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THE APPROXIMATE SOLUTION OF A DIFFERENTIAL EQUATION IN MANY STEPS

Djurdjica Takači

Prirodno-matematički fakultet. Institut za matematiku
21000 Novi Sad, ul.dr Ilije Djuričića br. 4, Jugoslavija

ABSTRACT

In this paper we construct the approximate solution of a certain linear partial differential equation with constant coefficients, using the field of Mikusiński operators M, in steps, on the interval $[0,T]$. We also give the error of approximation.

In papers [1] and [2] the following linear partial differential equation with constant coefficients was observed:

$$(1)\qquad \sum_{\mu=0}^{m}\sum_{\nu=0}^{n} \alpha_{\mu,\nu} \frac{\partial^{\mu+\nu} x(\lambda,t)}{\partial\lambda^{\mu}\,\partial t^{\nu}} = \phi(\lambda,t),\ \lambda_1 \le \lambda \le \lambda_2,\ 0 \le t < \infty .$$

Using the field of Mikusiński operators M the approximate solution of equation (1) was constructed in [1] and the error of approximation was given in [2] on the interval $[0,T]$. This error of approximation increases rather fast by enlarging T.

In this paper we shall find the approximate solution of (1) for $n=1$ and $\phi(\lambda,t)=0$, i.e.

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$$(2)\qquad \sum_{\mu=o}^{m}\sum_{\nu=o}^{n}\alpha_{\mu,\nu}\frac{\partial^{\mu+\nu}x(\lambda,t)}{\partial\lambda^{\mu}\,\partial t^{\nu}}=0$$

with the following conditions:

$$(3)\qquad \frac{\partial^{\mu}x(\lambda,0)}{\partial\lambda^{\mu}}=0,\qquad \lambda>0,\ \mu=0,\dots,m$$

$$(4)\qquad \frac{\partial^{\mu}x(0,t)}{\partial\lambda^{\mu}}=0,\qquad t>0,\ \mu=0,\dots,m-2$$

$$\frac{\partial^{m-1}x(0,t)}{\partial\lambda^{m-1}}=1,\qquad t>0$$

on the interval $[0,T]$. We shall divide this interval into two intervals ($[0,T_1]$ and $[T_1,T]$)and seek the approximate solution succesively. We shall prove that this enables us to obtain a better estimation of the error. At the same time we can construct the approximate solution on an arbitrary interval $[T_1,T]$, $0<T_1<T$. The method which we shall use can be applied in many steps in the same way as in two steps (dividing the interval $[0,T]$ into intervals $[0,T_1]\,[T_1,T_2]\dots[T_{n-1},T_n]$ where $T_n=T$).

In the field M the differential equation

$$(5)\qquad \sum_{\mu=o}^{m}\sum_{\nu=o}^{1}\alpha_{\mu,\nu}s^{\nu}x^{(\mu)}(\lambda)=f(\lambda)$$

where

$$(5')\qquad f(\lambda)=\sum_{\mu=o}^{m}\alpha_{\mu,1}\frac{\partial^{\mu}x(\lambda,t)}{\partial\lambda^{\mu}}\Big|_{t=o}$$

corresponds to equation (2). Conditions (3) imply $f(\lambda)=0$. The exact solution of equation (5) with (5') has the form ([1]):

$$(6)\qquad x(\lambda)=\sum_{j=1}^{m}\ell b_j\exp(\lambda\omega_j)\quad\text{where}\quad \omega_j=\sum_{i=o}^{\infty}c_{i,j}\,\ell^{\frac{i-p}{q}}$$

while the approximate one is ([1]):

$$(7)\qquad \tilde{x}(\lambda)=\sum_{s=1}^{m}\ell b_j\exp(\lambda\tilde{\omega}_j)\quad\text{where}\quad \tilde{\omega}_j=\sum_{i=o}^{i_o}c_{i,j}\,\ell^{\frac{i-p}{q}};$$

We shall suppose that $\tilde{b}_j\in C$, where $\{\tilde{b}_j\}=\ell b_j$.

In (6) and (7) an exponential operator $e^{\lambda\omega}$ appears . The conditions for the existence of that operator, together with its character, are given in [1]. Now, in this paper we suppose that $p/q < -1$, so then $(e^{\lambda\omega}-I)$ belongs to $\mathcal{C}$. $\mathcal{C}$ is the ring of continuous complex valued functions defined over $[0,\infty)$.

After a change of variables $t = \tau + T_1$, equation (2) becomes:

$$\text{(8)} \qquad \sum_{\mu=0}^{m} \sum_{\nu=0}^{1} \alpha_{\mu,\nu} \frac{\partial^{\mu+\nu} x(\lambda,\tau+T_1)}{\partial\lambda^{\mu}\,\partial\tau^{\nu}} = 0 .$$

Let us denote $x(\lambda,\tau+T_1) = y(\lambda,\tau)$ and observing that:

$$\text{(9)} \qquad e^{-T_1 s}\{y(\lambda,\tau)\} = \left\{ \begin{array}{ll} y(\lambda,\tau-T_1), & \tau > T_1 \\ 0, & \tau < T_1 \end{array} \right\} \equiv X(\lambda,\tau)$$

we obtain:

$$\{y(\lambda,\tau)\} = \{x(\lambda,\tau+T_1)\} = e^{T_1 s}\{X(\lambda,\tau)\} = e^{T_1 s} X(\lambda)$$

and

$$\{\frac{\partial x(\lambda,\tau+T_1)}{\partial\tau}\} = s\{x(\lambda,\tau+T_1)\} - x(\lambda,T_1)I = e^{T_1 s} sX(\lambda) - x(\lambda,T_1)I$$

Equation (8) corresponds in field $\mathcal{M}$ to the equation:

$$\text{(10)} \qquad e^{T_1 s} \sum_{\mu=0}^{m} \sum_{\nu=0}^{1} \alpha_{\mu,\nu} s^{\nu} X^{(\mu)}(\lambda) = F_1(\lambda)$$

where

$$\text{(11)} \qquad F_1(\lambda) = \sum_{\mu=0}^{m} \alpha_{\mu,1} \frac{\partial^{\mu} x(\lambda,t)}{\partial\lambda^{\mu}} \Big|_{t=T_1} .$$

The solution of equation (10) with conditions in $\mathcal{M}$:

$$X(0) = X'(0) = \ldots = X^{(m-2)}(0) = 0$$

$$\text{(12)} \qquad X^{(m-1)}(0) = \ell$$

(which follows from (4)) can be formed in the following way ([3]):

$$\text{(13)} \qquad X(\lambda) = e^{-T_1 s} \int_0^{\lambda} F(\chi) X_h(\lambda-\chi)\, d\chi ,$$

where $F(\chi) = \dfrac{F_1(\chi)}{\ell(\alpha_{m,0}+\alpha_{m,1}s)} = \dfrac{F_1(\chi)}{C}$

and $X_h(\lambda)$ is the solution of the homogeneous part of equation (10). In fact, it has the same form as the solution $x(\lambda)$ of equation (5) given by (6), so we have:

$$X(\lambda) = x(\lambda) + e^{-T_1 s} \int_0^{\lambda} F(\chi)x(\lambda-\chi)d\chi \; . \tag{14}$$

Integrals in (13) and (14) exist because $F(\chi)x(\lambda-\chi)$ is a continuous function.

Let us observe that the solution of equation (2) $x(\lambda,t)$ can be written as $X(\lambda,\tau)$ for $t=\tau+T_1$ and $t\in[T_1,T]$.

Now, let us replace the exact solution $x(\lambda)$ with an approximate one $\tilde{x}(\lambda)$ given by (7); so instead of equation (10) we have:

$$e^{T_1 s} \sum_{\mu=o}^{m} \sum_{\nu=o}^{1} \alpha_{\mu,\nu} s^{\nu} \tilde{X}^{(\mu)}(\lambda) = \tilde{F}_1(\lambda) \tag{15}$$

where

$$\tilde{F}_1(\lambda) = \sum_{\mu=o}^{m} \alpha_{\mu,1} \frac{\partial^{\mu}\tilde{x}(\lambda,t)}{\partial\lambda^{\mu}}\Big|_{t=T_1} \; . \tag{16}$$

The solution of equation (15) with conditions (12) has the from

$$\tilde{X}_1(\lambda) = X_h(\lambda) + e^{-T_1 s} \int_0^{\lambda} \tilde{F}(\chi)X_h(\lambda-\chi)d\chi, \; \tilde{F}(\chi) = \frac{\tilde{F}_1(\chi)}{C} \tag{17}$$

and the approximate solution of (15) is

$$\tilde{X}(\lambda) = \tilde{x}(\lambda) + \tilde{X}_p(\lambda) = \tilde{x}(\lambda) + e^{-T_1 s} \int_0^{\lambda} \tilde{F}(\chi)\tilde{x}(\lambda-\chi)d\chi \tag{18}$$

where $\tilde{x}(\lambda)$ is the approximate solution of the homogeneous part of (15). In fact it has the same form as the approximate solution of equation (5) given by (7).

Let us remark that equation (10) and (15) differ only in their right-hand sides. In the next section we shall prove that the solution of equation (10) depends continuously on the right-hand side. This fact enables us to use $\tilde{X}(\lambda,\tau)$, (where $\{\tilde{X}(\lambda,\tau)\} = \tilde{X}(\lambda)$, $\tilde{X}(\lambda)$ is given by (18)) as the approximate solution of equation (2) on the interval $[T_1,T]$ and to find the error of approximation on $[T_1,T]$ as the difference between $X(\lambda)$ and $\tilde{X}(\lambda)$.

MEASURE OF APPROXIMATION

In this section we use the notion of an absolute value (module) of certain operators from M. If an operator a is defined by a function $a(t), t \geq 0$ from $\mathcal{X}$, then its absolute value $|a| = |\{a(t)\}| = \{|a(t)|\}$ is again a function from $\mathcal{X}$ ([1]. [3]). $\mathcal{X}$ is the ring of locally integrable functions over $[0,\infty)$.

If $g(\lambda,\chi)$ is a continuous operator function, there exists $q \in M$ such that $\{\tilde{g}_1(\lambda,\chi,t)\} = qg(\lambda,\chi)$ and $\tilde{g}_1(\lambda,\chi,t)$ is a continuous function, then:

$$|\int_0^\lambda qg(\lambda,\chi)d\chi| \leq_T \ell \int_0^\lambda G(\lambda,\chi)d\chi$$

where $G(\lambda,\chi) = \max_{0\leq t\leq T} |\tilde{g}_1(\lambda,\chi,t)|$.

The other properties of the absolute value which we are going to use are given in [1].

First, let us estimate the difference between $F(\lambda)$ given by (11) and $\tilde{F}(\lambda)$ given by (16). For that purpose we note that in the field M the μ-th derivative of $x(\lambda)$ given by (6) has the following form:

$$x^{(\mu)}(\lambda) = \sum_{j=1}^{m} \ell b_j \omega_j^{\mu} \exp(\lambda\omega_j). \tag{19}$$

In the same manner we take:

$$\tilde{x}^{(\mu)}(\lambda) = \sum_{j=1}^{m} \ell b_j \tilde{\omega}_j^{\mu} \exp(\lambda\tilde{\omega}_j). \tag{20}$$

We shall also need the following inequalities (for $(p/q) < -1$), [1], [2]:

(21)
$$|\sum_{i=o}^{i_o} c_{i,j} \ell^{(i-p)/q}| \leq_T \ell \sum_{i=o}^{i_o} |c_{i,j}| \frac{T^{(i-p)/q-1}}{\Gamma((i-p)/q)} = \nu_j(T)\ell$$

$$|(\sum_{i=i_o+1}^{\infty} c_{i,j} \ell^{(i-p)/q})^k| \leq (\ell^{(i_o-p+1-q)/q} \rho_j^{i_o+1} \sum_{i=o}^{\infty} M\rho_j^i \ell^{i/q+1})^k \leq_T$$

$$\leq \left(\frac{T^{\frac{i_o+1-p-q}{q}} \cdot \rho^{i_o+1}}{\Gamma\left(\frac{i_o+1-p}{q}\right)} M \ell \sum_{i=o}^{\infty} \rho_j \frac{T^{i/q}}{\Gamma(i/q)+1} \right)^k \leq_T \gamma_j^k(T) \frac{T^{k-1}}{(k-1)!} \ell ,$$

$$|\ell \exp(\lambda \sum_{i=o}^{i_o} c_{i,j} \ell^{(i-p)/q})| \leq \ell \sum_{k=o}^{\infty} \frac{\lambda^k}{k!} |\sum_{i=o}^{i_o} c_{i,j} \ell^{(i-p)/q}|^k \leq_T$$

$$\leq_T \ell \sum_{k=o}^{\infty} \frac{\lambda^k}{k!} \nu_j^k(T) \frac{T^k}{k!} = N_j(\lambda,T)\ell .$$

$$|\exp(\lambda \sum_{i=i_o+1}^{\infty} c_{i,j} \ell^{(i-p)/q}) - I| \leq_T \lambda\gamma_j(T) \sum_{k=o}^{\infty} \frac{\lambda^k}{k!} \gamma_j^k(T) \frac{T^k}{k!} \ell =$$

$$= {}_T G_j(\lambda,T)\ell .$$

Since the solution $x(\lambda,t)$ and the approximate one $\tilde{x}(\lambda,t)$ together with their partial derivatives by λ up to the order m are continuous on the set $\{(\lambda,t) | \ 0 \leq \lambda \leq \lambda_o, \ 0 \leq t \leq T\}$, there exist numbers $R_\mu(\lambda,T)$ (which we shall determine later in Lemma 1.) so that:

(22) $$|x^{(\mu)}(\lambda) - \tilde{x}^{(\mu)}(\lambda)| \leq_T R_\mu(\lambda,T)\ell \quad , \quad \mu=0,\ldots,m .$$

This implies:

$$\left| \frac{\partial^\mu x(\lambda,t)}{\partial\lambda^\mu} \Big|_{t=T} - \frac{\partial^\mu \tilde{x}(\lambda,t)}{\partial\lambda^\mu} \Big|_{t=T} \right| \leq R_\mu(\lambda,T).$$

Using (11), (16) and (22) we have:

(23) $$|F_1(\lambda)-\tilde{F}_1(\lambda)| \leq \sum_{\mu=o}^{m} |\alpha_{\mu,1}| R_\mu(\lambda,T) = \varepsilon(\lambda,T)$$

From (2) and notation (21) follows (if $(p/q) < -1$):

(24) $$|x(\lambda)-\tilde{x}(\lambda)| \leq_T \ell \sum_{j=1}^{m} |b_j| N_j(\lambda,T) G_j(\lambda,T) T = {}_T R_o(\lambda,T)\ell$$

LEMMA 1. *The error of approximation of the μ-th derivative is:*

(25) $$|x^{(\mu)}(\lambda)-\tilde{x}^{(\mu)}(\lambda)| \leq_T \sum_{j=1}^{m} |b_j| N_j(\lambda,T) (\nu_j^\mu(T) G_j(\lambda,T) \frac{T^\mu}{\mu!} \ell +$$
$$+ \sum_{r=1}^{\mu} \binom{\mu}{r} \nu_j^{\mu-r}(T) \gamma_j^r(T) (\sum_{k=o}^{\infty} \frac{\lambda^k}{k!} \gamma_j^k(T) \frac{T^k}{k!}) \ell^{\mu-1}) \ell$$

P r o o f. In view of (19) and (20) we can write:

$$|x^{(\mu)}(\lambda)-\tilde{x}^{(\mu)}(\lambda)| = \sum_{j=1}^{m} |b_j||\omega_j^{\mu}\ell\exp(\lambda\omega_j)-\tilde{\omega}_j^{\mu}\ell\exp(\lambda\tilde{\omega}_j)| \le$$

$$\le \sum_{j=1}^{m}|b_j||\ell\exp(\lambda\tilde{\omega}_j)||(\tilde{\omega}_j + \sum_{i=i_o+1}^{\infty} c_{i,j}\ell^{(i-p)/q})^{\mu}\exp(\lambda \sum_{i=i_o+1}^{\infty} c_{i,j}$$

$$\cdot\ \ell^{(i-p)/q})-\tilde{\omega}_j^{\mu}| \le \sum_{j=1}^{m} |b_j||\ell\exp(\lambda\tilde{\omega}_j)|(|\tilde{\omega}_j^{\mu}|\ \cdot$$

$$\cdot\ |\exp(\lambda \sum_{i=i_o+1}^{\infty} c_{i,j}\ell^{(i-p)/q}) - I| +$$

$$+ |(\sum_{r=1}^{\mu} \binom{\mu}{r}\tilde{\omega}_j^{\mu-r}(\sum_{i=i_o+1}^{\infty} c_{i,j}\ell^{(i-p)/q})^{r})\exp(\lambda \sum_{i=i_o+1}^{\infty} c_{i,j}\ell^{(i-p)/q})|$$

$$\le T\sum_{j=1}^{m} |b_j|N_j(\lambda,T)(\nu_j^{\mu}(T)G_j(\lambda,T)\frac{T^{\mu}}{\mu!}\ell +$$

$$+ (\sum_{r=1}^{\mu} \binom{\mu}{r}\nu_j^{\mu-r}(T)\gamma_j^{r}(T))(\sum_{k=o}^{\infty}\frac{\lambda^k}{k!}\gamma_j^{k}(T)\frac{T^k}{k!})\ell^{\mu-1})\ell \equiv {}_T R_{\mu}(\lambda,T)\ell\ .$$

LEMMA 2. *The solution* $X(\lambda)$ *given by (13) of equation (10) with conditions (12) depends continuously on* $F(\lambda)$.

P r o o f. If $\tilde{X}_1(\lambda)$, given by (17), is the solution of equation (15) with conditions (12) (equations (10) and (15) differ only in their right-hand sides) then using relations (21) and (23) we have:

$$|X(\lambda)-\tilde{X}_1(\lambda)| = |\int_0^{\lambda} (F(\chi)-\tilde{F}(\chi))e^{-T_1 s}x(\lambda-X)dX| \le$$

$$\le {}_T\lambda\varepsilon(\lambda,T_1)\bar{N}_j(\lambda,T-T_1)(G_j(\lambda,T-T_1)\ell + \ell).$$

$$\text{where}\quad |\frac{\ell}{C}\exp(\lambda\sum_{i=0}^{i_o} C_{i,j}\ell^{\frac{\ell-p}{2}})| \le {}_T\bar{N}_j(\lambda,T)$$

Lemma 2 enables us to use $\tilde{X}(\lambda)$ from relation (18) as the approximate solution in the field M of equation (10).

PROPOSITION. *If* $X(\lambda)$ *is the solution of equation (10) given by (13) and* $\tilde{X}(\lambda)$ *is the approximate one of equation (15) given by (18) then the error of approximation is:*

$$(26) \quad |X(\lambda)-\tilde{X}(\lambda)| \leq_T \sum_{j=1}^{m} |b_j| \lambda \bar{N}_j(\lambda, T-T_1) \ell (\varepsilon(\lambda, T_1) + G_1(\lambda, T-T_1)(T-T_1)(\varepsilon(\lambda,T_1)+\tilde{F}(\lambda))) + R_o(\lambda, T-T_1)\ell$$

P r o o f. Using (13) and (18) we can write:

$$|X_p(\lambda)-\tilde{X}_p(\lambda)| = |\int_0^\lambda e^{-T_1 s}(F(\chi)x(\lambda-\chi)-\tilde{F}(\chi)\tilde{x}(\lambda-\chi))d\chi| =$$

$$= |\int_0^\lambda e^{-T_1 s}(F(\chi)x(\lambda-\chi)-\tilde{F}(\chi)x(\lambda-\chi)+\tilde{F}(\chi)x(\lambda-\chi)-\tilde{F}(\chi)\tilde{x}(\lambda-\chi))d\chi|$$

$$\leq \int_0^\lambda |(F(\chi)-\tilde{F}(\chi))e^{-T_1 s}\tilde{x}(\lambda-\chi)|d\chi + \int_0^\lambda |(F(\chi)-\tilde{F}(\chi))e^{-T_1 s}(x(\lambda-\chi) - \tilde{x}(\lambda-\chi))|d\chi + \int_0^\lambda |\tilde{F}(\chi)e^{-T_1 s}(x(\lambda-\chi)-\tilde{x}(\lambda-\chi))| \, d\chi .$$

In view of (21),(23),(24) and (25) we have:

$$|X(\lambda)-\tilde{X}(\lambda)| \leq_T \sum_{j=1}^{m} |b_j| \lambda \bar{N}_j(\lambda, T-T_1)(\varepsilon(\lambda, T_1)\ell + G_j(\lambda, T-T_1)(T-T_1)\ell(\varepsilon(\lambda,T_1)+\tilde{F}(\lambda))) + R_o(\lambda, T-T_1)\ell .$$

EXAMPLE. The following example will show the advantage of approximation in two steps compared to the older method from [1], [2].

Let us observe the partial differential equation:

$$(27) \quad \frac{\partial^2 x(\lambda,t)}{\partial\lambda\partial t} - \frac{\partial x(\lambda,t)}{\partial\lambda} - x(\lambda,t) = 0$$

with conditions:

$$(28) \quad \frac{\partial x(\lambda,0)}{\partial\lambda} = 0, \ \lambda > 0 \ ; \quad x(0,t) = 1, \ \ t > 0 .$$

In the field $\mathcal{M}$, equation

(29) $$(s-1)x'(\lambda) - x(\lambda) = 0$$

corresponds to the equation (27) with (28). The solution of equation (29) is:

$$x(\lambda) = \ell\exp(\lambda\omega), \quad \text{where } \omega = \sum_{i=0}^{\infty} \ell^{i+1} \quad \text{and} \quad x'(\lambda) = \ell\omega\exp(\lambda\omega)$$

while the approximate solution of equation (29) is:

$$\tilde{x}(\lambda) = \ell\exp(\lambda\tilde{\omega}), \quad \text{where } \tilde{\omega} = \sum_{i=0}^{i_0} \ell^{i+1} \quad \text{and} \quad \tilde{x}'(\lambda) = \ell\tilde{\omega}\exp(\lambda\tilde{\omega}).$$

After a change of variables $t = \tau + T_1$ equation (27) becomes:

(30) $$\frac{\partial^2 x(\lambda,\tau+T_1)}{\partial\lambda\,\partial\tau} - \frac{\partial x(\lambda,\tau+T_1)}{\partial\lambda} - x(\lambda,\tau+T_1) = 0$$

with conditions:

$$\frac{\partial x(\lambda,\tau+T_1)}{\partial\lambda}\Big|_{\tau=0} = \phi(\lambda), \qquad x(0,\tau) = 1 .$$

In the field $\mathcal{M}$, equation

(31) $$e^{T_1 s}(s-1)X'(\lambda) - X(\lambda) = F_1(\lambda)$$

corresponds to equation (30). The solution of equation (31) is:

(32) $$X(\lambda) = e^{-T_1 s}\int_0^{\lambda} F(X)x(\lambda-X)\,dX + x(\lambda) .$$

If we take $\tilde{\phi}(\lambda) = \dfrac{\partial\tilde{x}(\lambda,\tau+T_1)}{\partial\lambda}\Big|_{\tau=0}$ instead of $\phi(\lambda)$ we get equation:

(33) $$e^{T_1 s}(s-1)\tilde{X}'(\lambda) - \tilde{X}(\lambda) = \tilde{F}_1(\lambda).$$

From Lemma 2 and [1] the approximate solution of equation (33) given by:

(34) $$\tilde{X}(\lambda) = e^{-T_1 s}\int_0^{\lambda} \tilde{F}(X)\tilde{x}(\lambda-X)\,dX + \tilde{x}(\lambda)$$

can be observed as the approximate solution of equation (31) and the function $\tilde{X}(\lambda,\tau)$ $(\tilde{X}(\lambda) = \{\tilde{X}(\lambda,\tau)\})$ as the approximate solution of equation (27) on interval $[T_1,T]$.

Since m=1, $b_j = 1$, $M = \rho = 1$, we can write the entities in (21) without index j. Also, $F_1(\lambda) = \phi(\lambda)$ and $\tilde{F}_1(\lambda) = \tilde{\phi}(\lambda)$ and therefore we can obtain $\varepsilon(\lambda,T)$ using estimation (25) for $\mu = 1$. So we have:

$$(35). \quad |x'(\lambda)-\tilde{x}'(\lambda)| \leq_T N(\lambda,T)(\nu(T)G(\lambda,T)T + \\ + \gamma(T)(\sum_{k=0}^{\infty} \frac{\lambda^k}{k!}\gamma^k(T)\frac{T^k}{k!}))\ell = \varepsilon(\lambda,T)\ell .$$

If the exact solution of equation (31) is given by (32) and the approximate one is given by (34) then the measure of approximation is:

$$|X(\lambda)-\tilde{X}(\lambda)| \leq_T \bar{N}(\lambda,T-T_1)(\varepsilon(\lambda,T_1) + G(\lambda,T-T_1)(T-T_1)(\varepsilon(\lambda,T_1) + \\ + \tilde{\phi}(\lambda))\ell + R_o(\lambda,T-T_1)\ell .$$

In the following table there are two errors of approximation, the first (one step) on interval $[0,T]$ and the other (two steps) on the interval $[T_1,T]$. Let us remark that the error of approximation is smaller when we work in two steps, especially if T is bigger than 1.

T	one step $i_o=5, \lambda=1$	T_1	Two steps $i_o=5, \lambda=1$
0,1	$1\cdot 10^{-10}$		
0,2	$2,77\cdot 10^{-8}$	0,1	$2,\ 1\cdot 10^{-9}$
0,4	$6,17\cdot 10^{-6}$	0,2	$2,56\cdot 10^{-7}$
0,5	$4,08\cdot 10^{-5}$	0,1	$1,02\cdot 10^{-5}$
0,8	$3,80\cdot 10^{-3}$	0,4	$5,75\cdot 10^{-5}$
1	$5,73\cdot 10^{-2}$	0,5	$4,51\cdot 10^{-4}$
1,6	$5,85\cdot 10^{3}$	0,8	$1,14\cdot 10^{-1}$
2	$1,00\cdot 10^{-7}$	1	$5,75\cdot 10^{0}$

It is obvious that this method can be applied in the same way in many steps, by dividing the interval $[0,T]$ into n parts which are not neccessarily equal.

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REZIME

PRIBLIŽNO REŠENJE DIFERENCIJALNE JEDNAČINE U KORACIMA

U ovom radu se konstruiše približno rešenje diferencijalne jednačine:

$$\sum_{\mu=0}^{m} \sum_{\nu=0}^{1} \alpha_{\mu,\nu} \frac{\partial^{\mu+\nu} x(\lambda,t)}{\partial\lambda^{\mu}\, \partial t^{\nu}} = 0$$

sa sledećim uslovima:

$$\frac{\partial^{\mu} x(0,t)}{\partial\lambda^{\mu}} = 0 \qquad \lambda > 0,\ \mu = 0,\dots,m$$

$$\frac{\partial^{\mu} x(0,t)}{\partial\lambda^{\mu}} = 0 \qquad t > 0,\ \mu = 0,\dots,m-2$$

$$\frac{\partial^{m-1} x(0,t)}{\partial\lambda^{m-1}} = 1 \qquad t > 0$$

na intervalu $[0,T]$ u koracima. Znajući približno rešenje na intervalu $[0,T_1]$, $T_1 < T$, konstruiše se približno rešenje na intervalu $[T_1,T]$ i ocenjuje greška.