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CORRECTIONS TO "SOLVABILITY OF CONVOLUTION EQUATIONS IN $H'\{M_p\}$"

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Our paper "Solvability of Convolution Equations in $H'\{M_p\}$" was published in this Journal, Volume 11(1981),45-58.

In the proof of Theorem 5 we overlooked that β (p. 55) must not be bounded. So from this point up to the end of the paper we have to suppose the following additional assumption

(B) For every $p \in N$ there exist $p' \in N$, $\delta > 0$, and $x_\delta > 0$ such that

$$M_p^*(x) \geq M_{p'}^*(x^{1+\delta}) \quad \text{if} \quad x > x_\delta$$

To avoid missunderstendings we shall reformulate Theorem 5 and give the complete proof of it.

THEOREM 5. *Let* $F(\xi)$ *be an entire analytic function which is* M_q*-slowly decreasing for some* $q \in N$ *and let* $p \geq q'$ *where* q' *correspond to* q *in* (B). *If* $F(\xi)$ *satisfies an estimate* (9) *for some* c, n, *and this* p *then* $F(\xi)$ *is extremely slowly decreasing.*

P r o o f. We shall use the idea of the proof of Theorem 3′ from [4].

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There exists $L_1 > 0$ such that

$$\sup\{M_p^*(x)/M_q^*(x/A_1), \quad |x| \geq L_1\} \leq 1$$

holds (A_1 is from (13)). Namely if $0 < \delta_1 < \delta$ from (B) follows

$$M_q^*(x/A_1) \geq M_p^*((\frac{x}{A_1})^{1+\delta}) \geq M_p^*(x^{1+\delta_1}) \geq M_p^*(x)$$

for sufficiently large $|x|$.

Let us take $L \geq L_1$ so large that $\rho_1(\log(1+|\xi|)) > 1$ for each ξ with $|\xi| > L$. Let us fix ξ with $|\xi| > L$ and define

$$\beta := \frac{\log \rho}{\log(M_p^{*-1}(M_q^*(\rho/A_1))) - \log \rho}$$

where $\rho = \rho_1(\log(1+|\xi|)) > 1$. Observe that from (B) follows that $0 < \beta \leq \delta$. Let us put $\bar{R} := \rho^{(\beta+1)/\beta}$.

As in |4|, we apply Hadamard's Three Circles Theorem on the function $F(\xi+\lambda w)$ (λ-complex variable) for the circles with radiuses $1, \rho, \bar{R}$ and

$$\gamma := \frac{\log(\bar{R}/\rho)}{\log \bar{R}} = \frac{1}{\beta+1}.$$

All the time, w is a complex parameter. So we have

$$(14) \qquad \sup\{|F(\xi+w)|;\ |w| \leq 1\} \geq$$
$$\geq (\sup\{|F(\xi+\rho w)|;\ |w| \leq 1\})^{1+\beta}/(\sup\{|F(\xi+Rw)|;|w| \leq 1\})^{\beta}.$$

Using (9) we obtain

$$|F(\xi+\bar{R}w)| = |F(\xi+\bar{R}\cdot \mathrm{Re}w + i\cdot\bar{R}\cdot \mathrm{Im}w)| \leq$$
$$\leq c\cdot(1+|\xi|)^n\cdot(1+\bar{R})^n\cdot\exp(M_p^*(\bar{R})) \leq c'\cdot c\cdot(1+|\xi|)^n\cdot\exp(2\cdot M_p^*(\bar{R}))$$

where we have put $c' := \sup\{(1+\bar{R})^n \exp(-M_p^*(\bar{R}));\ \bar{R} \in R\} < \infty$.

Since we have construced $\bar{R}$ so that $M_p^*(\bar{R}) = M_q^*(\rho/A_1)$ we have

$$(15) \qquad \sup\{|F(\xi+Rw)|;\ |w| \leq 1\} \leq C\cdot(1+|\xi|)^{n+2}$$

for some $C > 0$. Returning to (14) using (11) we obtain the statement for $|\xi| \geq L$, because β is bounded.

Using the Maximum Principle we obtain for $|\xi| \leq L$

$$\sup\{|F(\xi+w)|;\ |w| \leq 1\} \geq C_1 > 0$$

and this togerher with (15) gives that $F(\xi)$ is extremely slowly decreasing.

Let us remark that after the condition (B) Theorem 5 is superfluous.

At last in Theorem 7 there is a miss print, namely $S \in O_C'(\mathcal{H}'\{M_p\})$.

We are indebted to Olaf von Grudzinski who noticed that β must not be bounded without additional conditions.

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ISPRAVKA RADA "REŠIVOST KONVOLUCIJE JEDNAČINE U $H'\{M_p\}$"

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