{\bar{R}}{}_{\mu}{}^{\kappa}{}_{\beta\gamma} = K \overset{2}{\bar{R}}{}_{\mu}{}^{\kappa}{}_{\beta\gamma}$$

$$(4.7)\qquad [\bar{D}_1\bar{D}_2]\ \overset{2}{\bar{R}}{}_{\mu}{}^{\nu}{}_{\beta\gamma} = K \overset{2}{\bar{R}}{}_{\mu}{}^{\nu}{}_{\beta\gamma}$$

From the above we can conclude.

If in the Riemannian space the curvature tensor R has the property (4.1) then the curvature tensor $\overset{2}{\bar{R}}{}_{\alpha}{}^{\kappa}{}_{\beta\gamma}$, $\overset{2}{\bar{R}}{}_{\alpha}{}^{\nu}{}_{\beta\gamma}$, $\overset{2}{\bar{R}}{}_{\mu}{}_{\beta\gamma}$, $\overset{2}{\bar{R}}{}_{\mu}{}^{\nu}{}_{\beta\gamma}$ defined by (3.7) have the property (4.4)-(4.7), where the left hand side of these formulas are defined by (3.8)-(3.11).

The formulas (4.4)-(4.7) are valid if instead of condition (4.1) we have the weaker condition

$$(4.8)\qquad [D_1D_2]R_j{}^i{}_{hk} = K R_j{}^i{}_{hk}$$

From (4.1) follows (4.8) for every motion d_1x^p, d_2x^q, but from (4.8) only (4.2) follows.

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REZIME

VEZA IZMEDJU DUPLOG ALTERNIRANOG APSOLUTNOG DIFERENCIJALA TENZORA KRIVINE FINSLEROVOG PROSTORA I INDUKOVANIH KRIVINA PODPROSTORA

U uvodu su date formule (1.15)-(1.20) koje povezuju tenzore krivi na prostoru F_n i potprostora F_m. U 2. su date veze izmedju duplog alterniranog apsolutnog diferencijala ovih tenzora krivina. Te formule se uprošćuju za Riemannov prostor, što je odredjeno u 3. U 4. je ispitivan 2 - rekurentni Riemannov prostor i njegov potprostor. Tada važe formule (4.4) - (4.7).