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ON THE EQUATION $Tx = F(x,Q(Tx))$ IN LOCALLY CONVEX SPACES

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ABSTRACT

In |6| two theorems on the existence of a solution of equations $Tx = F(x,Tx)$ and $Tx = F(x,Q(Tx))$ in locally convex spaces are proved. Using fixed point theorems from |4| and |7| we shall prove in this paper, some generalizations of Rzepecki's results from |6|.

In |1|,|2|,|7| and |3|,|4| some generalizations of the Banach contraction principle in locally convex spaces are proved. Some applications of such a result in the theory of Mikusinski operators are given in |3|.

Here we shall use the following fixed point theorem proved in |7|.

THEOREM A. *Let* $(X,\{d_i\}_{i\in I})$ *be a sequentially complete Hausdorff uniformizable space,* T *be continuous mapping* X *into* X *satisfying the following conditions:*

1) *For each* $i \in I$ *there exist* $f(i) \in I$ *and a nondecreasing function* $q_i : \mathbb{R}^+ = [0,\infty) \to [0,1]$ *so that*

   $$d_i(Tx,Ty) \le q_i(d_{f(i)}(x,y))d_{f(i)}(x,y)$$

   for every $x,y \in X$.
2) *For each* $i \in I$ *and* $t \in \mathbb{R}^+$, $\overline{\lim_{n\to\infty}}\, q_{f^n(i)}(t) < 1$.
3) *There is an element* $x_0 \in X$ *such that:*

   $$\sup_{n\in N} d_{f^n(i)}(x_0,Tx_0) = K_i \in \mathbb{R}, \text{ for every } i \in I.$$

Then there exists a unique $x^* \in X$ *such that* $x^* = Tx^*$ *and for each* $i \in I$:

$$\sup_{n\in N} d_{f^n(i)}(x_o,x^*) = S_i \in \mathbb{R} .$$

In the following Lemma A is an index set and $\{f_a\}_{a\in A}$ is a net of transformations.

LEMMA 1. *Let E be a Hausdorff locally convex topological vector space in which the topology is defined by the family* $\{p_i\}_{i\in I}$ *of seminorms and* $g : I \to I$. *Further let M be a closed sequentially complete subset of E, for every* $a\in A$, $f_a : M\to M$ *so that* $\lim_{a\in A} f_a x$ *exists for every* $x\in M$ *and the following conditions are satisfied:*

a) *For every* $i\in I$ *there exists* $q_i:\mathbb{R}^+\to[0,1]$ *which is a nondecreasing function for which is* $\overline{\lim_{n\to\infty}} q_{g^n(i)}(t) < 1$ *for every* $i\in I$ *and every* $t\in\mathbb{R}^+$ *and for every* $x,y\in M$:

(1) $p_i(f_a x - f_a y) \le q_i(p_{g(i)}(x-y))p_{g(i)}(x-y)$ *for every* $a\in A$.

b) *For every* $i\in I$ *there exists a continuous function* $h_i : E\to\mathbb{R}^+$ *such that* h_i *maps bounded set into bounded set* $h_i(0)=0$ *and* $:p_{g^n(i)}(x) \le h_i(x)$, *for every* $x\in E$ *and every* $n\in N\cup\{0\}$, *and for every* $x\in M$ *is* $\{f_a x | a\in A\}$ *bounded.*
If $f_o x = \lim_{a\in A} f_a x$, $x\in M$, $f_a x_a = x_a$ *for every* $a\in A$, $x_o = f_o x_o$ *then* $\lim_{a\in A} x_a = x_o$.

P r o o f. From (1) it follows that:

$$p_i(f_o x - f_o y) \le q_i(p_{g(i)}(x-y))p_{g(i)}(x-y)$$

for every $x,y\in M$ and every $i\in I$. Applying Theorem A, from b) we conclude that for every $b\in A\cup\{0\}$ there exists one and only one $x_b\in M$ so that $x_b = f_b x_b$. Further in |7| it is proved that for an arbitrary $y_o^a\in M$, $x_a = \lim_{n\to\infty} y_n^a$, where $y_n^a = f_a y_{n-1}^a$ $(n\ge 1)$ and that:

$$p_i(y_{k+1}^a - y_k^a) \le \prod_{s=o}^{k-1} q_{g^s(i)}(p_{g^{s+1}(i)}(y_{k-s}^a - y_{k-s-1}^a))p_{g^k(i)}(y_o^a - y_1^a)$$

for every $a\in A$, $k\in N$ and $i\in I$.

Since $p_{g^k(i)}(y_o^a - y_1^a) \leq h_i(y_o^a - y_1^a)$ for every $k \in N \cup \{0\}$, $a \in A$ and $i \in I$ and $q_i(t) \in [0,1]$ for every $i \in I$ and every $t \in \mathbb{R}^+$ it follows that:

$$p_j(y_n^a - y_{n-1}^a) \leq h_i(y_o^a - y_1^a)$$

for every $n \in N$, $a \in A$ and $j \in O(i,g) = \{i, g(i), g^2(i), \ldots\}$.

So we have:

$$p_i(y_{k+1}^a - y_k^a) \leq \prod_{s=o}^{k-1} q_{g^s(i)}(h_i(y_o^a - y_1^a))h_i(y_o^a - y_1^a), \quad i \in I, \quad a \in A.$$

From b) it follows that y_o^a can be an arbitrary element from M and so let $y_o^a = x_o$, for every $a \in A$. Then:

$$p_i(y_{k+1}^a - y_k^a) \leq \prod_{s=o}^{k-1} q_{g^s(i)}(h_i(x_o - f_a x_o))h_i(x_o - f_a x_o), \ k \in N, \ i \in I, \ a \in A.$$

Further, it follows that the set $\{x_o - f_a x_o | a \in A\}$ is bounded and by using b) we conclude that there exists $N_i \in \mathbb{R}^+$ so that

$$h_i(x_o - f_a x_o) \leq N_i, \quad \text{for every } a \in A \ (i \in I)$$

which implies that:

$$(2) \quad p_i(y_{k+1}^a - y_k^a) \leq \prod_{s=o}^{k-1} q_{g^s(i)}(N_i)h_i(x_o - f_a x_o) \quad (k \in N, \ a \in A, \ i \in I).$$

From (2) we have:

$$p_i(y_m^a - y_n^a) \leq \sum_{s=n}^{m-1} p_i(y_{s+1}^a - y_s^a) \leq \sum_{s=n}^{m-1} \left(\prod_{k=o}^{s-1} q_{g^k(i)}(N_i)\right)h_i(x_o - f_a x_o)$$

and when $m \to \infty$.

$$(3) \quad p_i(x_a - y_n^a) \leq \sum_{s=n}^{\infty} \left(\prod_{k=o}^{s-1} q_{g^k(i)}(N_i)\right)h_i(x_o - f_a x_o) \ (n \in N, \ i \in I, a \in A).$$

Since $\overline{\lim_{n\to\infty}} q_{g^n(i)}(t) < 1$ it follows that the series $\sum_{s=1}^{\infty} \left(\prod_{k=o}^{s-1} q_{g^k(i)}(N_i)\right)$ is convergent for every $i \in I$. From (3) we have:

$$p_i(x_a - y_1^a) \leq \sum_{s=1}^{\infty} \left(\prod_{k=o}^{s-1} q_{g^k(i)}(N_i)\right)h_i(x_o - f_a x_o)$$

and so:

$$p_i(x_a-x_o) \leq p_i(x_a-f_ax_o)+p_i(f_ax_o-x_o) \leq \sum_{s=1}^{\infty} (\prod_{k=o}^{s-1} q_{g^k(i)}(N_i)) \times$$
$$\times h_i(x_o-f_ax_o) + p_i(f_ax_o-x_o) \quad (i \in I,\ a \in A).$$

Since $x_o - f_a x_o = f_o x_o - f_a x_o$ and $\lim_{a\in A} f_o x_o - f_a x_o = 0$, by using the continuity of h_i and $h_i(0)=0$, it follows that:

$$\lim_{a\in A} p_i(x_a - x_o) = 0, \quad \text{for every } i \in I$$

which means that $\lim_{a\in A} x_a = x_o$.

In the following Lemma, E is as in Lemma 1. Lemma 2 is a generalization of Theorem 1 from |6|, for bounded $T(M)$.

LEMMA 2. *Let* M *be an arbitrary set,* T *be a transformation from* M *into* E *such that* $T(M)$ *is a sequentially complete, bounded closed set and let* $\{g_a\}_{a\in A}$ *be a net of transformations defined on* M *with the values in* E *and* $g_a(M) \subseteq T(M)$, *for all* $a \in A$. *Suppose that for every* $a \in A$ *and every* $x \in M$ *there exists* $\lim_{a\in A} g_a x = g_o x$. *Further, for every* $i \in I$ *there exists* $f(i) \in I$ *and* $q_i : \mathbb{R}^+ \to [0,1]$ *as in Lemma 1 so that:*

$$p_i(g_ax-g_ay) \leq q_i(p_{f(i)}(Tx-Ty))p_{f(i)}(Tx-Ty)$$

for every $x,y \in M$ *and condition* b) *of Lemma 1 for* $f=g$ *is satisfied. Let* $b \in A \cup \{0\}$. *Then:*

1. *For every* $y \in T(M)$ *the set* $g_b(T^{-1}y)$ *has only one element and the mapping* $f_b: y \to g_b(T^{-1}y)$ *has one and only one fixed point* $y_b \in T(M)$.

2. *For every* $x \in T^{-1}y$, $g_b x = Tx$ *and* $g_b x^{(i)} = Tx^{(i)}$ $(i \in \{1,2\})$ *implies that* $Tx^{(1)} = Tx^{(2)}$.

3. $\lim_{a\in A} Tx_a = Tx_o$ $(x_b \in T^{-1}y_b,\ b \in A \cup \{0\})$.

P r o o f. First, we shall prove that $g_b(T^{-1}y)$ contains only one element. Let $y \in T(M)$ and $Tx_i = y$ $(i \in \{1,2\})$. We shall prove that $v_1 = v_2$, where $v_i = g_b x_i$, $i \in \{1,2\}$. We have:

$$p_i(v_1 - v_2) = p_i(g_b x_1 - g_b x_2) \leq q_i(p_{f(i)}(Tx_1 - Tx_2))p_{f(i)}(Tx_1 - Tx_2).$$

Since $Tx_1 = Tx_2 = y$ it follows that $v_1 = v_2$. From this we conclude that f_b is a singlevalued mapping. Since $g_b(M) \subseteq T(M)$ it follows that $f_b : T(M) \to T(M)$. Further let $x,y \in T(M)$, $x = Tu$, $y = Tv$. Then:

$$p_i(f_b x - f_b y) = p_i(g_b u - g_b v) \leq q_i(p_{f(i)}(Tu-Tv))p_{f(i)}(Tu-Tv) =$$

$$= q_i(p_{f(i)}(x-y))p_{f(i)}(x-y)$$

and so for every $b \in A \cup \{0\}$ there exists one and only one element $y_b \in T(M)$ so that $f_b y_b = g_b(T^{-1}y_b) = y_b$. Applying Lemma 1 we conclude that $\lim_{a \in A} y_a = y_o$. The proofs of 2. and 3. are as in Theorem 1 |6|.

THEOREM 1. *Let* $M \subseteq E, K$ *be a closed and convex subset of* E, $T : M \to K, T(M)$ *be complete and bounded,* $Q : K \to E$ *be a compact mapping,* $F : M \times K \to T(M)$ *so that the following conditions are satisfied:*

1. *For every* $i \in I$ *there exist* $f(i) \in I$ *and* $q_i : \mathbb{R}^+ \to [0,1]$ *as in Lemma 1 so that:*

$$p_i(F(x_1,y) - F(x_2,y)) \leq q_i(p_{f(i)}(Tx_1 - Tx_2))p_{f(i)}(Tx_1 - Tx_2)$$

for every $x_1, x_2 \in M$ *and* $y \in K$ *and the condition* b) *of Lemma 1 is satisfied for* $g = f$.

2. *For every* $i \in I$ *there exists* $C(i) > 0$ *so that:*

$$p_i(F(x,y_1) - F(x,y_2)) \leq C(i)p_i(Qy_1 - Qy_2)$$

for every $x \in M$ *and every* $y_1, y_2 \in K$ *and for every* $i \in I$ *the serie*s:

$$\sum_{k=1}^{\infty} \left(\prod_{r=0}^{k-1} q_{f^r(i)}(t)\right) C(f^k(i))$$

is convergent, for every $t \in \mathbb{R}^+$.

Then there exists $x \in M$ *so that* $F(x,Tx) = Tx$.

P r o o f. From Lemma 2 it follows that for every $y \in K$ there exists $u(y) \in M$ such that $F(u(y),y) = Tu(y)$. If $\tilde{f}: y \mapsto Tu(y)$ $(y \in K)$ then $\tilde{f}(K) \subseteq K$. The continuity of $\tilde{f}$ follows as in |6| since all the conditions of Lemma 2 are satisfied, where $\{x_a\}_{a\in A}$ is a convergent net in K, $\lim_{a\in A} x_a = x_o$ and $g_a x = F(x,x_a)$, $g_o x = F(x,x_o)$, for $x \in M$. Let us prove that $\overline{\tilde{f}(K)}$ is compact which means that for every net $\{x_a\}_{a\in A}$ in K there exists a convergent subnet of the net $\{fx_a\}_{a\in A} = \{F(u_{x_a},x_a)\}_{a\in A}$. We have $(a,c \in A)$:

$$p_i(F(u_{x_a},x_a) - F(u_{x_c},x_c)) \le p_i(F(u_{x_a},x_a) - F(u_{x_c},x_a)) +$$

$$+ p_i(F(u_{x_c},x_a) - F(u_{x_c},x_c)) \le q_i(p_{f(i)}(Tu_{x_a}-Tu_{x_c}))p_{f(i)}(Tu_{x_a}-Tu_{x_c})$$

$$+ C(i)p_i(Qx_a-Qx_c) \le q_i(p_{f(i)}(Tu_{x_a}-Tu_{x_c}))\left[q_{f(i)}(p_{f^2(i)}(Tu_{x_a}-Tu_{x_c}))\right.$$

$$\left. \times\, p_{f^2(i)}(Tu_{x_a}-Tu_{x_c}) + C(f(i))p_{f(i)}(Qx_a-Qx_c))\right] + C(i)p_i(Qx_a-Qx_c) \le$$

$$\le \dots \le \prod_{s=0}^{n} q_{f^s(i)}(p_{f^{s+1}(i)}(Tu_{x_a}-Tu_{x_c}))p_{f^{n+1}(i)}(Tu_{x_a} - Tu_{x_c}) +$$

$$+ \sum_{k=0}^{n-1} \left(\prod_{s=0}^{k} q_{f^s(i)}(p_{f^{s+1}(i)}(Tu_{x_a}-Tu_{x_c}))\right) C(f^{k+1}(i))p_{f^{k+1}(i)}($$

$(Qx_a-Qx_c) + C(i)p_i(Qx_a-Qx_c)$. Further there exists $M(i) > 0$ $(i \in I)$ so that

$$p_{f^{s+1}(i)}(Tu_{x_a}-Tu_{x_c}) \le M(i) \quad (s \in \mathbb{N} \cup \{0\},\ i \in I,\ a,c \in A).$$

Then:

$$p_i(F(u_{x_a},x_a) - F(u_{x_c},x_c)) \le \prod_{s=0}^{n} q_{f^s(i)}(M(i))M(i) + C(i)p_i(Qx_a-Qx_c) +$$

$$\sum_{k=0}^{n-1} \left(\prod_{s=0}^{k} q_{f^s(i)}(M(i))C(f^{k+1}(i))h_i(Qx_a-Qx_b)\right.$$

If $n \to \infty$ then:

$$p_i(F(u_{x_a},x_a)-F(u_{x_c},x_c)) \leq C(i)p_i(Qx_a-Qx_c) + \sum_{r=o}^{\infty} (\prod_{s=o}^{r} q_{f^s(i)} (M(i)) \cdot$$

$$\times\ C(f^{r+1}(i))h_i(Qx_a-Qx_c)\ .$$

The set $\overline{Q(K)}$ is compact and so there exists a convergent subnet $\{Qx_a\}_{a\in A'}$ of the net $\{Qx_a\}_{a\in A}$. Now, we shall show that $\{F(u_{x_a},x_a)\}_{a\in A'}$ is a Cauchy net which means that there exists $a_o \in A'$, such that $a,c \geq a_o(i,\varepsilon)$ implies $(\varepsilon > 0)$:

$$p_i(F(u_{x_a},x_a) - F(u_{x_c},x_c)) < \varepsilon\ .$$

Since h_i is continuous and $h_i(0) = 0$ it follows that for every $\varepsilon' > 0$ there exists a neighbourhood $V_{i,\varepsilon'}(0)$ of zero so that

$$z \in V_{i,\varepsilon'} \Rightarrow h_i(z) < \frac{\varepsilon'}{K(i)}$$

where $K(i) = \sum_{r=o}^{\infty} (\prod_{s=o}^{r} q_{f^s(i)} (M(i))C(f^{r+1}(i))+C(i)$. Since $\{Qx_a\}_{a\in A'}$ is convergent there exists $a_o(i,\varepsilon)$ so that $a,c \geq a_o$ $(a,c \in A')$ implies:

$$Qx_a - Qx_c \in V_{i,\varepsilon}\ .$$

Then we have that $a,c \geq a_o\ (i,\varepsilon)\ (a,c \in A')$ implies that:

$$p_i(F(u_{x_a},x_a) - F(u_{x_c},x_c)) < \varepsilon.$$

The rest of the proof is as in |6|.

As in |6| and using a similar method as in Theorem 1 we can prove the following theorem.

THEOREM 2. *Let* $M \subsetneq E$, K *be a closed and convex subset of* E, $T: M \to E$, $\overline{T(M)}$ *be bounded complete* $\overline{T(M)} \subseteq K$, $Q: K \to E$ *be a compact mapping and* $F: M \times \overline{Q(K)} \to E$ *so that for every* $x \in M$ *the mapping*

$y \to F(x,y)$ $(y \in \overline{QK})$ *is continuous,* $F(M,\overline{Q(K)}) \subseteq T(M)$ *and the following conditions are satisfied:*

For every $i \in I$ *there exist* $f(i) \in I$ *and* $q_i : \mathbb{R}^+ \to [0,1]$ *as in Lemma 1 so that:*

$$p_i(F(x_1,y)-F(x_2,y)) \le q_i(p_{f(i)}(Tx_1-Tx_2))p_{f(i)}(Tx_1-Tx_2)$$

for every $x_1,x_2 \in M$ *and every* $y \in \overline{Q(K)}$ *and condition* b) *of Lemma 1 is satisfied for* $g = f$.

Then there exists $x \in M$ *so that* $F(x, Q(Tx)) = Tx$.

From Theorem 2 we obtain the following Corollary, which is a generalization of Krasnoseljski's fixed point theorem.

COROLLARY *Let* $K=\bar{K}$ *be a bounded, convex and complete subset of* E, $Q : K \to E$ *be a compact mapping,* $F : K \times \overline{Q(K)} \to E$ *so that for every* $x \in K$ *the mapping* $y \to F(x,y)$ $(y \in \overline{Q(K)})$ *is continuous,* $F(K,\overline{Q(K)}) \subseteq K$ *and the following condition is satisfied:*

For every $i \in I$ *there exist* $f(i) \in I$ *and* $q_i : \mathbb{R}^+ \to [0,1]$ *as in Lemma 1 so that:*

$$p_i(F(x_1,y)-F(x_2,y)) \le q_i(p_{f(i)}(x_1-x_2))p_{f(i)}(x_1-x_2)$$

for every $x_1,x_2 \in K$ *and every* $y \in \overline{Q(K)}$ *and condition* b) *of Lemma 1 is satisfied for* $g = f$.

Then there exists $x \in K$ *so that* $F(x,Qx) = x$.

P r o o f. It is enough to take that $T = IdK$, $M = K$.

REMARK. In Krasnoseljski's fixed point theorem is $F(x,y) = Sx + y$, where S is a contraction type mapping.

Using a similar method as in |6| we can prove the following Lemma.

LEMMA 3. *Let* $(E, \{p_i\}_{i\in I})$ *be a Hausdorff locally convex topological vector space in which the topology is defined by the family* $\{p_i\}_{i\in I}$ *od seminorm,* $g : I \to I$, M *be a closed sequentially complete subset of* E, *for every* $a \in A$, $f_a : M \to M$ *so that* $\lim_{a\in A} f_a x$ *exists for every* $x \in M$ *and the following conditions are satisfied:*

1. *For every* $i \in I$ *there exists* $q(i) > 0$ *so that:*

$p_i(f_a x - f_a y) \leq q(i) p_{g(i)}(x-y)$, *for every* $x,y \in M$ *and every* $a \in A$.

2. *For every* $i \in I$ *and every* $n \in N$ *there exist* $a_n(i) \geq 0$ *and* $f(i) \in I$ *so that* $p_{g^n(i)}(x) \leq a_n(i) p_{f(i)}(x)$, *for every* $x \in E$ *and the series:*

$$\sum_{n=1}^{\infty} \left(\prod_{r=0}^{n-1} q(g^r(i)) \right) a_n(i)$$

is convergent for every $i \in I$.

If $f_0 x = \lim_{a\in A} f_a x$, *for every* $x \in M$ *then* $\lim_{a\in A} x_a = x_0$, *where* $f_a x_a = x_a$, *for every* $a \in A \cup \{0\}$.

Using Lemma 3 instead of Lemma 1 it is easy to prove the following theorem on the existence of a solution of the equation $Tx = F(x,Q(Tx))$.

THEOREM 3. *Let* $E, M, K, T, \overline{T(M)}, Q$ *and* F *be as in Theorem 2 and the following condition be satisfied:*

For every $i \in I$ *there exist* $q(i) > 0$ *and* $g(i) \in I$ *so that:*

$$p_i(F(x_1,y) - F(x_2,y)) \leq q(i) p_{g(i)}(Tx_1 - Tx_2)$$

for every $x_1, x_2 \in M$ *and every* $y \in \overline{Q(K)}$ *and condition 2. of Lemma 3 is satisfied.*

Then there exists $x \in M$ *so that* $F(x,Q(Tx)) = Tx$.

Suppose that E is a vector space in which the topology is defined by the family $\{p_i\}_{i\in I}$ of functionals such that the following conditions are satisfied:

1. $p_i(0) = 0$, for every $i \in I$.
2. $p_i(-x) = p_i(x)$, for every $x \in E$ and every $i \in I$.
3. $p_i(x+y) \leq p_i(x)+p_i(y)$, for every $x,y \in E$ and every $i \in I$.
4. If $p_i(x_a-x)\to 0$, ($\{x_a\}$ is a net from E) and $t_a \to t$ then $p_i(t_a x_a - tx) \to 0$, $a \in A$.

Then E is a topological vector space in which the fundamental system of neighbourhoods of zero in E is given by the family:

$$V(i,r) = \{x \mid p_i(x) < r\} \quad (i \in I,\ r > 0).$$

Suppose now that $(E, \{p_i\}_{i\in I})$ is a topological vector space in which the topology is defined by the family of functionals $\{p_i\}_{i\in I}$ so that 1.,2.,3. and 4. are satisfied. We shall suppose that E is Hausdorff which means that $p_i(x) = 0$, for every $i \in I$ implies that $x = 0$.

Similarly as in |8|, it is easy to prove the following fixed point theorem.

THEOREM 4. *Let* $(E, \{p_i\}_{i\in I})$ *be a Hausdorff topological vector space in which the topology is defined by the family of functionals* $(i \in I)$ *so that* 1.,2.,3. *and* 4. *is satisfied. Let* M *be a closed and convex subset of* E *and* $F: M \to M$ *be a compact mapping such that the following condition is satisfied:*

For every $i \in I$ *there exists* $C_i > 0$ *so that:*

$$p_i(tx) \leq C_i t p_i(x)\ ,\ \textit{for every } t \in [0,1] \textit{ and } x \in F(M) - F(M).$$

Then there exists $x \in M$ *so that* $x = Fx$.

P r o o f. First, we shall show that for every V from the fundamental system of neighbourhoods of zero in E there exists a neighbourhood of zero U so that:

$$co(U \cap (F(M) - F(M)) \subseteq V$$

where co is convex hull. Let $V = V(i,r)$ $(i \in I,\ r > 0)$. Then $U = V(i, \frac{r}{C_i})$. Indeed, suppose that $z \in co(U \cap (F(M)-F(M))$. Then there exist $s_i \geq 0$ $(i \in \{1,2,\dots,n\})$ so that $\sum_{i=1}^{n} s_i = 1$ and $u_i \in U \cap (F(M)-F(M))$ $(i \in \{1,2,\dots,n\})$ so that:

$$z = \sum_{i=1}^{n} s_i u_i \ .$$

Then we have:

$$p_i(z) \leq \sum_{j=1}^{n} p_i(s_j u_j) \leq \sum_{j=1}^{n} C_i s_j p_i(u_j) < \sum_{j=1}^{n} C_i s_j \frac{r}{C_i} = r.$$

Then |5| there exists $x \in M$ such that $x = Fx$.

Now, using Theorem 4 instead of Tihonov's fixed point theorem, it is easy to prove the following generalization of Theorem 1.

THEOREM 1'. *Let* $(E, \{p_i\}_{i \in I})$ *be a Hausdorff topological vector space in which the topology is defined by the family of functionals* p_i $(i \in I)$ *so that* 1.,2.,3. *and* 4. *are satisfied. Let* M,K,T,Q *and* F *be as in Theorem 1 and conditions* 1. *and* 2. *be satisfied as in Theorem 1. Suppose that for every* $i \in I$ *there exists* $C_i > 0$ *so that:*

$$p_i(tx) \leq C_i t p_i(x) \quad , \textit{ for every } t \in [0,1] \textit{ and every } x \in T(M)-T(M).$$

Then there exists $x \in M$ *so that* $F(x,Tx) = Tx$.

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REZIME

O JEDNAČINI Tx=(Fx,Q(Tx)) U LOKALNO KONVEKSNIM PROSTORIMA

U ovom radu su, korišćenjem teorema o nepokretnoj tački iz rada |4| i |7|, dokazana neka uopštenja teorema o egzistenciji rešenja jednačina Tx=F(x,Tx), i Tx=F(x,Q(Tx)) u lokalno konveksnim prostorima koje su dokazane u radu |6|.