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SOME PROPERTIES OF SYNTAX-DIRECTED TRANSLATIONS

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ABSTRACT

In the paper are defined the following concepts: syntax-directed translation scheme (SDTS), form of SDTS, rule of SDTS, syntax-directed translation (SDT), input grammar of SDTS, input language of SDTS, output grammar of SDTS, output language of SDTS, bi-unique SDT, conditions of composition of SDTs, conditions of equality of SDTs, identical SDT, inverse SDT, simple SDTS, simple SDT and compound SDT, and are proved 13 theorems about some properties (equivalence, composition, commutation, association and unity) of SDTs.

INTRODUCTION

Syntax-directed translation has been known for quite a long time. One of the first who has advocated its use was Irons |1|,|2|. There exists a great variety of definition models for syntax directed translations. Two of them are:

- the syntax-directed translation scheme |3| and
- the pushdown transducer |4|.

Syntax-directed translation forms the basis for many compiler generators and compiler writing systems |5|,|6|,|7|, |8|,|9|.

THE BASIC DEFINITIONS AND THEOREMS

DEFINITION 1. *A syntax-directed translation scheme (abbreviation used: SDTS) is a 5-tuple* $T = (N, \Sigma, \Delta, R, S)$, *where:*

(1) N *is a finite set of nonterminal symbols.*

(2) Σ *is a finite set of terminal symbols (input alphabet).*

(3) Δ *is a finite set of output symbols (output alphabet), and* $N \cap (\Sigma \cup \Delta) = \emptyset$

(4) R *is a finite set of rules of the form:*

$A \to \alpha, \beta, \pi$

where:

$A \in N$,

$\alpha \in (N \cup \Sigma)^*$,

$\beta \in (N \cup \Delta)^*$,

strings α *and* β *contain the same number of nonterminal symbols* m, $\emptyset \leq m \leq \min(|\alpha|, |\beta|)$,

(a) *if* $m \geq 1$ π *is a permutation of indices of the nonterminal symbols of strings* α *and* β, *so that the* i-*th nonterminal symbol of string* α *is equal to the* $\pi(i)$-*th nonterminal symbol of string* β , $1 \leq i, \pi(i) \leq m$. b) *if* $m = \emptyset$ π *is* [ ] *and usually is omitted.*

(5) S *is the start symbol,*

$S \in N$.

DEFINITION 2. *Let* $T = (N, \Sigma, \Delta, R, S)$ *be a SDTS. A form of the SDTS* T *is a triple* $F = (\alpha, \beta, \pi)$, *where* α, β *and* π *satisfy condition (4) of definition 1.*

DEFINITION 3. *Let*

$T = (N, \Sigma, \Delta, R, S)$ *be a SDTS,* T

$F_1 = (\alpha_1, \beta_1, \pi_1)$ *and*

$F_2 = (\alpha_2, \beta_2, \pi_2)$ *be the forms of SDTS* T ,

A *be* i-*th nonterminal symbol of string* α_1 *and* $\pi_1(i)$-*th nonterminal symbol of string* β_1 *and*

$A \to \alpha, \beta, \pi$ *be a rule of the SDTS* T .

Let the string α and β contain m nonterminal symbols

A binary relation of direct derivation of the forms $\Rightarrow_T$ is defined on the set $(N \cup \Sigma)^* \times (N \cup \Delta)^* \times \Pi$, where Π is the set of all the permutations of indices of nonterminal symbols, in the following way:

- the string α_2 is derived directly by the substitution of i-th nonterminal symbol of the string α_1 with the string α,

- the string β_2 is derived directly by the substitution of $\pi_1(i)$-th nonterminal symbol of the string β_1 with the string β,

- the permutation π_2 is obtained by the permutation π_1 and π in the following way:

(a) if $m \geq 1$ then

(1) for every $j < i$

if $\pi_1(j) < \pi_1(i)$ then $\pi_2(j) = \pi_1(j)$

if $\pi_1(j) > \pi_1(i)$ then $\pi_2(j) = \pi_1(j) + m - 1$;

(2) for every $j > i$

if $\pi_1(j) < \pi_1(i)$ then $\pi_2(j + m - 1) = \pi_1(j)$,

if $\pi_1(j) > \pi_1(i)$ then $\pi_2(j + m - 1) = \pi_1(j) + m - 1$;

(3) $\pi_2(i + d - 1) = \pi_1(i) + \pi(d) - 1$

for every d, $1 \leq d \leq m$.

(b) if $m = \emptyset$ then

$\pi = [\ ]$ but π_2 is obtained by the permutation π_1, applying the rules (1) and (2).

THEOREM 1. *The relation of direct derivation is:*

- *reflexive,*
- *transitive and*
- *nonsymmetric.*

P r o o f. By definition, a binary relation R on a set S is reflexive if and only if (abbreviation used: iff) for every $s \in S$ valid sRs.

From here the binary relation $\Rightarrow_T$ on the set $(N \cup \Sigma)^* \times (N \cup \Delta)^* \times \Pi$ is reflexive iff for every form $(\alpha, \beta, \pi) \in (N \cup \Sigma)^* \times (N \cup \Delta)^* \times \Pi$ valid $(\alpha, \beta, \pi) \Rightarrow_T (\alpha, \beta, \pi)$.

Let us suppose the opposite, i.e. that there exists the form $(a, b, p) \in (N \cup \Sigma)^* \times (N \cup \Delta)^* \times \Pi$ for which the binary relation $\Rightarrow_T$ is not valid. As follows from rule $A \to A, A, [1]$ applied on the form (a,b,p) gives the form (a′,b′, p′) thus the minimum one of the following three inequalities:

$$a \neq a'$$
$$b \neq b'$$
$$p \neq p' \ ; \quad p' = p \cdot [1]$$

must be valid, but it is impossible.

In a similar way we can prove two other properties.

The transitive closure of the relation $\Rightarrow_T$ is denoted by $\overset{+}{\Rightarrow}_T$.

The reflexive and transitive closure of the relation $\Rightarrow_T$ is denoted by $\overset{*}{\Rightarrow}_T$.

The k-th product of the relation $\Rightarrow_T$ is denoted by $\overset{k}{\Rightarrow}_T$.

In the cases when a mistake can not be made, it is possible to omit the symbol T of SDTS.

DEFINITION 4. *Syntax-directed translation (abbreviation used: SDT) is the translation defined by SDTS.*

Let $T = (N, \Sigma, \Delta, R, S)$ be a SDTS.
SDT $\tau(T) = \{ (x,y) \mid (S,S) \overset{*}{\Rightarrow}_T (x,y) ;\ x \in \Sigma^*,\ y \in \Delta^* \}$.

DEFINITION 5. *Let* $T = (N, \Sigma, \Delta, R, S)$ *be a SDTS.*
The grammar $G_i = (N, \Sigma, P_i, S)$ *, where* $P_i = \{A \to \alpha \mid A \to \alpha, \beta, \pi$ $\pi, \in R\}$*, is the input grammar of SDTS* T *. The language* $L_i = L(G_i)$ *defined by the grammar* G_i*, that is,* $L_i = \{x \mid S \overset{*}{\Rightarrow} x, x \in \Sigma^*\}$ *is the input language of SDTS* T *, that is, a language from which is translated.*

The grammar $G_o = (N, \Delta, P_o, S)$*, where* $P_o = \{A \to \beta \mid A \to \alpha, \beta, \pi \in$ $\in R\}$*, is the output grammar of SDTS* T*. The language* $L_o = L(G_o)$ *defined by the grammar* G_o*, that is,* $L_o = \{y \mid S \overset{*}{\Rightarrow} y,\ y \in \Delta^*\}$ *is the output language of SDTS* T*, namely a language into which is translated.*

The derivation rules of the grammar G_i are in the form:

$$A \to \alpha; \quad A \in N, \quad \alpha \in (N \cup \Sigma)^* ,$$

so the grammar G_i is contex-free. Consequently, the language L_i is contex-free.

The derivation rules of the grammar G_o are in the form:

$$A \to \beta; \quad A \in N, \ \beta \in (N \cup \Delta)^* ,$$

so the grammar G_o is contex-free. Consequently, the language L_o is the contex-free.

THEOREM 2. *The domain of defining* $X \subseteq \Sigma^*$ *and the set of value* $Y \subseteq \Delta^*$ *of the SDT* $\tau(T)$ *are equal, respectively, to the input and the output language of the SDTS* T .

P r o o f. Let $T = (N, \Sigma, \Delta, R, S)$ be an SDTS. The SDTD T defines the SDT $\tau(T)$:

$$\tau(T) = \{(x,y) \mid (S,S) \overset{*}{\underset{T}{\Longrightarrow}} (x,y), \quad x \in \Sigma^* , \quad y \in \Delta^*\} .$$

The domain of defining the SDT $\tau(T)$ is the set X:

$$X = \{x \mid S \overset{*}{\underset{G_i}{\Longrightarrow}} x, \quad x \in \Sigma^*\} .$$

The set of values of the SDT $\tau(T)$ is the set Y:

$$Y = \{y \mid S \overset{*}{\underset{G_o}{\Longrightarrow}} y, \quad y \in \Delta^*\} .$$

It follows directly that:

$$X = L_i \quad \text{and}$$
$$Y = L_o$$

as we wanted to prove.

COROLLARY. *Let* X *and* Y *be some languages If* min(tip(X), tip(Y)) ≥ 2 *then, in a general case, there exist a SDT* $\tau_1 = \tau(T_1)$ *and a SDT* $\tau_2 = \tau(T_2)$, *so it is valid:*

$$\tau_1 : X \to Y$$
$$\tau_2 : Y \to X ,$$

besides, there exist τ_1 *and* τ_2 *but only in special cases.*

Speaking differently the theory of SDT is valid in a general case for contex-free and regular languages.

IMPORTANT PROPERTIES OF SDT

THEOREM 3. *Let* $\tau_1 = \tau(T_1)$ *and* $\tau_2 = \tau(T_2)$ *be the SDT, respectively defined by the SDTS* T_1 *and* T_2*:*

$$T_1 = (N_1, \Sigma, \Delta, R_1, S_1)$$

$$T_2 = (N_2, \Sigma, \Delta, R_2, S_2)$$

If:

$$L_i^{(1)} = L_i^{(2)} = L_i \subseteq \Sigma^* \quad and$$

$$\tau_1(x) = \tau_2(x)$$

for every word x *from the domain* L_i*, then it is:*

$$L_o^{(1)} = L_o^{(2)} = L_o \subseteq \Delta^* .$$

P r o o f. By definition, we have:

$$\tau_1 = \tau(T_1) = \{(x,y) \mid (S_1, S_2) \underset{T_1}{\overset{*}{\Longrightarrow}} (x,y),\ x \in L_i,\ \ y \in L_o^{(1)}\}$$

and

$$\tau_2 = \tau(T_2) = \{(x,y) \mid (S_2, S_2) \underset{T_2}{\overset{*}{\Longrightarrow}} (x,z),\ x \in L_i,\ z \in L_o^{(2)}\} .$$

Let us have:

$$\tau_1(x) = y \in L_o^{(1)} \subseteq \Delta^*$$

$$\tau_2(x) = z \in L_o^{(2)} \subseteq \Delta^*$$

Since:

$$\tau_1(x) = \tau_2(x)$$

it follows:

$$y = z.$$

As, it is valid for every word $x \in L_i$, and it follows immediately:

$$L_o^{(1)} = L_o^{(2)} .$$

THEOREM 4. *In a general case, the problem of equivalence of two SDT is undecidable.*

P r o o f. Let $\tau_1 = \tau(T_1)$ and $\tau_2 = \tau(T_2)$ be two SDT, respectively, defined by the SDTS T_1 and T_2:

$$T_1 = (N_1,\Sigma,\Delta,R_1,S_1)$$

and

$$T_2 = (N_2,\Sigma,\Delta,R_2,S_2)$$

Further, let:

$$L_i^{(1)},\ L_o^{(1)},\ L_i^{(2)} \text{ and } L_o^{(2)}$$

be input and output language, that are the domains and the ranges of SDT τ_1 and τ_2.

The mentioned languages are contex-free. The problem of equivalence of two SDT can be reduced to the problem of equivalence:

$$L_i^{(1)} = L_i^{(2)}$$

and

$$L_o^{(1)} = L_o^{(2)}$$

of two pairs of contex-free languages, which are undecidable in a general case, as it is well-known.

DEFINITION 6. *Let τ be a SDT. τ is a bi-unique translation iff the following relation of equivalence is valid:*

$$\forall(x_1,x_2)(x_1 \cdot L_i,\ x_2 \cdot L_i)\tau(x_1) = \tau(x_2) \Longleftrightarrow x_1 = x_2 .$$

THEOREM 5. *Let $\tau = \tau(T)$ be a SDT defined by the SDTS T:*

$$T = (N,\Sigma,\Delta,R,S)$$

τ is a bi-unique translation iff for every rules of the SDTS T of the form:

$$A \to \alpha_1,\beta_1,\pi_1$$

and

$$A \to \alpha_2,\beta_2,\pi_2$$

is valid in the following equivalence relation:

$$\alpha_1 = \alpha_2 \Longleftrightarrow \beta_1 = \beta_2 .$$

π_1 can be equal to π_2, but it is not necessary.

P r o o f. Let τ be a bi-unigue translation. Let:

$$(S,S,[1]) = F_o \Longrightarrow F_1 \Longrightarrow \dots \Longrightarrow F_n = (x_1,\tau(x_1),[\ ])$$

also be the derivation and translation of the word x_1.

Let us suppose, that by using the rule $A \to \alpha_1, \beta_1, \pi_1$ the form F_{i+1} is directly derived from the form F_i, namely:

$$F_i = (a_i, b_i, p_i) \Longrightarrow (a_{i+1}, b_{i+1}, p_{i+1}) = F_{i+1}$$

and by using the rule $A \to \alpha_2, \beta_2, \pi_2$ the form F'_{i+1} is directly derived from the form F_i, namely:

$$F_i = (a_i, b_i, p_i) \Longrightarrow (a'_{i+1}, b'_{i+1}, p'_{i+1}) = F'_{i+1}.$$

If $\alpha_1 = \alpha_2$ and $\beta_1 \neq \beta_2$, it follows that $b_i \neq b_{i+1}$, so as the result of the derivation and translation, $(x_1, \tau(x_1)'$, $[\,])$ is obtained, where $\tau(x_1) \neq \tau(x_1)'$. And that is impossible, since τ is a bi-unique translation. So it results that if $\alpha_1 = \alpha_2$ then $\beta_1 = \beta_2$.

Analogously, for the assumption $\alpha_1 \neq \alpha_2$, $\beta_1 \neq \beta_2$ we obtain the contradiction too, hence we exclude this assumption.

DEFINITION 7. *Let* $\tau_i = \tau(T_i)$, $i=1,\dots,n$ *be an SDT defined by the SDTS* $T_i = (N_i, \Sigma_i, \Delta_i, R_i, S_i)$, $i=1,\dots,n$.

The conditions:

- $N_i = N_{i+1} = N$, $i=1,\dots,n-1$
- $\Delta_i = \Sigma_{i+1}$, $i=1,\dots,n-1$
- *for every rule* $A \to \beta_{i-1}, \beta_i, \pi_i \in R_i$ *there exists the rule* $A \to \beta_i, \beta_{i+1}, \pi_{i+1} \in R_{i+1}$, $i=2,\dots,n-1$
- $S_i = S_{i+1} = S$, $i=1,\dots,n-1$

are called the conditions of composition SDT τ_i, $i=1,\dots,n$.

THEOREM 6. *The composition* $\tau = \tau_1 \cdot \ldots \cdot \tau_n$ *of the SDT* τ_i, $i=1,\dots,n$, *which satisfies the conditions of the composition is SDT.*

P r o o f. First of all, let us prove that the composition $\tau = \tau_1 \cdot \tau_2$ of SDT τ_1 and τ_2 is SDT satisfying the conditions of the composition.

In fact, by using the definition of SDTS, SDT and the conditions of the composition of SDT, we obtain:

$$T_1 = (N, \Sigma_1, \Delta, R_1, S)$$

$$T_2 = (N, \Delta, \Delta_2, R_2, S)$$

$$\tau_1 = \tau(T_1) = \{(x,y) \mid (S,S) \overset{*}{\underset{T_1}{\Longrightarrow}} (x,y),\ x \in \Sigma_1^*,\ y \in \Delta^*\}$$

$$\tau_2 = \tau(T_2) = \{(y,z) \mid (S,S) \overset{*}{\underset{T_2}{\Longrightarrow}} (y,z),\ y \in \Delta^*,\ z \in \Delta_2^*\} .$$

Let be:

$$T = (N, \Sigma_1, \Delta_2, R, S),$$

where

$$R = \{A \to \alpha, \gamma, \pi \mid A \to \alpha, \beta, \pi_1 \in R_1,\ A \to \beta, \gamma, \pi_2 \in R_2,\ \ \pi = \pi_1 \cdot \pi_2\}$$

and

$$\tau = \tau(T) = \{(x,z) \mid (S,S) \overset{*}{\underset{T}{\Longrightarrow}} (x,z),\ x \in \Sigma_1^*,\ z \in \Delta_2^*\} .$$

T is the SDTS of τ. The SDT τ is the composition of SDT τ_1 and τ_2.

In fact, let:

$x \in \Sigma_1^*$

$y = \tau_1(x) \in \Delta_1^*$ and

$z = \tau_2(y) \in \Delta_2^*$.

$(x,y) \in \tau_1$ is obtained by $y = \tau_1(x)$, that means that there exists the derivation:

$$(S,S) = F_o \Longrightarrow F_1 \Longrightarrow \ \dots \ \Longrightarrow F_i = (\alpha_i, \beta_i, p_i) \Longrightarrow \dots \Longrightarrow F_n = (x,y).$$

Analogously, $(y,z) \in \tau_2$ is obtained by $z = \tau_2(y)$, and that also means that there exists the derivation:

$$(S,S) = G_o = \ G_1 \Longrightarrow \dots \Longrightarrow G_i = (\beta_i, \gamma_i, q_i) \Longrightarrow \dots \Longrightarrow G_n = (y,z).$$

So, it follows from here that exists the following derivation:

$$(S,S) = H_o \Longrightarrow H_1 \Longrightarrow \dots \Longrightarrow H_i = (\alpha_i, \gamma_i, r_i) \Rightarrow \dots \Longrightarrow H_n = (x,z)$$

which signify that $(x,z) \in \tau = \tau_1 \cdot \tau_2$, and that $z = \tau(x) = \tau_2(\tau_1(x)) =$ $= (\tau_1 \cdot \tau_2)(x)$ and τ is SDT.

According to the mathematical induction there follows directly the proof of the theorem for n SDT.

DEFINITION 8. *Let* $\tau_i = \tau(T_i)$, $i=1,\dots,n$ *be an SDT defined by the SDTS* $T_i = (N, \Sigma_i, \Delta_i, R_i, S)$, $i=1,\dots,n$.

The conditions:

$$\Sigma_1 = \dots \Sigma_n = \Delta_1 = \dots = \Delta_n$$

are called the conditions of equality of SDT τ_i, $i=1,\dots,n$.

The SDTSs which satisfy the conditions of equality are of the form:

$$T_i = (N, \Sigma, \Delta, R_i, S), \ i=1,\dots,n.$$

THEOREM 7. *Let* $\tau_1 = \tau(T_1)$ *and* $\tau_2 = \tau(T_2)$ *be the SDTs which satisfy the conditions of composition and equality.*

In a general case the law of commutation is not valied for SDTs, i.e. $\tau_1 \cdot \tau_2 \neq \tau_2 \cdot \tau_1$.

P r o o f. Let, us suppose the opposite and then prove the claim of the theorem by quoting the contra-example.

Let:

T_1 = ({<bb>,<bc>},{0,1},{0,1},R_1<bb>)

where

R_1 = {<bb> → <bc>,<bc>
<bb> → <bb><bc>,<bb><bc>
<bc> → 0,0
<bc> → 1,0}

be an SDTS.

$$\tau_1 = \tau(T_1) = \{(x,y) \mid x \in \{0,1\}^+,\ y \in \{0\}^+,\ |x| = |y|\}$$

is the SDT defined by the SDTS T_1.

Also, let:

T_2 = ({<bb>,<bc>},{0,1},{0,1},R_2,<bb>)

where:

R_2 = {<bb> → <bc>, <bc>
<bb> → <bb><bc>,<bb><bc>
<bc> → 0,1
<bc> → 1,1}

be another SDTS.

$\tau_2 = \tau(T_2) = \{(x,y) \mid x \in \{0,1\}^+,\ y \in \{1\}^+,\ |x| = |y|\}$

is the SDT defined by the SDTS T_2.

For every x, $x \in \{0,1\}^+$

$$(\tau_1 \cdot \tau_2)(x) = \tau_2(\tau_1(x)) = \tau_2(y) = z$$

$y \in \{0\}^+$, $|x| = |y|$ and
$z \in \{1\}^+$, $|y| = |z|$.

$$(\tau_2 \cdot \tau_1)(x) = \tau_1(\tau_2(x)) = \tau_1(z) = y$$

$z \in \{1\}^+$, $|x| = |z|$ and
$y \in \{0\}^+$, $|z| = |y|$.

Since $z \neq y$, it follows that $\tau_1 \cdot \tau_2 \neq \tau_2 \cdot \tau_1$.

THEOREM 8. *Let $\tau_1 = \tau(T_1)$, $\tau_2 = \tau(T_2)$ and $\tau_3 = \tau(T_3)$ be the SDTs which satisfy the conditions of the composition.*

In general case the law of association is validated for the SDT, i.e. $(\tau_1 \cdot \tau_2) \cdot \tau_3 = \tau_1 \cdot (\tau_2 \cdot \tau_3)$.

P r o o f. Let:

$T_1 = (N, \Sigma_1, \Delta_1, R_1, S)$,

$T_2 = (N, \Delta_1, \Delta_2, R_2, S)$ and

$T_3 = (N, \Delta_2, \Delta_3, R_3, S)$

be the corresponding SDTSs.

For every derivation and translation rule:

$A \to \alpha, \beta, \pi_1 \in R$

there are derivation and translation rules:

$A \to \beta, \gamma, \pi_2 \in R_2$ and

$A \to \gamma, \delta, \pi_3 \in R_3$.

Furthermore, let:

(1) $\tau_1(x) = y$

(2) $\tau_2(y) = z$

(3) $\tau_3(z) = u$.

From (1) it follows that there is the derivation:

(1') $(S,S) = F_0 \Rightarrow \ldots \Rightarrow (\alpha_i, \beta_i) = F_i \Rightarrow \ldots \Rightarrow (x,y)$.

Analogously, from (2) and (3) it follows:

(2´) $(S,S) = G_o \Rightarrow \ldots \Rightarrow (\beta_i,\gamma_i) = G_i \Rightarrow \ldots \Rightarrow (y,z)$

and

(3´) $(S,S) = H_o \Rightarrow \ldots \Rightarrow (\gamma_i,\delta_i) = H_i \Rightarrow \ldots \Rightarrow (z,u)$.

From (1´) and (2´) it follows:

(12) $(S,S) \Rightarrow \ldots \Rightarrow (\alpha_i,\gamma_i) \Rightarrow \ldots \Rightarrow (x,z)$

and from (2´) and (3´) it follows:

(23) $(S,S) \Rightarrow \ldots \Rightarrow (\beta_i,\delta_i) \Rightarrow \ldots \Rightarrow (y,u)$.

Further that means that

$$(\tau_1 \cdot \tau_2)(x) = z$$

and

$$(\tau_2 \cdot \tau_3)(y) = u$$

so:

$$((\tau_1 \cdot \tau_2) \cdot \tau_3)(x) = \tau_3((\tau_1 \cdot \tau_2)(x)) = \tau_3(z) = u$$

$$(\tau_1 \cdot (\tau_2 \cdot \tau_3))(x) = (\tau_2 \cdot \tau_3)(\tau_1(x)) = (\tau_2 \cdot \tau_3)(y) = u$$

and finally:

$$((\tau_1 \cdot \tau_2) \cdot \tau_3)(x) = (\tau_1 \cdot (\tau_2 \cdot \tau_3))(x)$$

and this is the claim of the theorem.

DEFINITION 9. *SDT τ_I which is defined by the SDTS T_I :*

$$T_I = (N,\Sigma,\Sigma,R_I,S)$$

where:

$R_I = \{A \to \alpha,\alpha,\pi_I\}$,

$A \in N$,

$\alpha \in (N \cup \Sigma)^*$

$\pi_I = [1,\ldots,m]$, *where* m *is the number of the nonterminal symbols of the string* α *is an identical translation.*

DEFINITION 10. *Let* τ *and* θ *be the SDTs. If:*

$$\tau \cdot \theta = \tau_I \quad \text{and} \quad \theta \cdot \tau = \tau_I$$

then the translation θ *is the inverse translation of the translation* τ .

The inverse translation is denoted by the same letter and upper index -1, i.e. $\theta = \tau^{-1}$.

$\tau \cdot \tau^{-1}$ is the identical translation of the language Σ^*.

$\tau^{-1} \cdot \tau$ is the identical translation of the language Δ^*.

THEOREM 9. *Let* $T = (N,\Sigma,\Delta,R,S)$ *be an SDTS of a bi-unique translation*

$$\tau = \tau(T)$$

$$\tau = \{(x,y) \mid (S,S) \overset{*}{\underset{T}{\Rightarrow}} (x,y),\ x \in \Sigma^*,\ y \in \Delta^*\} .$$

$T^{-1} = (N,\Delta,\Sigma,R^{-1},S)$ *is the SDTS of the SDT*

$$\tau^{-1} = \tau(T^{-1})$$

$$\tau^{-1} = \{(y,x) \mid (S,S) \overset{*}{\underset{T^{-1}}{\Rightarrow}} (y,x),\ y \in \Delta^*,\ x \in \Sigma^*\}$$

where:

$$R^{-1} = \{A \to \beta,\alpha,\Pi^{-1} \mid A \to \alpha,\beta,\Pi \in R,\ \Pi \cdot \Pi^{-1} = \Pi_I\} .$$

P r o o f. The translation τ^{-1} is the inverse translation of the translation τ, if the following cases are satisfied:

$$\tau \cdot \tau^{-1} = \tau_I \quad \text{and}$$

$$\tau^{-1} \cdot \tau = \tau_I .$$

Since:

$$T = (N,\Sigma,\Delta,R,S) \quad \text{and}$$

$$\tau(T) = \{(x,y) \mid (S,S) \overset{*}{\underset{T}{\Rightarrow}} (x,y),\ x \in \Sigma^*,\ y \in \Delta^*\}$$

and

$$T^{-1} = (N,\Delta,\Sigma,R^{-1},S) \quad \text{and}$$

$$\tau(T^{-1}) = \{(y,x) \mid (S,S) \overset{*}{\underset{T^{-1}}{\Rightarrow}} (y,x),\ y \in \Delta^*,\ x \in \Sigma^*\}$$

it follows that:

$$T \cdot T^{-1} = (N,\Sigma,\Sigma,R \cdot R^{-1},S) \quad \text{and}$$

$$\tau(T) \cdot \tau(T^{-1}) = \{(x,x) \mid (S,S) \overset{*}{\underset{T \cdot T^{-1}}{\Longrightarrow}} (x,x),\ x \in \Sigma^*\} = \tau(T \cdot T^{-1})$$

and

$$T^{-1} \cdot T = (N,\Delta,\Delta,R^{-1} \cdot R,S) \quad \text{and}$$

$$\tau(T^{-1}) \cdot \tau(T) = \{(y,y) \mid (S,S) \overset{*}{\underset{T^{-1} \cdot T}{\Longrightarrow}} (y,y),\ y \in \Delta^*\} = \tau(T^{-1} \cdot T).$$

Since:

$$R = \{A \to \alpha, \beta, \Pi \mid A \in N,\ \alpha \in (N \cup \Sigma)^*,\ \beta \in (N \cup \Delta)^*\} \quad \text{and}$$

$$R^{-1} = \{A \to \beta, \alpha, \Pi^{-1} \mid A \to \alpha, \beta, \Pi \in R,\ \Pi \cdot \Pi^{-1} = \Pi_I\}$$

it follows that:

$$R \cdot R^{-1} = \{A \to \alpha, \alpha, \Pi_I\} = R_I$$

and

$$R^{-1} \cdot R = \{A \to \beta, \beta, \Pi_I\} = R_I$$

then:

$$T \cdot T^{-1} = (N, \Sigma, \Sigma, R_I, S) = T_I$$

and

$$T^{-1} \cdot T = (N, \Delta, \Delta, R_I, S) = T_I$$

then

$$\tau \cdot \tau^{-1} = \tau(T) \cdot \tau(T^{-1}) = \tau(T \cdot T^{-1}) = \tau(T_I) = \tau_I$$

and

$$\tau^{-1} \cdot \tau = \tau(T^{-1}) \cdot \tau(T) = \tau(T^{-1} \cdot T) = \tau(T_I) = \tau_I ,$$

and that is the claim of the theorem.

THEOREM 10. *Let τ be a bi-unique SDT. The inverse translation τ^{-1} is unique.*

P r o o f. If we suppose the opposite, i.e. let τ_1 and τ_2 be two mutually different inverse translations of the translation τ.

$\tau_1 \cdot \tau$ and $\tau_2 \cdot \tau$ are identical translations of the language Δ^*, i.e. $\tau_1 \cdot \tau(y) = \tau_I(y) = y$, for every $y \in \Delta^*$

$$\tau_2 \cdot \tau(y) = \tau_I(y) = y, \text{ for every } y \in \Delta^*$$

then
$$\tau_1 \cdot \tau(y) = \tau_2 \cdot \tau(y), \text{ for every } y \in \Delta^*$$

$$\tau(\tau_1(y)) = \tau(\tau_2(y)), \text{ for every } y \in \Delta^*.$$

Since τ is a bi-unique SDT, it follows, that:

$$\tau_1(y) = \tau_2(y), \text{ for every } y \in \Delta^*$$

then
$$\tau_1 = \tau_2$$

and that is the opposite to the initial assumption.

It follows, that:

$$\tau_1 = \tau_2 = \tau^{-1} .$$

THEOREM 11. *Let* τ *not be a bi-unique SDT. The SDT* τ^{-1} *is not unique.*

P r o o f. We shall prove the theorem by quotation of a contra-example.

Let:

T = ({<bb>,<bc>},{0,1},{0,1},R,<bb>)

be an SDTS, where:

R = {<bb> → <bc>,<bc>
<bb> → <bb><bc>,<bb><bc>
<bc> → 0,0
<bc> → 1,0} .

$$\tau = \tau(T) = \{(x,y) \mid x \in \{0,1\}^{+},\ y \in \{0\}^{+},\ |x| = |y|\},$$

is the SDT, which is defined by the SDTS T.

T^{-1} = ({<bb>,<bc>},{0,1},{0,1},R^{-1},<bb>)

is the inverse SDTS, where:

R^{-1} ={<bb> → <bc>,<bc>
<bb> → <bb><bc>,<bb><bc>
<bc> → 0,0
<bc> → 0,1} .

The following rules:

<bc> → 0,0 and
<bc> → 0,1

are the elements of the set R^{-1} and they cause the inuniqueness.

For example:

$\tau(01) = 00$, but

$$\tau^{-1}(00) = \begin{cases} 00 \\ 01 \\ 10 \\ 11 \end{cases} .$$

DEFINITION 11. *SDTS* $T = (N, \Sigma, \Delta, R, S)$ *is a simple SDTS if in every rule* $A \to \alpha, \beta$ *of the set* R, *the nonterminal symbols of the strings* α *and* β *follow in the same order.*

DEFINITION 12. *SDT* τ *is simple iff it is defined by a simple SDTS* $T(\tau)$.

THEOREM 12. *The composition of:*

- *a simple and compound,*
- *a compound and simple*

SDT is a compound SDT.

The composition of a finite number simple SDT is a simple SDT.

The composition of a compound and compound SDT is a compound SDT if the given translations are not mutually inverse. In the opposite case the composition is a simple SDT.

P r o o f. Let $T=(N,\Sigma,\Delta,R,S)$ be an SDTS, and $\tau=\tau(T)$ the corresponding SDT.

If every rule of SDTS T has the form:

$$A\to\alpha,\beta,\Pi_I$$

then SDTS T and SDT τ are simple.

Let:

$$A\to\alpha,\beta,\Pi_1 \text{ and}$$

$$A\to\beta,\gamma,\Pi_2$$

be, in that order, the rules of SDTS T_1 and SDTS T_2.

The composition of the first and second rule is the following rule:

$$A\to\alpha,\gamma,\Pi=\Pi_1\cdot\Pi_2$$

of the SDTS $T=T_1\cdot T_2$

$$\Pi=\Pi_I\cdot\Pi=\Pi\cdot\Pi_I,\ \Pi\neq\Pi_I$$

$$\Pi_I=\Pi_I\cdot\Pi_I$$

$$\Pi_I\neq\Pi=\Pi_1\cdot\Pi_2,\ \Pi_1\neq\Pi_I,\ \Pi_2\neq\Pi_I$$

$$\Pi_I=\Pi=\Pi_1\cdot\Pi_2,\ \Pi_1\neq\Pi_I,\ \Pi_2\neq\Pi_I,\ \Pi_1=\Pi_2^{-1},\ \Pi_2=\Pi_1^{-1}$$

which follows the truth of the claim of the theorem.

THEOREM 13. *Let τ be a simple SDT. The inverse translation of a simple SDT is a simple SDT.*

P r o o f. Let $T=(N,\Sigma,\Delta,R,S)$ be the scheme of a simple SDT $\tau=\tau(T)$. The rules of the SDTS T have the following form:

$$A\to\alpha,\beta,\Pi_I\ .$$

The scheme T^{-1} of the inverse SDT $\tau^{-1} = \tau(T^{-1})$ has the following form:

$$T^{-1} = (N, \Delta, \Sigma, R^{-1}, S)$$

where is:

$$R^{-1} = \{A \to \beta, \alpha, \Pi^{-1} \mid A \to \alpha, \beta, \Pi_I \in R,\ \Pi_I \cdot \Pi^{-1} = \Pi_I\}.$$

Since:

$\Pi = \Pi_I \cdot \Pi$ for every permutation Π, it follows, that:

$$\Pi_I \cdot \Pi^{-1} = \Pi^{-1} = \Pi_I .$$

The rules of the set R^{-1} have the following form:

$$A \to \beta, \alpha, \Pi_I$$

then the scheme T^{-1} and the translation τ^{-1} are simple.

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REZIME

NEKE OSOBINE SINTAKSNO-VODJENIH PREVODJENJA

U radu su definisani sledeći pojmovi: šema SVP (sintaksno-vodjenog prevodjenja), forma, SVP, obostrano-jednoznačno SVP, indentičko SVP, inverzno SVP, prosto SVP, složeno SVP kao i uslovi kompozicije i jednakosti SVP.

Dokazano je više teorema u vezi osobina SVP:

- Oblast definisanosti i skup vrednosti SVP su KS-jezici.
- Problem ekvivalentnosti dva SVP je nerešiv.
- Kompozicija SVP, koja zadovoljavaju uslove kompozicije je SVP.
- Ne važi zakon komutacije za SVP.
- Važi zakon asocijacije za SVP, koja zadovoljavaju uslove kompozicije.
- Inverzno prevodjenje, obostrano-jednoznačnog SVP je jedinstveno.
- Inverzno prevodjenje SVP je u opštem slučaju višeznačno.
- Kompozicija prostog i složenog (odnosno složenog i prostog) SVP je složeno SVP.
- Kompozicija konačno mnogo prostih SVP je prosto SVP.
- Kompozicija složenog i složenog SVP je složeno SVP ako prevodjenja nisu medjusobno inverzna - u suprotnom je prosto SVP.
- Inverzno prevodjenje prostog SVP je prosto SVP.