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A NOTE ON ALMOST CLOSED MAPPINGS AND NEARLY PARACOMPACTNESS

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ABSTRACT

In this note it will be shown that the Lemma 1.1 in [4] and the proofs of some theorems in [3], [4], [7] and [10], where this lemma was used, are not correct.

It will be shown that this lemma and these theorems are correct with new additional condition.

Moreover, some new characterizations of almost closed mappings will be shown.

All definitions could be find in the paper [4].

In [11] T.Noiri has proved:

LEMMA A. *If a mapping* $f: X \to Y$ *is almost continuous and almost open, then:*

a) For each regularly open set V *of* Y, $f^{-1}(V)$ *is regularly open in* X,

b) For each regularly closed set V *of* Y, $f^{-1}(V)$ *is regularly closed in* X.

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In [3] the author has proved:

LEMMA B. *(Lemma 1.1) If a mapping* $f:X \to Y$ *is almost continuous and almost closed, then:*

a) For each regularly closed set F *of* $Y, f^{-1}(F)$ *is regularly closed in* X,

b) For each regularly open set V *of* $Y, f^{-1}(V)$ *is regularly open open in* X.

From the following example it follows that Lemma B is not correct.

EXAMPLE 1. Let

$$X = \{a,b,c,d,e\}, \quad \tau_X = \{\emptyset,\{a\},\{b\},\{a,b\},\{a,c\},\{a,b,c\},\{a,b,d\},\{a,b,c,d\},X\} ;$$

$$Y = \{a,b,c\}, \quad \tau_Y = \{\emptyset,\{a\},\{b\},\{a,b\},Y\} .$$

Let $f:X \to Y$ be a mapping of a space X onto a space Y defined by

$$f(a) = b, f(b) = a, \quad f(c) = f(d) = f(e) = c .$$

f is almost continuous and almost closed. $\{b\}$ is regularly open in Y, since $\alpha(\{b\}) = \{\bar{b}\}^o = \{b,c\}^o = \{b\}$. But $f^{-1}(\{b\}) = \{a\}$ is not regularly open in X, since

$$\alpha(\{a\}) = \{a,c,d,e\}^o = \{a,c\} \neq \{a\} .$$

However, we can show that the Lemma B is necessarily true if a new condition is added.

LEMMA 1. *If* $f:X \to Y$ *is almost continuous and almost closed surjection, such that for each regularly closed set* F *of* Y

$$f^{-1}(f(\overline{[f^{-1}(F)]^o})) = \overline{[f^{-1}(F)]^o} , \qquad \textit{then}$$

a) For each regularly closed set F *of* $Y, f^{-1}(F)$ *is regularly closed in* X;

b) For each regularly open set V *of* $Y, f^{-1}(V)$ *is regularly open in* X.

P r o o f. a) Let F be any regularly closed subset of Y. Then $f^{-1}(F)$ is closed. Hence we have

$$\overline{[f^{-1}(F)]^{o}} \subset f^{-1}(F) .$$

On the other hand, since f is almost countinuous and F^{o} is non empty regularly open subset of Y, $f^{-1}(F^{o})$ is open, hence

$$f^{-1}(F^{o}) \subset [f^{-1}(F)]^{o} \subset \overline{[f^{-1}(F)]^{o}} .$$

Since f is almost closed and $\overline{[f^{-1}(F)]^{o}}$ is regularly closed, then $f(\overline{[f^{-1}(F)]^{o}})$ is closed. Since $f^{-1}(F^{o}) \subset \overline{[f^{-1}(F)]^{o}}$, then

$$F^{o} \subset f(\overline{[f^{-1}(F)]^{o}}) \text{ , i.e. } F = \overline{F^{o}} \subset f(\overline{[f^{-1}(F)]^{o}}) .$$ Hence we

have $$f^{-1}(F) \subset f^{-1}(f(\overline{[f^{-1}(F)]^{o}})) = \overline{[f^{-1}(F)]^{o}} .$$

Hence we have

$$f^{-1}(F) = \overline{[f^{-1}(F)]^{o}} .$$

b) Let U be any regularly open subset of Y. Then $Y \setminus U$ is a regularly closed subset of Y. $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is regularly closed, hence $f^{-1}(U)$ is regularly open.

A mapping $f: X \to Y$ is said to be α-irreducible iff for every regularly closed subset F of Y there is no proper regularly subset of $f^{-1}(F)$ mapped onto the whole of F.

COROLLARY 1. *If* f *is an almost closed, almost continuous and α-irreducible mapping of a space* X *onto a space* Y, *then for each regularly open (regularly closed) subset* F *of* Y, $f^{-1}(F)$ *is regularly open (regularly closed) in* X .

The following example shows that there exists a mapping with the properties as in Lemma 1, which is not almost open.

EXAMPLE 2. Let

$$X = \{a,b,c,d\} \text{ , } \tau_X = \{\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\},X\} ;$$

$$Y = \{a,b,c\}, \quad \tau_Y = \{\emptyset,\{a\},\{c\}.\{a,c\},Y\} .$$

Let $f:X \to Y$ be a mapping of a space X onto a space Y defined by:

$$f(a) = a,\ f(b) = f(d) = b,\ f(c) = c\ .$$

f is almost closed and almost continuous. f is not almost open, since $f(\{b\}) = \{b\}(\{b\}$ is a regularly open set in X, since $\alpha(\{b\}) = \{b,d\}^{o} = \{b\})$ is not open in Y.

The proper regularly closed subsets in the space Y are the subsets

$$F_1 = \{a,b\} \quad \text{and} \quad F_2 = \{b,c\}\ .$$

$$\overline{[f^{-1}(F_1)]^{o}} = \overline{\{a,b,d\}^{o}} = \{a,b,d\};\ \overline{[f^{-1}(F_2)]^{o}} = \overline{\{b,c,d\}^{o}} = \{b,c,d\}.$$

$$f^{-1}(f(\overline{[f^{-1}(F_1)]^{o}})) = f^{-1}(f(\{a,b,d\})) =$$

$$= f^{-1}(\{a,b\}) = \{a,b,d\}\ ;$$

$$f^{-1}(f(\overline{[f^{-1}(F_2)]^{o}})) = f^{-1}(f(\{b,c,d\})) = f^{-1}(\{b,c\}) = \{b,c,d\}\ .$$

f is not α-irreducible, since $\overline{f^{-1}(F_1^{o})} = \{a,d\}$ is a proper regularly closed subset of $f^{-1}(F_1) = \{a,b,d\}$ such that $f(\overline{f^{-1}(F_1^{o})}) = F_1$.

The following example shows that there exists a mapping with the properties as in Lemma 1, which is not almost open and continuous.

EXAMPLE 3. Let

$X = \{a,b,c,d,e\}$, $\tau_X = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, X\}$;

$Y = \{a,b\}$, $\tau_Y = \{\emptyset, \{a\}, Y\}$.

Let $f:X \to Y$ be a mapping of a space X onto a space Y defined by:

$$f(a) = f(d) = a, \qquad f(c) = f(b) = f(e) = b\ .$$

f is almost closed and almost continuous . f is not almost open, since $f(\{b\}) = \{b\}$, nor continuous, since $f^{-1}(\{a\}) = \{a,d\}$.

By using Lemma B, the author has proved:

Theorem A ([3], [4], [7]). Let $f:X\to Y$ be any almost closed, almost continuous mapping of a space X onto a space Y. Then:

a) if for each point $y\in Y, f^{-1}(y)$ is α-nearly paracompact (α-nearly compact) and if X is almost regular, Y is almost regular.

b) if for each point $y\in Y, f^{-1}(y)$ is α-nearly compact and if X is almost regular nearly paracompact, Y is almost regular nearly paracompact,

c) if K is α-nearly compact, f(K) is α-nearly compact,

d) if for each point $y\in Y, f^{-1}(y)$ is α-nearly compact and if X is a Hausdorff locally nearly compact space, Y is Hausdorff locally nearly compact.

The proof of this Theorem is not correct, since we are used that the inverse image of every regularly open set is regularly open. We do not know if the formulation of this Theorem is true.

However, we can show that Theorem B is necessarily true if a new condition is added.

THEOREM 1. *Let $f:X\to Y$ be any almost closed, almost continuous mapping of a space X onto a space Y, such that for every regularly closed set F of Y, $f^{-1}(f(\overline{[f^{-1}(F)]^{o}}))=\overline{[f^{-1}(F)]^{o}}$. Then:*

a) if for each point $y\in Y, f^{-1}(y)$ is α-nearly paracompact and if X is almost regular, Y is almost regular,

b) if for each point $y\in Y, f^{-1}(y)$ is α-nearly compact and if X is almost regular nearly paracompact, Y is almost regular nearly paracompact,

c) if K is α-nearly compact, f(K) is α-nearly compact,

d) if for each point $y\in Y, f^{-1}(y)$ is α-nearly compact and if X is a Hausdorff locally nearly compact space, Y is Hausdorff locally nearly compact.

P r o o f. a) It is identical with the proof of Theorem 2.3 in [4].

b) It is identical with the proof of Theorem 2.1 in [3].

c) It is identical with the proof of Lemma 2.1 in [4].

d) It is identical with the proof of Theorem 2.9 in [4].

REMARK 1. Theorem 1 is true if $f:X \to Y$ is an almost closed, almost continuous and α-irreducible surjection.

THEOREM 2. *If f is a closed almost continuous mapping of a regular space X onto a space Y such that $f^{-1}(y)$ is α-paracompact for each point $y \in Y$, then Y is almost regular.*

P r o o f. It is similar to the proof of Theorem 2.2 in [4].

COROLLARY 2. *If f is an almost closed almost continuous mapping of a regular space X onto a space Y, such that $f^{-1}(y)$ is α-nearly paracompact for each point $y \in Y$, then Y is almost regular.*

P r o o f. f is closed (Theorem 1, [9]). In a regular space every α-nearly paracompact is α-paracompact.

COROLLARY 3. ([11]) *If X is regular and $f:X \to Y$ is an almost continuous and almost closed surjection such that $f^{-1}(y)$ is compact for each point $y \in Y$, then Y is almost regular.*

THEOREM 3. *If f is an almost closed continuous mapping of an almost normal space X onto a space Y such that for each regularly closed subset F of Y, $f^{-1}(f(\overline{[f^{-1}(F)]^{o}})) = \overline{[f^{-1}(F)]^{o}}$, then Y is almost normal.*

P r o o f. Let U be an open and V an regularly open set in Y such that $U \cup V = Y$. Then $f^{-1}(U)$ is open and $f^{-1}(V)$ is a regularly open set in X such that $f^{-1}(U) \cup f^{-1}(V) = X$. Since X is almost normal, then by Lemma 2.1 in [5], there exist the regularly closed subsets A_o and B_o of X such that $A_o \subset f^{-1}(U)$, $B_o \subset f^{-1}(V)$ and $A_o \cup B_o = X$. Since f is almost closed, then $A = f(A_o)$ and $B = f(B_o)$ are closed sets, such that $A \subset U$, $B \subset V$

and $A \cup B = Y$. Hence, by Lemma 2.1 in [5], Y is almost normal.

LEMMA 2. *Let f be any mapping of an almost regular space X onto a space Y such that for each point* $y \in Y$, $f^{-1}(y)$ *is* α*-nearly paracompact, then the following are equivalent:*

a) f is almost closed,

b) f is star closed,

c) for any subset K in Y and any star open set U containing $f^{-1}(K)$, *there exists an open set V in Y such that* $K \subset V$ *and* $f^{-1}(V) \subset U$.

P r o o f. (a) → (b). First, it will be shown that $f:(X,\tau^*) \to Y$ is almost closed (τ^* is semiregularization of τ). Let F be any τ^*-regularly closed set. Then, there exists τ^* - open set U such that $F = \bar{U}_{\tau^*} = \bar{U}_{\tau}$, hence F is τ-regularly closed. Since $f:(X,\tau) \to Y$ is almost closed, then f(F) is closed. f is the almost closed mapping of the regular space (X,τ^*) onto a space Y such that $f^{-1}(y)$ is α-paracompact for each point $y \in Y$, hence, by Theorem 1 in [9] $f:(X,\tau^*) \to Y$ is closed. Hence the implication is proved.(b) → (a), which is obvious.

(b) → (c). Let K be any subset in Y and U be any star open subset in X containing $f^{-1}(K)$. Let $V = Y \setminus f(X \setminus U)$. Since f is star closed, $f(X \setminus U)$ is closed. Hence V is open in Y such that

$$f^{-1}(K) \subset f^{-1}(V) = X \setminus f^{-1}(f(X \setminus U)) \subset U .$$

(c) → (b). Let A be any star closed subset in X and $y \in Y \setminus f(A)$ be any point. Then we have $f^{-1}(y) \subset X \setminus A$. Since $X \setminus A$ is star closed, there exists an open set V in Y such that $f^{-1}(y) \subset f^{-1}(V) \subset X \setminus A$, i.e. $y \in Y \subset Y \setminus f(A)$, hence $Y \setminus f(A)$ is open in Y. Hence f(A) is closed.

LEMMA 3. *Let X be almost regular, and A and B be any disjoint sets such that A is* α*-nearly paracompact and B star closed. Then, there exist disjoint regularly open sets containing A and B respectively.*

P r o o f. $X \setminus B$ is τ^*-open set containing A which is α-paracompact in (X,τ^*). Since (X,τ^*) is regular, there exists a τ^*-open set C such that $A \subset C \subset \bar{C}_{\tau^*} = \bar{C}_{\tau} \subset X \setminus B$. Let $U = \alpha(C)$. Then, U is a regularly open set such that $A \subset U \subset \bar{U} \subset X \setminus B$. Let $V = X \setminus U$. U and V are disjoint regularly open sets containing A and B respectively.

COROLLARY 4. *Let X be any Hausdorff almost regular space. Then, for any disjoint* α*-nearly paracompact sets* A *and* B*, there exist the disjoint regularly open sets* U *and* V *containing* A *and* B *respectively.*

P r o o f. In a Hausdorff space every α-nearly paracompact is star closed (Theorem 1. [7]).

THEOREM 4. *If* $f: X \to Y$ *is an almost closed mapping of an almost regular space* X *onto a space* Y *such that* $f^{-1}(y)$ *is star closed for each point* $y \in Y$ *and* $f^{-1}(F)$ *is* α*-nearly paracompact for each proper regularly closed set* F *of* Y*, then* Y *is almost regular.*

P r o o f. Let F be any regularly closed set in Y and $y \notin F$ be any point. Then, by the preceding Lemma, there exist the disjoint regularly open sets U and V such that $f^{-1}(F) \subset U$ and $f^{-1}(y) \subset V$. Since f is almost closed, then there exist the open sets U_1 and V_1 in Y such that $F \subset U_1$, $y \in V_1$, $f^{-1}(F) \subset f^{-1}(U_1) \subset U$ and $f^{-1}(y) \in f^{-1}(V_1) \subset V$. Hence, Y is almost regular.

COROLLARY 5. *If* $f: X \to Y$ *is an almost closed mapping of a Hausdorff almost regular space* X *onto a space* Y *such that* $f^{-1}(y)$ *and* $f^{-1}(F)$ *are* α*-nearly paracompact for each point* $y \in Y$ *and each proper regularly closed set* F *in* Y*, then* Y *is Hausdorff almost regular.*

P r o o f. Y is almost regular. Now, we shall show that Y is Hausdorff. Let a and b be different points of Y. $f^{-1}(a)$ and $f^{-1}(b)$ are disjoint α-nearly paracompact sets in X, hence there exist the disjoint regularly open sets U and V containing $f^{-1}(a)$ and $f^{-1}(b)$ respectively.

Since f is almost closed, then there exist open sets U_1 and V_1 such that $a \in U_1$, $b \in V_1$, $f^{-1}(U_1) \subset U$ and $f^{-1}(V_1) \subset V$, hence Y is Hausdorff.

THEOREM 5. *If f is an almost closed mapping of a Hausdorff space X onto a space Y such that $f^{-1}(y)$ is α-nearly compact for each point $y \in Y$ and $f^{-1}(F)$ is α-nearly paracompact for each proper regularly closed set F of Y, then Y is Hausdorff almost regular.*

P r o o f. By the Theorem 3 in [13] Y is Hausdorff. Now, we shall show that Y is almost regular. Let F be any regularly closed subset of Y and $y \notin F$ any point. Since $f^{-1}(y)$ is α-nearly compact and $f^{-1}(F)$ is α-nearly paracompact, then by Theorem 2.1 in [4] there exist disjoint regularly open sets U an V containing $f^{-1}(y)$ and $f^{-1}(F)$ respectively. Since f is almost closed, there exist disjoint open sets U_1 and V_1 containing F and y respectively, hence Y is almost regular.

THEOREM 6. *If $f: X \to Y$ is an almost closed and almost continuous mapping of a Hausdorff nearly paracompact (almost regular nearly paracompact) space X onto a space Y such that for each proper regularly closed set F of Y, $f^{-1}(F)$ is α-nearly paracompact and for each point $y \in Y$, $f^{-1}(y)$ is α-nearly compact (star closed α-nearly compact) then, Y is Hausdorff almost regular nearly paracompact.*

P r o o f. Y is Hausdorff almost regular. We shall show that Y is nearly paracompact. Let $\mathcal{U} = \{U_i : i \in I\}$ be any regularly open covering of Y. Since f is almost continuous, $f^{-1}(\mathcal{U}) = \{f^{-1}(U_i) : i \in I\}$ is open covering of a space X. Since X is nearly paracompact, there exists regularly open locally finite refinement $\mathcal{V} = \{V_j : j \in J\}$ of $\{\alpha(f^{-1}(U_i)) : i \in I\}$.Then by Lemma 2 in [12] $\{f(V_j) : j \in J\}$ is locally finite covering of Y. $\{\overline{f(V_j)} : j \in J\}$ is closed locally finite covering of Y. For each $j \in J$ there exists $i(j) \in I$ such that $V_j \subset \alpha(f^{-1}(U_{i(j)})) \subset f^{-1}(\overline{U_{i(j)}})$, hence $\overline{f(V_j)} \subset f(\overline{V_j}) \subset \overline{U_{i(j)}}$.

Now, $\{\overline{f(V_j)}: j \in J\}$ is a closed locally finite refinement of $\{\bar{U}_i: i \in I\}$.

By Lemma 1.1 in [20], the family $\{\overline{[\overline{f(V_j)}]^o}: j \in J\}$ is a locally finite regularly closed cover of X. Since $[\overline{f(V_j)}]^o \subset \subset \alpha(U_{i(j)}) = U_{i(j)}$, then $\{[\overline{f(V_j)}]^o: j \in J\}$ is a locally finite family of open sets which refines $\mathcal{U}$ and the closures of whose members cover the space Y. Hence, by Lemma 1.3 in [6], Y is almost paracompact. Since every almost regular almost paracompact is nearly paracompact, then Y is nearly paracompact.

THEOREM 7. *If $f: X \to Y$ is an almost closed mapping of a Hausdorff locally nearly compact space onto a space Y such that $f^{-1}(F)$ is α-nearly paracompact for each proper regularly closed subset $F \subset Y$ and $f^{-1}(y)$ is α-nearly compact for each point $y \in Y$, then Y is a Hausdorff locally nearly compact.*

P r o o f. Y is a Hausdorff almost regular space. We shall show that Y is locally nearly compact. Since X is a Hausdorff nearly compact, for each point $x \in f^{-1}(y)$, there exists a regularly open neighbourhood K_x such that $\bar{K}_x$ is α-nearly compact. Now, the family

$$K = \{K_x: x \in f^{-1}(y)\}$$

is an X-regularly open cover of $f^{-1}(y)$, hence there exist a finite number of points $x_1, x_2, \ldots, x_n$ in $f^{-1}(y)$ such that

$$f^{-1}(y) \subset \bigcup\{K_{x_i}: i=1,2,\ldots,n\} .$$

Let

$$K = \bigcup\{\bar{K}_{x_i}: i=1,2,\ldots,n\} .$$

$f^{-1}(y) \subset K^o$. Since f is almost closed, then there exists an open set V_y containing y such that $f^{-1}(V_y) \subset K^o$. Hence, we have

$$y \in V_y \subset f(K^o) \subset f(K) .$$

Since in a Hausdorff space every α-nearly paracompact is star closed, then f is almost continuous. Since f is almost continuous and almost closed and K is α-nearly compact, then f(K) is almost compact, i.e. α-nearly compact (X is almost regular). f(K) is closed, hence $\bar{V}_y \subset f(K)$. $\bar{V}_y$ is α-nearly compact, hence Y is locally nearly compact.

By using Lemma B the author has proved:

THEOREM B. ([4], [10]) *Let X be a topological space. Let $\mathcal{D}$ be an almost-upper semicontinuous decomposition of X and let $\mathcal{D}$ have a quotient topology. Then:*

a) if X is almost normal, $\mathcal{D}$ is almost normal ;

b) if the members of $\mathcal{D}$ are α-nearly paracompact subsets of X and if X is almost regular, $\mathcal{D}$ is almost regular;

c) if the members of $\mathcal{D}$ are α-nearly compact subsets of X and if X is almost regular (almost regular nearly paracompact Hausdorff locally nearly compact) $\mathcal{D}$ is almost regular (almost regular nearly paracompact, Hausdorff locally nearly compact).

The proof of this Theorem is not correct, since we have used that the inverse image of every regularly open set is regularly open.

EXAMPLE 4. Let

$$X = \{a,b,c,d,e\},\ \tau_X = \{\emptyset,\{a\},\{b\},\{a,b\},\{a,c\},\{a,b,c\},\{a,b,d\},\{a,b,c,d\},\ X\}\ ;$$

$$\mathcal{D} = \{\{a\},\{b\},\{c,d,e\}\};\ \tau_{\mathcal{D}} = \{\emptyset,\{\{a\}\},\{\{b\}\},\{\{a,b\}\},\mathcal{D}\},\ R = \bigcup\{D\times D : D\in\mathcal{D}\}\ .$$

The projection $P: X \to X/R$ is almost closed (the decomposition is almost-upper semicontinuous) such that $P^{-1}(\{\{a\}\}) = \{a\}$ is not regularly open in X.

LEMMA 4. *Let X be a topological space. Let $\mathcal{D}$ be an almost-upper semicontinuous decomposition of X such that for each proper regularly closed set A of X, $R[A]$ is α-nearly paracompact ($R = \bigcup\{D\times D:\ D\in\mathcal{D}\}$) . Let $\mathcal{D}$ have a quotient topology. Then, for each proper regularly closed subset A of $\mathcal{D}$, $P^{-1}(A)$ is α-nearly paracompact.*

P r o o f. Let A be any proper regularly closed subset of $\mathcal{D}$. Then we have $P^{-1}(A^{o}) \subset \overline{[P^{-1}(A)]^{o}} \subset P^{-1}(A)$, hence $P(P^{-1}(A^{o})) \subset$ $\subset P(\overline{[P^{-1}(A)]^{o}}) \subset P(P^{-1}(A))$, i.e. $A^{o} \subset P(\overline{[P^{-1}(A)]^{o}}) \subset A$. Since $P(\overline{[P^{-1}(A)]^{o}})$ is closed we have $A = P(\overline{[P^{-1}(A)]^{o}})$, i.e.

$$P^{-1}(A) = P^{-1}(P(\overline{[P^{-1}(A)]^{o}})) = R[\overline{P^{-1}(A)^{o}}]$$

Since $\overline{[P^{-1}(A)]^{o}}$ is regularly closed, hence $P^{-1}(A)$ is α-nearly paracompact.

THEOREM 8. *Let* X *be a topological space. Let* $\mathcal{D}$ *be an almost-upper semicontinuous decomposition of* X. *Let* $\mathcal{D}$ *have a quotient topology. Then:*

a) if the members of $\mathcal{D}$ *are star closed in* X *such that for each proper regularly closed set* A *of* X, R[A] *is* α*-nearly paracompact* ($R = \bigcup\{D \times D:\ D \in \mathcal{D}\}$) *and if* X *almost regular,* $\mathcal{D}$ *is almost regular ;*

b) if the members of $\mathcal{D}$ *are* α*-nearly paracompact in* X *such that for each proper regularly closed set* A *of* X, R[A] *is* α*-nearly paracompact and if* X *is Hausdorff almost regular, then* $\mathcal{D}$ *is Hausdorff almost regular;*

c) if the members of $\mathcal{D}$ *are* α*-nearly compact of* X *such that for each proper regularly closed set* A *of* X, R[A] *is* α*-nearly paracompact and if* X *is Hausdorff locally nearly compact (Hausdorff nearly paracompact),* $\mathcal{D}$ *is Hausdorff almost regular locally nearly compact (Hausdorff almost regular nearly paracompact);*

d) if the members of $\mathcal{D}$ *are star closed* α*-nearly compact of* X *such that for each proper regularly closed set* A *of* X, R[A] *is* α*-nearly paracompact and if* X *is almost regular nearly paracompact,* $\mathcal{D}$ *is Hausdorff almost regular nearly paracompact.*

P r o o f. a) This follows from Theorem 4.

b) This follows from Corollary 5.

c) This follows from Theorem 6 and Theorem 7.

d) This follows from Theorem 6.

THEOREM 9. *Let X be a topological space. Let $\mathcal{D}$ be an almost-upper semicontinuous decomposition of X, such that for each proper regularly closed subset A of X,* $(R = \bigcup\{D \times D : D \in \mathcal{D}\})$ $R[A] = \{R[x] : x \in A\}$ *is locally finite. Let $\mathcal{D}$ have a quotient topology. Then:*

a) if the members of $\mathcal{D}$ are closed in X then a subset A of $\mathcal{D}$ is regularly open iff it is regularly closed;

b) if the members of $\mathcal{D}$ are closed in X then $P^{-1}(A)$ is regularly open (regularly closed) for every regularly open (regularly closed) subset of $\mathcal{D}$,

c) if the members of $\mathcal{D}$ are closed in X and if X is almost normal, $\mathcal{D}$ is almost normal;

d) if X is Hausdorff locally nearly compact (Hausdorff nearly paracompact) and if the members of $\mathcal{D}$ are α-nearly compact in X, $\mathcal{D}$ is Hausdorff almost regular locally nearly compact (Hausdorff almost regular nearly paracompact);

e) if the members of $\mathcal{D}$ are closed α-nearly paracompact in X and if X is almost regular, $\mathcal{D}$ is almost regular.

P r o o f. a) Let F be any regularly closed subset of $\mathcal{D}$. Then $[P^{-1}(F)]^o$ is regularly closed, such that $P^{-1}(P(\overline{[P^{-1}(F)]^o})) = P^{-1}(F) = R[\overline{P^{-1}(F)^o}] = \bigcup\{R[x] : x \in \overline{[P^{-1}(F)]^o}\}$. Since $\{R[x] : x \in \overline{[P^{-1}(F)]^o}\}$ is locally finite then, the family $\{R[x] : x \in P^{-1}(F^o)\}$ is locally finite. Thus, we have $\overline{P^{-1}(F^o)} = \bigcup\{\overline{R[x]} : x \in P^{-1}(F^o)\} = P^{-1}(F^o)$. Hence, F is open and closed.

b) This is obvious.

c) This is similar to the proof Theorem 3.

d) This is similar to the proof of b) and d) in Theorem 1.

e) This is similar to the proof of a) in Theorem 1.

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REZIME

SKORO ZATVORENA PRESLIKAVANJA U BLIZU PARAKOMPAKTNOST

U radu se posmatraju osobine skoro zatvorenih preslikavanja. Pokazuje se da postoji skoro zatvoreno i skoro neprekidno preslikavanje sa osobinom da inverzna slika skoro otvorenog (skoro zatvorenog) skupa nije uvek skoro otvoren (skoro zatvoren) skup. Medjutim, ako je f skoro zatvoreno i skoro neprekidno preslikavanje prostora X na prostor Y sa osobinom da je $f^{-1}(f([\overline{f^{-1}(F)}]^{o})) = [f^{-1}(F)]^{o}$ tada je inverzna slika svakog skoro otvorenog (skoro zatvorenog) skupa skoro otvoren (skoro zatvoren)skup.

Dalje se ispituje kako se odnose skoro regularni, Hausdorffovi, skoro normalni i blizu parakompaktni prostori pri skoro zatvorenim preslikavanjima.

Na kraju se daju neke osobine skoro odozgo poluneprekidnog razlaganja datog prostora.