Zbornik radova Prirodno-matematičkog fakulteta-Univerzitet u Novom Sadu
knjiga 12 (1982)
Review of Research Faculty of Science-University of Novi Sad, Volume 12(1982)

AN ANALYTIC CHARACTERIZATION OF THE SPACE OF GENERALIZED FUNCTIONS WHICH HAVE A LAGUERRE EXPANSION

Stevan Pilipović
Prirodno-matematički fakultet. Institut za matematiku
21000 Novi Sad, ul. dr Ilije Djuričića br.4, Jugoslavija

ABSTRACT

The space LG' of generalized functions, whose elements have a Laguerre orthonormal expansion into a series, are investigated in |2| and |3| . In this paper we define the space LG by using a suitable family of seminorms. This implies some properties of the space LG' and the representation theorem for some elements from LG'. Also, by using a convolution in LG' and a Laplace transform we give expansions into a series of some important generalized functions from LG'.

I In monograph |4|, Zemanian investigated spaces of generalized functions whose elements have an orthonormal expansion into a series. These spaces are denoted by A'.

In |4| various examples of such spaces are given. These spaces correspond to various spaces $L^2(I)$, (I is an interval in R) and their orthonormal bases (ψ_n). The best known space of A'-type is the space S'. Among the examples, the space of the test functions LG_α and of the generalized functions LG'_α, $\alpha > -1$, are given and these correspond to the space $L^2(0,\infty)$ and the generalized Laguerre orthonormal bases (ℓ_n^α) of $L^2(0,\infty)$, $\alpha > -1$, where

$$\ell_n^\alpha(x) := \left(\frac{\Gamma(n+1)}{\Gamma(\alpha+n+1)}\right)^{1/2} x^{\alpha/2} e^{-x/2} L_n^\alpha(x), \quad n \in N_o \ (= N \cup \{0\}) .$$

Laguerre polynomials are given by

$$L_n^\alpha(x) := \sum_{m=o}^{n} \binom{n+\alpha}{n-m} \frac{(-x)^m}{m!} \quad \left(\binom{y}{\nu} = \frac{\Gamma(y+1)}{\Gamma(\nu-1)\Gamma(y-\nu-1)}\right) .$$

For a fixed $\alpha > -1$, the functions $\ell_n^\alpha(x)$, $n \in N_o$, are the eigenfunctions for the self-adjoint operator in $L^2(0,\infty)$

$$R_\alpha := x^{-\alpha/2}e^{x/2}Dx^{\alpha+1}e^{-x}Dx^{-\alpha/2}e^{x/2} \quad (D = \frac{d}{dx})$$

for which

(1) $$R_\alpha \ell_n^\alpha = - n\ell_n^\alpha, \quad n = 0,1,\ldots$$

hold.

In |2| and |3| we investigated the spaces LG and LG´ which correspond to $\alpha = 0$, LG:=LG_o, LG´:=LG_o', because the base (ℓ_n), $\ell_n = \ell_n^o$, $n \in N_o$, gives some conveniences. For example in LG´ the convolution and Laplace transform are defined and for them an exchange formula holds (|3|).

In this paper we are going to define the space LG by using the suitable family of seminorms. This will imply some properties of the space LG´.

We shall also give the expansions of some of the important generalized functions from LG´ by using the convolution and Laplace transform in the space LG´.

1. We introduce in this paper the space LG by the following definition:

DEFINITION 1. *The space* LG *is the subspace of* $L^2(0,\infty) \cap C^\infty(0,\infty)$ *for which elements*

$$r_k(\phi) := (\int_o^\infty |R^k\phi(x)|^2 dx)^{1/2} < \infty, \quad k \in N_o, \quad (R := R_o)$$

$$(R^k\phi, \ell_n) = (\phi, R^k\ell_n), \quad k \in N_o, \ n \in N_o,$$

hold.

(If $\phi, \psi \in L^2(0,\infty)$ *then* $(\phi,\psi) := \int_o^\infty \phi(t)\bar{\psi}(t)dt = \langle \phi, \bar{\psi} \rangle$*)* .

The convergence structure in this space is given by the sequence of seminorms (r_k), $k \in N_o$.

THEOREM 1. (i) *A function* ϕ *from* $L^2(0,\infty)$ *of the form*

(2) $$\phi \overset{2}{=} \sum_{n=o}^\infty a_n\ell_n \qquad (\overset{2}{=} \text{ means: in square mean})$$

is in LG *iff for every* $k \in N_o$ *there exists* $C_k > 0$ *such that*

(3) $$n^k|a_n| \leq C_k\ ,\quad n \in N_o$$

(ii) *A sequence* (ϕ_m) *from* $L^2(0,\infty)$ *of the form*

(4) $$\phi_m \overset{2}{=} \sum_{n=o}^{\infty} a_{m,n}\ell_n$$

converges in LG *to* $\phi \in$ LG *of the form (2), iff for any* $k \in N$ *there exists* $C_k > 0$ *such that*

(5) $$n^k|a_{m,n}| \leq C_k\ ,\quad n \in N_o \text{ and}$$

(6) *for every* $n \in N_o$, $a_{m,n} \to a_n$ *as* $m \to \infty$.

P r o o f. (i) Since (3) holds iff for every $k \in N_o$

$$\sum_{n=o}^{\infty} n^{2k}|a_n|^2 < \infty\ ,$$

the assertion (i) directly follows from |4| Lemma 9.3.3. .

(ii) The conditions (5) and (6) are equivalent to the condition:

$$\text{for every } k \in N_o,\quad \sum_{n=o}^{\infty} n^{2k}|a_{m,n}-a_n|^2 \to 0 \quad \text{as} \quad m \to \infty\ .$$

This condition is equivalent to $r_k(\phi_n-\phi) \to 0$ for every $k \in N$.

Let us put $\tilde{D}^{2k}\phi := D(\underbrace{xD(xD(\ldots}_{k-1}(xD\phi)\ldots)))$; for example $\tilde{D}^2\phi = D(xD\phi)$, $\tilde{D}^4\phi = D(xD(xD\phi))$ $(\tilde{D}^o\phi = \phi)$.

We are going to prove the following theorem:

THEOREM 2. (i) *A function* $\phi \in$ LG *iff* $\phi \in C^\infty[0,\infty)$ *and*

$$\gamma_k(\phi) := \sup\{x^k[D^\ell\phi(x)];\ x \in 0,\infty),\ \ell \leq k\} < \infty,\ k \in N_o\ ;$$

$$\rho_{2k}(\phi) := \sup\{[\tilde{D}^{2k}\phi(x)];\ x \in 0,\infty)\} < \infty\ ,\quad k \in N\ .$$

(ii) *A sequence* (ϕ_m) *from* LG *converges to* 0 *iff for every* $k \in N_o$ $\gamma_k(\phi_m) \to 0$ *and* $\rho_{2k}(\phi_m) \to 0$ *as* $m \to \infty$.

First we are going to prove two lemmas.

LEMMA 3. *For the functions* (ℓ_n), $n \in N_o$, *the following formulas and estimates hold:*

(i) *If* $k,j \in N_o$ *and* $j \leq k$ *then*

$$x^k D^j \ell_n(x) = \sum_{i=0}^{2k} c_{n,k,j,i}\ell_{n-k+i}, \quad n \in N_o$$

and there exists $C_{k,j} > 0$ *such that*

$$|c_{n,k,j,i}| \leq C_{k,j}\hat{n}^k, \quad i = 0,\dots,k \quad (\hat{n} = \begin{cases} n & n \neq 0 \\ 1 & n = 0 \end{cases}) \quad .$$

(ii) *If* $k \in N$

$$\tilde{D}^{2k}\ell_n = \sum_{i=0}^{2k} \tilde{c}_{n,k,i}\ell_{n-k+i}, \quad n \in N_o$$

and there exists $\tilde{C}_k > 0$ *such that*

$$|\tilde{c}_{n,k,i}| \leq \tilde{C}_k\hat{n} \quad .$$

P r o o f. From the formula from |1| ,p. 188. for $\alpha = 0$ and $n \in N_o$, we have

$$(7) \quad x\ell_n(x) = -(n+1)\ell_{n+1}(x) + (2n+1)\ell_n(x) - n\ell_{n-1}; \quad (\ell_{-1} = 0)$$

$$(8) \quad xD\ell_n(x) = \frac{1}{2}(n+1)\ell_{n+1} - \frac{1}{2}\ell_n - \frac{n}{2}\ell_{n-1} \quad .$$

So, by induction, one can prove (i).

For the assertion of (ii) we ought to combine (7), (8) with (1) for $\alpha = 0$. In this way we obtain

$$(9) \quad D(xD\phi) = -\frac{1}{4}(n+1)\ell_{n+1} - \frac{1}{4}(2n+1)\ell_n - \frac{1}{4}n\ell_{n-1} \quad , n \in N_o,$$

from which (ii) follows by induction.

LEMMA 4. *If* $\phi \in LG$ *is of the form*

$$\phi \overset{2}{=} \sum_{n=0}^{\infty} a_n\ell_n$$

then this series converges uniformly to ϕ *in* $[0,\infty)$.

Moreover, for any $k \in N_o$ *and* $j \in N_o$

$$(10) \quad x^k D^j\phi(x) = \sum_{n=0}^{\infty} a_n x^k D^j \ell_n(x)$$

In (10) *the series converges uniformly in* $[0,\infty)$.

P r o o f. For the proof we need the following formula and estimate from |1|.

(11) $$DL_n^{\alpha} = - L_{n-1}^{\alpha+1} , \quad n \in N_o$$

(12) $$|L_n^{\alpha}(x)| \leq \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)} \cdot \frac{1}{n!} e^{x/2}, \quad x \in [0,\infty), \; \alpha \geq 0, \; n \in N_o .$$

From (12) for $\alpha=0$ and $x \in [0,\infty)$, we have

$$\sum_{n=o}^{\infty} |a_n||\ell_n(x)| \leq \sum_{n=o} |a_n| < \infty .$$

This means that $\sum_{n=o}^{\infty} a_n \ell_n$ converges uniformly in $[0,\infty)$ to ϕ.

Let us prove (10) for $(k,j)=(1,0)$ and $(k,j)=(0,1)$ because for arbitrary (k,j), (10) follows by induction.

From (11) and (12) it follows that for every $n \in N_o$

$$|D\ell_n| = |e^{-x/2} \left(-\tfrac{1}{2} L_n(x) - L_{n-1}^1(x)\right)| \leq n+1 , \; x \in [0,\infty),$$

holds.

Since for (a_n), $n \in N_o$, (3) holds, we obtain that $\sum_{n=o}^{\infty} a_n \ell_n'$ converges uniformly on $[0,\infty)$.

Similarly, from (7) and (3) it follows that $\sum_{n=o}^{\infty} a_n x \ell_n(x)$ converges uniformly in $[0,\infty)$.

P r o o f of Theorem 2. (i) Let $\phi \in C^{\infty}[0,\infty)$ and for every $k \in N_o$, $\gamma_k(\phi) < \infty$, and $\rho_{2k}(\phi) < \infty$. Since the functions $x \mapsto x^k D^j \phi(x)$, $x \mapsto \tilde{D}^{2k}\phi(x)$, $j,k \in N_o$, $j \leq k$, are bounded on $[0,1)$, we have

(13) $$\int_0^{\infty} |x^k D^j \phi(x)|^2 dx \leq \int_0^1 |x^k D^j \phi(x)|^2 dx + \int_1^{\infty} \frac{1}{x^2} |x^{k+1} D^j \phi(x)|^2 dx < \infty ,$$

(14) $$\int_0^{\infty} |\tilde{D}^{2k}\phi(x)|^2 dx \leq \int_0^1 |\tilde{D}^{2k}\phi(x)|^2 dx + \int_1^{\infty} \frac{1}{x^2} |\tilde{D}^{2k}\phi(x)|^2 dx < \infty .$$

So we obtain that $x^k D^j \phi(x)$, $\tilde{D}^{2k}\phi(x) \in L^2(0,\infty)$, $j,k \in N_o$, $j \leq k$.

We may prove by induction that $R^k\phi$, $k \in N_o$, is a linear combination of factors of the form $x^r D^i \phi(x)$, $r,i \in N_o$, $i \leq r \leq k$, and $\tilde{D}^{2s}\phi(x)$, $s \in N$, $s \leq k$. So $R^k \phi \in L^2(0,\infty)$ for every $k \in N_o$. By induction we may, as well, prove that for every $k \in N_o$ and $n \in N_o$

$$(R^k\phi, \ell_n) = \int_0^{\infty} R^k\phi(t)\ell_n(t)dt = \int_0^{\infty} \phi(t) R^k \ell_n(t)dt , \quad \phi \in LG .$$

From Definition 1 it follows that $\phi \in LG$. If $\phi \in L^2(0,\infty)$ such that ϕ is of the form (2) and (3) holds, then from Lemma 4, using (7), (11) and (12), we obtain $\gamma_k(\phi) < \infty$ and $\rho_{2k}(\phi) < \infty$, $k \in N_o$.

(ii) From (13) and (14) with ϕ_m instead of ϕ we have

$$x^k D^j \phi_m(x) \overset{2}{\to} 0, \quad \tilde{D}^{2k}\phi_m(x) \overset{2}{\to} 0, \; i,k \in N_o, \; j \leq k, \text{ as } m \to \infty .$$

This means that for any $k \in N_o$, $R^k \phi_m \overset{2}{\to} 0$ as $m \to \infty$. If ϕ_m is of the form (4), $m \in N$, is a sequence from LG which converges to $0 \in$ LG, using (5), (6), Lemma 4, (7), (11) and (12) we obtain that

$$\gamma_k(\phi_m) \to 0 \quad \text{and} \quad \rho_{2k}(\phi_m) \to 0, \; k \in N_o, \text{ as } m \to \infty .$$

We see that we do not need Lemma 3 for the proof of Theorem 2. Yet this Lemma and the appropriate assertion for the transposed mappings directly imply:

THEOREM 5. *The mappings from LG′ to LG′ defined by*

$$f \to D^j(x^k f) \quad j,k \in N_o, \; j \leq k;$$

(15) $$f \to D(x(D^2(x(\ldots D^2(x(D^2(x(Df))))\ldots))))$$

are continuous with respect to the strong (weak) topologies in LG′.

Let as remark that the convergence structure of LG′ is investigated in |2|.

2. Theorem 2 implies that LG is a subspace of $E(0,\infty)$ and that the convergence in LG implies the convergence in $E(0,\infty)$. Since $\mathcal{D}(0,\infty) \subset$ LG and $\mathcal{D}(0,\infty)$ is a dense subspace of $E(0,\infty)$ we have

$$E'(0,\infty) \subset LG' \quad \text{(see also |4| p. 319.).}$$

If $F \in L^1_{loc}(0,\infty)$ and for some $k \in N$, $F(t)t^{-k} \in L^1(1,\infty)$ then by

$$\phi \to \langle F,\phi\rangle := \int_0^\infty F(t)\phi(t)dt = \int_0^1 F(t)\phi(t)dt + \int_1^\infty F(t)t^{-k}(t^k\phi(t))dt$$

an element from LG′ is defined.

So Theorem 5 and Theorem 9.6.2. from |4| implies

THEOREM 6. (i) |4| $f \in LG'$ *iff* f *is of the form*

$$f = R^k F + c_o \exp(-x/2)$$

for some $F \in L^2(0,\infty)$, $k \in N$ *and* c_o*-complex number.*

(ii) *If* $F \in L^1_{loc}(0,\infty)$ *and* $F(t)t^{-k} \in L^1(1,\infty)$ *for some* $k \in N$ *then*

$$\tilde{D}^{2k}F \quad \text{and} \quad D^j x^r F$$

are from LG' *for every* $k \in N$ *and every* $j,r \in N_o$, $j \leq r$.

There is an open problem concerning the connections between the space LG' and $S'(0,\infty)$.

3. In |3| the convolution of elements $f = \sum_{n=o}^{\infty} a_n \ell_n$ and $g = \sum_{n=o}^{\infty} b_n \ell_n$ from LG' is defined by

$$f \odot g = \sum_{n=o}^{\infty} c_n \ell_n$$

where $c_n = \sum_{p+q=n} a_p b_q - \sum_{p+q=n-1} a_p b_q \quad \left(\sum_{p+q=-1} = 0 \right)$.

With this operation, LG' is a convolution algebra and in |3| we have proved that the mapping from $LG' \times LG'$ into LG' defined by

$$(f,g) \to f \odot g$$

is sequentially continuous (in the sense of weak topology).

If $f \in L^2$ and $g \in L^2$ then in LG'

$$(f \odot g)(x) = \int_o^{\infty} f(x-t)g(t)dt$$

holds (|3|).

The Laplace transform in LG' is defined by

$$\mathscr{L}(f) := \langle f, e^{-st} \rangle = \langle \sum_{n=o}^{\infty} a_n \ell_n, e^{-st} \rangle = \sum_{n=o}^{\infty} a_n \mathscr{L}(\ell_n)$$

$$= \sum_{n=o}^{\infty} a_n \frac{(s-1/2)^n}{(s+1/2)^{n+1}}, \quad \operatorname{Re} s > 0 \quad (\text{see } |4|).$$

We have shown that $\mathscr{L}$ is a sequentially continuous bijection of LG' (in relation to the weak topology in LG') and the space of functions L, analytic in the half-space $\operatorname{Re} s > 0$ of the form

$$F(s) = \sum_{n=0}^{\infty} a_n \frac{(s-1/2)^n}{(s+1/2)^{n+1}}$$

such that coefficients a_n, $n \in N_0$, satisfy the condition

$$|a_n| \le M\hat{n}^k$$

for some $M > 0$ and $k \in N$. In L we have a topology induced by the topology of the uniform convergence on a compact subsets of the half-space $\mathrm{Re}\, s > 0$.

We have shown, as well in |3|, that

$$\mathscr{L}(f \circledast g) = \mathscr{L}(f)\mathscr{L}(g) .$$

Let $f = \sum_{n=0}^{\infty} c_n \ell_n \in L^2(0,\infty)$ and $f_\nu = \sum_{n=0}^{\nu} c_n \ell_n$. Since $e^{-st} \in L^2(0,\infty)$, for any $s = \xi + i\eta$, $\xi > 0$, in the sense of the uniform convergence on compact subsets on the half-space $\mathrm{Re}\, s > 0$, we have

$$\sum_{n=0}^{\nu} a_n \frac{(s-1/2)^n}{(s+1/2)^{n+1}} = \int_0^\infty e^{-st} f_\nu(t)dt \to \int_0^\infty e^{-st} f(t)dt = \mathscr{L}(f)(s)$$

as $\nu \to \infty$.

This means that the Laplace transformation in LG' is a generalization of the Laplace transformation in $L^2(0,\infty)$.

4. We are going to give the expansions into a series of some elements from LG' in the sense of the convergence in LG'.

Since

$$\int_0^\infty e^{-st} t^\alpha dt = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}}, \quad \mathrm{Re}\, s > 0,\ \alpha \neq -1, \tag{16}$$

we have (for $s = \frac{1}{2}$) that $\int_0^\infty \ell_n(t)dt = 2(-1)^n$. This implies the expansion of function $f(x) = 1$, $x \in [0,\infty)$:

$$1 = 2 \sum_{n=0}^{\infty} (-1)^n \ell_n . \tag{17}$$

Let the δ-distribution $\delta(t-x)$, $x \ge 0$, be defined by $\langle\delta(t-x), \phi(t)\rangle := \phi(x)$. It is clear that $\delta(t-x) \in LG'$. We are going to find the coefficients a_n, $n \in N_0$, in the expansion

$$\delta(t-x) = \sum_{n=o}^{\infty} a_n \ell_n(t).$$

We have

(18) $$\mathscr{L}(\delta(t-x))(s) = <\delta(t-x), e^{-st}> = e^{-xs}$$

(19) $$\mathscr{L}(\delta(t-x))(s) = \sum_{n=o}^{\infty} a_n \mathscr{L}(\ell_n(t)) = \sum_{n=o}^{\infty} a_n \frac{(s-1/2)^n}{(s+1/2)^{n+1}}$$

In LG′ we have

$$e^{-xs} = \sum_{n=o}^{\infty} \frac{(s-1/2)^n}{(s+1/2)^{n+1}} \ell_n(x)$$

Thus from (18) and (19) we obtain

$$\sum_{n=o}^{\infty} a_n \frac{(s-1/2)^n}{(s+1/2)^{n+1}} = \sum_{n=o}^{\infty} \frac{(s+1/2)^n}{(s+1/2)^{n+1}} \ell_n(x).$$

Since $\mathscr{L}$ transformation is a bijection from LG′ to L, we have

(20) $$\delta(t-x) = \sum_{n=o}^{\infty} \ell_n(x)\ell_n(t).$$

Particularly, because $\ell_n(0) = 1$, $n \in N_o$, for $x = 0$, we have $\delta(t) = \sum_{n=o}^{\infty} \ell_n(t)$. If $f(t) \in LG'$ and $x \geq 0$, let us define the operation of the translation in LG′ by:

$$<f(t-x), \phi(t)> := <f(t), \phi(t+x)> \quad , \quad \phi \in LG .$$

Since the mapping form LG to LG defined by

$$\phi(t) \to \phi(x+t)$$

is continuous, it follows that $f(t-x) \in LG'$. If $f \in LG'$ is a function in $(0,\infty)$, then a generalized function $f(t-x)$ is a function which is equal to zero in $[0,x]$ and to $f(t-x)$ in (x,∞).

Let us prove: $\delta(t-x) \circledcirc f(t) = f(t-x)$. In order to prove this formula, some formulas for the Laguerre polynomials are needed.

LEMMA 7. *For* $x \geq 0$ *and* $t \in [0,\infty)$ *we have*

(21) $$L_n(t+x) = \sum_{p+q=n} L_p(x)L_p(t) - \sum_{p+q=n-1} L_p(x)L_q(t),$$

$$\left(\sum_{p+q=-1} = 0\right).$$

P r o o f. The coefficient of t^{ℓ}, $0 \le \ell \le n$, on the left side of (21) is

$$\sum_{r=0}^{n-\ell} \binom{n}{\ell+r}\binom{\ell+r}{r}\frac{(-1)^{\ell+r}}{(\ell+r)!}x^r . \tag{22}$$

The coefficient of t^{α} on the right side of (21) is

$$\frac{(-1)^{\ell}}{\ell!}\left[\sum_{r=0}^{n-\ell-1}\sum_{i=0}^{n-(\ell+r)}\binom{n-(\ell+r)}{i}\frac{(-1)^{i}x^{i}}{i!}\binom{\ell+r}{\ell} - \right. \tag{23}$$

$$\left. - \sum_{r=0}^{n-1-(\ell+r)}\binom{n-1-(\ell+r)}{i}\frac{(-1)^{i}x^{i}}{i!}\binom{\ell+r}{\ell}\right] .$$

The coefficients of x^i in (22) and (23) are:

$$\binom{n}{\ell+i}\frac{(-1)^{\ell+i}}{(\ell+i)!}\binom{\ell+i}{i} \tag{24}$$

$$\frac{(-1)^{\ell}}{\ell!i!}\sum_{r=0}^{n-\ell-1}\binom{n-\ell-r}{i}\binom{\ell-1+r}{\ell-1} . \tag{25}$$

The equality of (24) and (25) follows from the known identity:

$$\sum_{r=0}^{n-\ell-i}\binom{\ell-1+r}{\ell-1}\binom{n-\ell-r}{i} = \binom{n}{\ell+i} .$$

From this Lemma, by a simple calculation of the coefficients of $f(t-x)$ and using (20), we obtain

$$\delta(t-x) \odot f(t) = f(t-x) \tag{26}$$

From (17) and (26) we have:

$$h(t-x) := \begin{cases} 0 & 0 \le t \le x \\ 1 & t > x \end{cases} = \sum_{n=0}^{\infty}\left(\sum_{i=0}^{n-1} 4(-1)^{n-i}\ell_i(x) + 2\ell_n(x)\right)\ell_n(t) .$$

From (26) and the identity

$$\int_0^{\infty} e^{-st}x^m L_n(t)dt = (-1)^{m+n}\frac{d^m}{ds^m}\left[(1-1/s)1/s\right] , \quad n \in N_0$$

$$m \in N_0,$$

one may easily obtain the expansions of the generalized functions

$$(t-x)_+^m = \begin{cases} 0 & 0 \le t \le x \\ (t-x)^m & t > x . \end{cases}$$

REFERENCES

|1| A. Erdelyi, W.Magnus, F. Oberhettinger, F.G.Tricomi, *Higher Transcedental Functions, Vol. 2, Mc Graw Hill, New York, (1953).*

|2| S.Pilipović, *Spaces of Generalized Functions whose Elements have Laguerre's Expansion-Sequential Approach (in Serbo Croation) Rev. of Res.Fac. of Sci.-Univ. of Novi Sad, Vol.8, (1978), 33-39.*

|3| S.Pilipović, *A Convolution and Laplace Transformation in L'(in Serbo Croatian), Rev. of Res.Fac. of Sci.-Univ. of Novi Sad, Vol. 9. (1979), 53-58.*

|4| А.Г.Земанян, Интегральные преобразования обобщеньх функции "Наука" , Москва, (1974).

Received by the editors May 5,1983.

REZIME

ANALITIČKA KARAKTERIZACIJA PROSTORA UOPŠTENIH FUNKCIJA KOJE IMAJU LAGEROVU EKSPANZIJU

Prostor uopštenih funkcija čiji elementi imaju Lagerovu ekspanziju u redove su ispitivani u |2| i |3|. U ovom radu smo definisali prostor L pomoću odgovarajuće familije seminormi. Na taj način smo dobili odgovarajuće osobine prostora LG' kao i teoremu o reprezentaciji elemenata iz LG'. Takodje, koristeći konvoluciju u LG' kao i Laplasovu transformaciju dajemo razlaganje u redove važnih uopštenih funkcija iz LG' .