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CONNEXIONS IN f-MANIFOLDS

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1. K.Yano |6| introduced the notion of an f-structure on a C^∞ manifold N as a tensor field f of type (1,1) and rank 2m $(2m \leq n = \dim N)$ satisfying $f^3 + f = 0$, the existence of which is equivalent to a reduction of the structural group of the tangent bundle to the group $Gl(m,C) \times Gl(n-2m,R)$. A manifold N with an f-structure is called an f-manifold. Almost complex $(2m = n)$ and almost contact $(2m = n-1)$ structures are well-known examples of f-structures.

Let N be an n-dimensional manifold with an f-structure of rank 2m. If there exists on N vector fields ξ_α, $\alpha = 1,\dots,n-2m$, such that if η_α are dual 1-forms, then

(1) $$\eta_\alpha(\xi_\beta) = \delta_{\alpha\beta}$$

(2) $$f^2 = -I + \Sigma_\alpha \, \eta_\alpha \otimes \xi_\alpha$$

we say that the f-structure has complemented frames. As an immediate consequence of (1) and (2), we obtain

$$f(\xi_\alpha) = 0$$

$$\eta_\alpha \circ f = 0$$

On a manifold N the existence of an f-structure with complemented frames is equivalent to a reduction of the structural group of the tangent bundle to the group $Gl(m,C) \times E_{n-2m}$, where E_{n-2m} denotes (n-2m) x (n-2m) unit matrix (see |4|).

The purpose of this paper is to consider affine connexions in f-manifolds.

2. Let N be an n-dimensional manifold of class C^{∞} with an f-structure of rank 2m. Then the structural group of the tangent bundle can be reduced to the group $Gl(m,C) \times Gl(n-2m,R)$, and conversely.

Let $(fB(N), Gl(m,C) \times Gl(n-2m,R), N)$ be the principal bundle of adapted frames of f-structure on N, i.e. the principal bundle of frames with respect to which f has components

$$(3) \qquad f = \begin{bmatrix} 0 & E_m & 0 \\ -E_m & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where E_m denotes m x m unit matrix. With respect to every connexion on the bundle fB(N) the tensor field f is parallel. Indeed, let $\gamma : [0,1] \to N$ be a curve of class C^{∞} on the manifold N, and let $\tilde{\gamma}$ be a horizontal lift of γ in the bundle fB(N) with respect to the given connexion. If we denote $\tilde{\gamma}(0) = (\gamma(0), u_1(0), \ldots, u_n(0))$ and $\tilde{\gamma}(t) = (\gamma(t), u_1(t), \ldots, u_n(t))$, where $(u_1(0), \ldots, u_n(0))$ and $(u_1(t), \ldots, u_n(t))$ are any frames from fB(N) at points $\gamma(0)$ and $\gamma(t)$ respectively, then it is obvious that components of f with respect to frames $(u_1(0), \ldots, u_n(0))$ and $(u_1(t), \ldots, u_n(t))$ are equal and given by (3), which means that tensor field f is parallel along the curve γ.

If H' is a connexion on fB(N), then it can be extended to a connexion H on B(N) by the right action of the group $Gl(n,R)$ (B(N) is bundle of basses of N). Hence, H gives rise to a parallel translation along curves in N. Further, it is clear that with respect to this parallel translation, the tensor field f is parallel. Indeed, let we have parallel translation along a curve γ. We may choose a horizontal lift $\tilde{\gamma}$ of γ so that $\tilde{\gamma}(0) \in fB(N)$. Then we have $\tilde{\gamma} \subset fB(N)$, which follows from the definition of the connexion H.

Conversely, if H is a connexion on B(N), such that the tensor field f is parallel with respect to H, then H comes from a connexion H´ on fB(N) in the above manner. Indeed, let $b=(m,u_1,\dots,u_m)\in fB(N)$ and let $\tilde{\gamma}$ be a horizontal curve in B(N) passing through b. Then every point on $\tilde{\gamma}$ must belong to fB(N), since f is parallel along the curve $\gamma=\pi\circ\tilde{\gamma}$ (π is the projection $B(N)\to N$), by assumption. Therefore $H_b\subset T_b(fB(N))$, so we may define H´ by $H'_b=H_b$.

Thus we have

THEOREM 1. *For an affine connexion* H *on a manifold* N *with* f-*structure, the following conditions are equivalent:*

(a) H *is the extension of a connexion of* fB(N)

(b) The tensor field f *is parallel with respect to* H .

An affine connexion on N is said to be an f-connexion, if it satisfies any one (and hence both) of the conditions above.

From the general theory of connexions (see |1|) we know thet every principal bundle (P, G, N), with N paracompact, admits a connexion. This means that every paracompact manifold N with an f-structure, admits an f-connecion.

Let the f-structure on N have complemented frames. Then the structural group of the tangent bundle can be reduced to the group $Gl(m,C)\times E_{n-2m}$, and conversely.

Let $((f,\xi_\alpha,\eta_\alpha)B(N),\ Gl(m,C)\times E_{n-2m},N)$ be the principal bundle of adapted frames of f-structure with complemented frames on N, i.e. the principal bundle of frames with respect to which f has components (3), while η_α and ξ_α have components

$$\eta_\alpha = (\underbrace{0,\dots,0}_{2m},\underbrace{0,\dots 1}_{\alpha},\dots 0,) \quad \xi_\alpha = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} 2m \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 1\end{matrix}}\right\} \alpha \\ \vphantom{\begin{matrix}\vdots\\ 0\end{matrix}} \end{matrix}$$

It is obvious, that the tensor field f, vector fields ξ_α and 1-forms η_α are parallel with respect to every connexion in the bundle $(f,\xi_\alpha,\eta_\alpha)B(N)$.

The proof of the following theorem is analogous to that of Theorem 1.

THEOREM 2. *For an affine connexion* H *on a manifold* N *with* f*-structure with complemented frames, the following conditions are equivalent:*

a) H *is the extension of a connexion of* $(f,\xi_\alpha,\eta_\alpha)B(N)$

b) *The tensor field* f*, vector fields* ξ_α *and* 1*-forms* η_α *are parallel with respect to* H.

An affine connexion on N *is said to be an* $(f,\xi_\alpha,\eta_\alpha)$ *-conexion, if it satisfies any one (and hence both) of the conditions above.*

As a consequence of the above theorem, we have that every paracompact manifold N with an f-structure with complemented frames, admits an $(f,\xi_\alpha,\eta_\alpha)$ -connexion.

We shall now prove the existence of a connexion of a more special type.

THEOREM 3. *Every manifold* N *with an* f*-structure with complemented frames admits an* $(f,\xi_\alpha,\eta_\alpha)$ *-connexion such that its torsion* T *is given by*

$$4T(X,Y) = 4\,\Sigma_\alpha\, d\eta_\alpha(X,Y)\xi_\alpha - \Sigma_\alpha \eta_\alpha([f,f](X,Y))\xi_\alpha + \\ + [f,f](X + \Sigma_\alpha\eta_\alpha(X)\xi_\alpha, Y + \Sigma_\beta\eta_\beta(Y)\xi_\beta) \;,$$

where $[f,f]$ *is the Nijenhuis torsion of* f, *and* $X,Y \in \mathfrak{X}(N)$ ($\mathfrak{X}(N)$ *is the set of all vector fields of class* C^∞ *on* N).

P r o o f. Consider an arbitrary symmetric affine connexion on N (if N is paracompact, such a connexion exists) with covariant differentiation B. Let ∇ be the covariant differention with respect to a desired $(f,\xi_\alpha,\eta_\alpha)$ -connexion, and let $X,Y \in \mathfrak{X}(N)$ Then

$$\nabla_X\xi_\alpha = 0, \quad \text{for any } \alpha$$

$$\nabla_X\eta_\alpha = 0, \quad \text{for any } \alpha$$

(4) $$\nabla_X f = 0.$$

From (4) we obtain

(5) $$\nabla_X(fY) = f\nabla_X Y .$$

It may be written

(6) $$\nabla_X Y = B_X Y - H(X,Y)$$

where H is a tensor field of type (1,2). Now we have

(7) $$H(X,\xi_\alpha) = B_X\xi_\alpha$$

(8) $$\eta_\alpha(H(X,Y)) = \eta_\alpha(B_X Y) - X\eta_\alpha(Y).$$

From (5) and (6) we obtain

$$H(X,fY) - fH(X,Y) = B_X(fY) - fB_X Y .$$

After applying f to this equation and to Y, we obtain

$$fH(X,f^2Y) - f^2H(X,fY) = fB_X(f^2Y) - f^2B_X(fY).$$

Since by applying f to this equation and to Y again we have the same equation, the general solution of this equation is given by

$$4fH(X,f^2Y) = 2fB_X(f^2Y) - 2f^2B_X(fY) - fW(X,fY) + f^2W(X,f^2Y) ,$$

where W is a tensor field of type (1,2). From this equation after applying f, and from (2),(7) and (8), we obtain

$$4H(X,Y) = 4\Sigma_\alpha\ (\eta_\alpha(B_X Y) - X\eta_\alpha(Y))\xi_\alpha - 4\ \Sigma_\alpha \eta_\alpha(Y) f^2 B_X \xi_\alpha + $$
$$+\ 2f^2(B_X f)(fY) - f^2 W(X,fY) - fW(X,f^2 Y).$$

Let W be defined by

$$W(X,Y) = (B_Y f)X - \Sigma_\alpha \eta_\alpha(X) B_{fY}\xi_\alpha .$$

Then we have

$$\begin{aligned}
4T(X,Y) &= 4\nabla_X Y - 4\nabla_Y X - 4[X,Y] \\
&= 4B_X Y - 4B_Y X - 4[X,Y] - 4H(X,Y) + 4H(Y,X) \\
&= -4\Sigma_\alpha(\eta_\alpha(B_X Y) - X\eta_\alpha(Y))\xi_\alpha + 4\Sigma_\alpha \eta_\alpha(Y) f^2 B_X \xi_\alpha - \\
&- 2f^2(B_X f)(fY) + f^2(B_{fY} f)X - \Sigma_\alpha \eta_\alpha(X) f^2 B_{f^2Y}\xi_\alpha + \\
&+ f(B_{f^2Y} f)X + \Sigma_\alpha \eta_\alpha(X) fB_{fY}\xi_\alpha + \\
&+ 4\Sigma_\alpha(\eta_\alpha(B_Y X) - Y\eta_\alpha(X))\xi_\alpha - 4\Sigma_\alpha \eta_\alpha(X) f^2 B_Y \xi_\alpha + \\
&+ 2f^2(B_Y f)(fX) - f^2(B_{fX} f)Y + \Sigma_\alpha \eta_\alpha(Y) f^2 B_{f^2X}\xi_\alpha - \\
&- f(B_{f^2X} f)Y - \Sigma_\alpha \eta_\alpha(Y) fB_{fX}\xi_\alpha \\
&= 4\Sigma_\alpha d\eta_\alpha(X,Y)\xi_\alpha + 4\Sigma_\alpha \eta_\alpha(Y) f^2 B_X \xi_\alpha - \\
&- 4\Sigma_\alpha \eta_\alpha(X) f^2 B_Y \xi_\alpha - 2f^2 B_X(f^2 Y) - \\
&- 2fB_X(fY) + 2f^2 B_Y(f^2 X) + 2fB_Y(fX) + \\
&+ f^2 B_{fY}(fX) + fB_{fY}X + fB_{f^2Y}(fX) - \\
&- f^2 B_{f^2Y}X - f^2 B_{fX}(fY) - fB_{fX}Y - fB_{f^2X}(fY) + \\
&+ f^2 B_{f^2X}Y + \Sigma_\alpha \eta_\alpha(X) f^2 B_Y \xi_\alpha - \\
&- \Sigma_\alpha \eta_\alpha(X) f^2 B_{\Sigma_\beta \eta_\beta(Y)\xi_\beta}\xi_\alpha + \Sigma_\alpha \eta_\alpha(X) fB_{fY}\xi_\alpha - \\
&- \Sigma_\alpha \eta_\alpha(Y) f^2 B_X \xi_\alpha + \Sigma_\alpha \eta_\alpha(Y) f^2 B_{\Sigma_\beta \eta_\beta(X)\xi_\beta}\xi_\alpha - \\
&- \Sigma_\alpha \eta_\alpha(Y) fB_{fX}\xi_\alpha .
\end{aligned}$$

Since it may be written

$$-2f^2B_X(f^2Y) = 2f^2B_XY - 2f^2B_X(\Sigma_\alpha\eta_\alpha(Y)\xi_\alpha)$$

$$= 2f^2B_XY - 2\Sigma_\alpha\eta_\alpha(Y)f^2B_X\xi_\alpha$$

$$2f^2B_Y(f^2X) = -2f^2B_YX + 2\Sigma_\alpha\eta_\alpha(X)f^2B_Y\xi_\alpha,$$

we have

$$\begin{aligned}
4T(X,Y) &= 4\Sigma_\alpha d\eta_\alpha(X,Y)\xi_\alpha + \Sigma_\alpha\eta_\alpha(Y)f^2B_X\xi_\alpha - \Sigma_\alpha\eta_\alpha(X)f^2B_Y\xi_\alpha + \\
&+ 2f^2B_XY - 2f^2B_YX - 2f_BX(fY) + 2fB_Y(fX) - f^2[fX,fY] + \\
&+ fB_{fY}X - fB_{fX}Y - fB_Y(fX) + \Sigma_\alpha\eta_\alpha(Y)fB_{\xi_\alpha}(fX) + \\
&+ f^2B_YX - \Sigma_\alpha\eta_\alpha(Y)f^2B_{\xi_\alpha}X + fB_X(fY) - \\
&- \Sigma_\alpha\eta_\alpha(X)fB_{\xi_\alpha}(fY) - f^2B_XY + \Sigma_\alpha\eta_\alpha(X)f^2B_{\xi_\alpha}Y + \\
&+ \Sigma_\alpha\Sigma_\beta\eta_\alpha(X)\eta_\beta(Y)f^2(B_{\xi_\alpha}\xi_\beta - B_{\xi_\beta}\xi_\alpha) + \\
&+ \Sigma_\alpha\eta_\alpha(X)fB_{fY}\xi_\alpha - \Sigma_\alpha\eta_\alpha(Y)fB_{fX}\xi_\alpha \\
&= 4\Sigma_\alpha d\eta_\alpha(X,Y)\xi_\alpha + \Sigma_\beta\eta_\beta(Y)f^2[X,\xi_\beta] + \\
&+ \Sigma_\alpha\eta_\alpha(X)f^2[\xi_\alpha,Y] - \Sigma_\beta\eta_\beta(Y)f[fX,\xi_\beta] - \\
&- \Sigma_\alpha\eta_\alpha(X)f[\xi_\alpha,fY] + \Sigma_\alpha\Sigma_\beta\eta_\alpha(X)\eta_\beta(Y)f^2[\xi_\alpha,\xi_\beta] - \\
&- f^2[fX,fY] + f^2[X,Y] - f[X,fY] - f[fX,Y] = \\
&= 4\Sigma_\alpha d\eta_\alpha(X,Y)\xi_\alpha - f^2([fX,fY] - f[X,fY] - f[fX,Y] + \\
&+ f^2[X,Y]) + \Sigma_\beta\eta_\beta(Y)[f,f](X,\xi_\beta) + \\
&+ \Sigma_\alpha\eta_\alpha(X)[f,f](\xi_\alpha,Y) + \Sigma_\alpha\Sigma_\beta\eta_\alpha(X)\eta_\beta(Y)[f,f](\xi_\alpha,\xi_\beta) \\
&= 4\Sigma_\alpha d\eta_\alpha(X,Y)\xi_\alpha - f^2[f,f](X,Y) + \\
&+ [f,f](X,\Sigma_\beta\eta_\beta(Y)\xi_\beta) + [f,f](\Sigma_\alpha\eta_\alpha(X)\xi_\alpha,Y) + \\
&+ [f,f](\Sigma_\alpha\eta_\alpha(X)\xi_\alpha,\Sigma_\beta\eta_\beta(Y)\xi_\beta) = \\
&= 4\Sigma_\alpha d\eta_\alpha(X,Y)\xi_\alpha - \Sigma_\alpha\eta_\alpha([f,f](X,Y))\xi_\alpha + \\
&+ [f,f](X,Y + \Sigma_\beta\eta_\beta(Y)\xi_\beta) + [f,f](\Sigma_\alpha\eta_\alpha(X)\xi_\alpha,Y + \\
&+ \Sigma_\beta\eta_\beta(Y)\xi_\beta) = \\
&= 4\Sigma_\alpha d\eta_\alpha(X,Y)\xi_\alpha - \Sigma_\alpha\eta_\alpha([f,f](X,Y))\xi_\alpha + \\
&+ [f,f](X + \Sigma_\alpha\eta_\alpha(X)\xi_\alpha, Y + \Sigma_\beta\eta_\beta(Y)\xi_\beta) .
\end{aligned}$$

Now we can prove the next theorem.

THEOREM 4. *A manifold* N *with an* f*-structure with complemented frames admits a symmetric* $(f,\xi_\alpha,\eta_\alpha)$*-connexion, if and only if*

$$[f,f] = 0 \tag{9}$$

$$[f,f] + \Sigma_\alpha d\eta_\alpha \otimes \xi_\alpha = 0 \tag{10}$$

(If an f*-structure with complemented frames satisfies (10), we say it is normal).*

P r o o f. (9) and (10) imply $d\eta_\alpha = 0$, so that Theorem 4 is a special case of Theorem 3, i.e. the $(f,\xi_\alpha,\eta_\alpha)$ -connexion from Theorem 3 has the torsion $T = 0$.

Assume that N admits a symmetric $(f,\xi_\alpha,\eta_\alpha)$ -connexion and denote its covariant differentiation by ∇. Then, for some $X,Y \in \mathfrak{X}(N)$, we obtain

$$\begin{aligned}[f,f](X,Y) &= [fX,fY] - f[fX,Y] - f[X,fY] + f^2[X,Y] \\ &= \nabla_{fX}(fY) - \nabla_{fY}(fX) - f\nabla_{fX}Y + f\nabla_Y(fX) - \\ &- f\nabla_X(fY) + f\nabla_{fY}X + f^2\nabla_X Y - f^2\nabla_Y X \\ &= (\nabla_{fX}f)Y - (\nabla_{fY}f)X + f(\nabla_Y f)X - f(\nabla_X f)Y = 0\end{aligned}$$

$$d\eta_\alpha(X,Y) = X\eta_\alpha(Y) - Y\eta_\alpha(X) - \eta_\alpha([X,Y])$$

$$\begin{aligned}d\eta_\alpha(\xi_\beta,\xi_\gamma) &= \xi_\beta\eta_\alpha(\xi_\gamma) - \xi_\gamma\eta_\alpha(\xi_\beta) - \eta_\alpha([\xi_\beta,\xi_\gamma]) \\ &= \eta_\alpha(\nabla_{\xi_\beta}\xi_\gamma - \nabla_{\xi_\gamma}\xi_\beta) = 0\end{aligned}$$

$$\begin{aligned}d\eta_\alpha(-f^2X,\xi_\gamma) &= (-f^2X)\,\eta_\alpha(\xi_\gamma) - \xi_\gamma\eta_\alpha(-f^2X) - \xi_\alpha([-f^2X,\xi_\gamma]) \\ &= -\eta_\alpha(\nabla_{-f^2X}\xi_\gamma - \nabla_{\xi_\gamma}(-f^2X)) \\ &= -\eta_\alpha(\nabla_{\xi_\gamma}(f(fX))) \\ &= -\eta_\alpha((\nabla_{\xi_\gamma}f)(fX)) - \eta_\alpha(f(\nabla_{\xi_\gamma}(fX))) = 0\end{aligned}$$

$$d\eta_\alpha(-f^2X,-f^2Y) = (-f^2X)\eta_\alpha(-f^2Y) - (f^2Y)\eta_\alpha(-f^2X) -$$
$$- \eta_\alpha([-f^2X, -f^2Y]) =$$
$$= -\eta_\alpha([f(fX), f(fY)])$$
$$= -\eta_\alpha(f[f^2X,fY] + f[fX,f^2Y] - f^2[fX,fY] +$$
$$+ [f,f](fX,fY)) = 0$$

For any $X \in \mathfrak{X}(N)$ we have $X = -f^2X + \Sigma_\alpha \eta_\alpha(X)\xi_\alpha$, so that we have $d\eta_\alpha(X,Y) = 0$ for any $X,Y \in \mathfrak{X}(N)$.

THEOREM 5. *Let* N *be a manifold with an* f*-structure. Then the torsion* T *and the curvature* R *of an* f*-connexion satisfy the following identities:*

(1)′ $$T(fX,fY) - fT(fX, Y) - fT(X,fY) + f^2T(X,Y) = -[f,f](X,Y),$$

(2)′ $$R(X,Y) \circ f = f \circ R(X,Y),$$

where $X,Y \in \mathfrak{X}(N)$. *If the* f*-structure has complemented frames, then the torsion* T *and the curvature* R *of an* $(f,\xi_\alpha,\eta_\alpha)$*-connexion satisfy some more identities:*

(3)′ $$\eta_\alpha \circ T = d$$

(4)′ $$R(X,Y)\xi_\alpha = 0$$

(5)′ $$\eta_\alpha \circ R(X,Y) = 0$$

where $X,Y \in \mathfrak{X}(N)$.

P r o o f. Let $X,Y,Z \in \mathfrak{X}(N)$. Then we have

(1)′ Follows from $T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y]$

(2)′ Follows from $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$

(3)′ $(\eta_\alpha \circ T)(X,Y) = \eta_\alpha(\nabla_X Y) - \eta_\alpha(\nabla_Y X) - \eta_\alpha([X,Y]) = \nabla_X\eta_\alpha(Y) - \nabla_Y\eta_\alpha(X) - \eta_\alpha([X,Y]) = X\eta_\alpha(Y) - Y\eta_\alpha(X) - \eta_\alpha([X,Y]) = d\eta_\alpha(X,Y)$

$$(4)' \quad R(X,Y)\xi_\alpha = \nabla_X\nabla_Y\xi_\alpha - \nabla_Y\nabla_X\xi_\alpha - \nabla_{[X,Y]}\xi_\alpha = 0$$

$$(5)' \quad (\eta_\alpha \circ R(X,Y))Z = \eta_\alpha(\nabla_X\nabla_Y Z) - \eta_\alpha(\nabla_Y\nabla_X Z) - \eta_\alpha(\nabla_{[X,Y]}Z)$$

$$= \nabla_X\nabla_Y\eta_\alpha(Z) - \nabla_Y\nabla_X\eta_\alpha(Z) - \nabla_{[X,Y]}\eta_\alpha(Z)$$

$$= X(Y\eta_\alpha(Z)) - Y(X\eta_\alpha(Z)) - [X,Y]\eta_\alpha(Z) = 0$$

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REZIME

KONEKSIJE f-MNOGOSTRUKOSTI

U ovom radu se razmatraju afine koneksije na mnogostrukostima sa f-strukturom, kao i na mnogostrukostima sa globalno generisanom f-strukturom. Prva teorema daje potreban i dovoljan uslov da afina koneksija na mnogostrukosti sa f-strukturom bude f-koneksija (tj. u odnosu na koju je tenzorsko polje f paralelno). Druga teorema daje potreban i dovoljan uslov da afina koneksija na mnogostrukosti sa globalno generisanom f-strukturom bude $(f,\xi_\alpha,\eta_\alpha)$-koneksija (tj. koneksija u odnosu na koju su,

pored tenzorskog polja f, paralelna i vektorska polja ξ_α, i polja 1-formi η_α). U trećoj teoremi se pokazuje da svaka mnogostrukost sa globalnom f-strukturom dopušta $(f,\xi_\alpha,\eta_\alpha)$-koneksiju sa tenzorom torzije koji zavisi od same strukture. U četvrtoj teoremi se daje potreban i dovoljan uslov da mnogostrukost sa globalno generisanom f-strukturom dopušta simetričnu $(f,\xi_\alpha,\eta_\alpha)$-koneksiju. U petoj teoremi se daju neke osobine tenzora torzije i tenzora krivine neke f-koneksije na mnogostrukosti, odnosno $(f,\xi_\alpha,\eta_\alpha)$-koneksije na mnogostrukosti sa globalno generisanom f-strukturom.