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ON A NUMERICAL SOLUTION OF A TYPE OF SINGULARLY PERTURBED BOUNDARY VALUE PROBLEM BY USING A SPECIAL DISCRETIZATION MESH

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ABSTRACT

This paper presents a generalization of a mesh construction from [1] for a finite-difference discretization of a singularly perturbed problem (1). We give a class of functions that generate mesh points, enabling a quadratic convergence uniform in small perturbation parameter ε.

The possibilities of linear interpolation of numerical results is investigated as well, and the method is shown to be uniform in ε and to retain the accuracy order of numerical results.

1. INTRODUCTION

We consider the problem

(1a) $\quad Tu := -\varepsilon^2 u'' + b(x,u) = 0, \qquad x \in I = [0,1],$

(1b) $\quad Bu := (u(0),u(1)) = (U_0,U_1)$,

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under the basic assumptions:

$$b(x,u) \in C^k(I \times \mathbb{R}), \quad k \in \mathbb{N} ;$$
$$b_u(x,u) > \beta^2 > 0 , \quad (x,u) \in I \times \mathbb{R} ; \quad 0 < \varepsilon \leq \varepsilon_o ;$$
$$\beta, \quad \varepsilon_o, \quad U_o, \quad U_1 \in \mathbb{R} ,$$

where ε is a small perturbation parameter.

A problem of this type was considered, among the others, in [2] and the linear case of it in [1], [4], [5], [8]. It is well known that (T,B) is an inverse monotone operator and that there exists a unique solutions $u_\varepsilon \in C^{k+2}(I)$ to problem (1), see [2], [3]. The corresponding reduced problem

$$b(x,u) = 0 , \quad x \in I,$$

also has a unique solution in $C^k(I)$, which, in general, does not satisfy the boundary conditions (1b). Therefore u_ε shows two boundary layers at the endpoints of the interval I.

We use a classical finite-difference scheme on a non-uniform mesh to solve (1) numerically. The discretization mesh is constructed in a special way, which generalizes the idea from [1], see [4] as well. This enables the second order convergence, uniform in ε, the result of which we shall state in section 4 and prove in section 5. To obtain this we have to know the estimates of u_ε and its derivatives and that is the subject of section 2.

In section 3, we shall give a general mesh construction, where the mesh points are obtained via $x_i = \lambda(i/n)$, $i = 0,1,\dots,n$, $n \in \mathbb{N}$, with some suitable function λ.

In section 6 we shall show that our discretization mesh is suitable to get the approximation of u_ε at any point $x \in I$ by interpolating numerical results. The linear interpolation retains the second order accuracy and informity in ε.

Section 7 contains some numerical results. They agree fully with the theoretical ones.

Throughout the paper M will denote each positive constant independent of ε and of the discretization mesh.

2. ESTIMATES OF u_ε AND ITS DERIVATIVES

Define the linear operator as:

$$L_o z := -\varepsilon^2 z'' + g_\varepsilon(x) z, \quad x \in I, \quad z \in C^2(I),$$

where

$$g_\varepsilon(x) = b(x, u_\varepsilon(x)) - b(x,0) = \int_0^1 b_u(x, s u_\varepsilon(x))ds > \beta^2 > 0.$$

Obviously (L_o, B) is inverse monotone and we have

$$L_o(\pm u_\varepsilon) = \mp b(x,0). \tag{2}$$

Now we can easily get:

LEMMA 1. *For the solution* u_ε *to problem* (1) *we have* $|u_\varepsilon^{(i)}(x)| \le M\varepsilon^{-i}$, $i = 0,1,\dots,k+2$, $x \in I$.

P r o o f. For $i = 0$ the proof follows immediately from (2). For $i = 2$ we get the desired inequality directly from (1a) and for $i = 1$ we can use Lemma 1 from [1]. Further inequalities can be proved by differentiating (1a). We just have to use the formula for differentiating $b(x,u(x))$ from [2], page 35. □

LEMMA 2. *For the solution* u_ε *to problem* (1) *the following estimates hold:*

$$|u_\varepsilon^{(i)}(x)| \le M(1 + \varepsilon^{-i} V_\varepsilon(x)), \quad i = 1,\dots,k, \quad x \in I, \tag{3}$$

where $V_\varepsilon(x) = v_\varepsilon(x) + w_\varepsilon(x)$,

$$v_\varepsilon(x) = \exp(-\beta x/\varepsilon), \quad w_\varepsilon(x) = \exp(-\beta(1-x)/\varepsilon).$$

P r o o f. For $z \in C^2(I)$ we take

$$Lz = -\varepsilon^2 z'' + b_u(x,u_\varepsilon(x))\cdot z \quad .$$

Then:

$$L(\pm u'_\varepsilon) = \mp b_x(x,u_\varepsilon) \quad .$$

Because of the inverse monotonicity of (L,B) we can get (3) for $i = 1$. Here we use $|u'_\varepsilon(s)| \le M/\varepsilon$, $s = 0,1$, from Lemma 1.

Now suppose that (3) holds for $i = 1,2,\dots,j-1$, $2 \le j \le k$. We shall prove (3) for $i = j$. Consider

(4) $$L(\pm u_\varepsilon^{(j)}) = \mp((b(x,u_\varepsilon))^{(j)} - b_u(x,u_\varepsilon)\cdot u_\varepsilon^{(j)})$$

and use the already mentioned formula from [2].
We get

$$L(\pm u_\varepsilon^{(j)}) \le M(1 + \varepsilon^{-j}v_\varepsilon) \quad .$$

We could use the inductive hypothesis since on the right hand side of (4) we have derivatives of u_ε up to the order $j-1$. The proof now follows from the inverse monotonicity of (L,B).□

The following theorem is proved in [4] in the linear case.

THEOREM 1. The solution u_ε to problem (1) can be represented in the following way:

$$u_\varepsilon = m + y_\varepsilon \ ,$$

where for $i = 0,1,\dots,k$ *and* $x \in I$ *we have*

(5) $$|m^{(i)}(x)| \le M \quad ,$$

(6) $$|y_\varepsilon^{(i)}(x)| \le M\varepsilon^{-i}v_\varepsilon(x) \quad .$$

P r o o f. Consider the operator L_0. We can extend $g_\varepsilon(x)$ to the interval $[-1,2]$ in such a way that the smoothness and the property $g_\varepsilon(x) > \beta^2$ still hold. Denote this extension by $\bar{g}_\varepsilon(x)$. In the same way we make the extension $\bar{b}(x,0)$ of $b(x,0)$.

Let $m(x)$ be the unique solution to the problem

$$-\varepsilon^2 m'' + \overline{g}_\varepsilon(x)m = -\overline{b}(x,0), \quad x \in [-1,2] ,$$
$$m(-1) = m(2) = 0 .$$

Then (5) is obvious.

Now $y_\varepsilon = u_\varepsilon - m$ and we have

$$L_o y_\varepsilon = 0, \quad x \in I, \quad y_\varepsilon(s) = U_s - m(s), \quad s = 0,1 .$$

From the inverse monotonicity of (L_o,B) we get (6) for $i = 0$. Suppose that (6) holds for all $i = 0,1,\dots,j-1$, $1 \le j \le k$. We have

$$L_o(\pm y_\varepsilon^{(j)}) = \mp((g_\varepsilon(x)y_\varepsilon)^{(j)} - g_\varepsilon(x)y_\varepsilon^{(j)}) \quad .$$

Because of Lemma 2 it follows

$$|g_\varepsilon^{(i)}(x)| \le M(1 + \varepsilon^{-1}v_\varepsilon(x)), \quad i = 0,1,\dots,j, \quad x \in I$$

and

$$L_o(\pm y_\varepsilon^{(j)}) \le M\varepsilon^{-j}v_\varepsilon \quad ,$$

so, we can prove (6) for $i = j$ in the same way as we have proved (3) in Lemma 2. □

3. MESH CONSTRUCTION

From now on we shall take $k = 4$.

Let $q \in (0,1/2)$. Consider the function $\phi \in C^3\ [0,q)$ with the properties

$$\phi^{(i)}(t) > 0, \quad i = 0,1,2,3 , \qquad t \in (0,q)$$
$$\phi(0) = 0, \quad \phi(q) = +\infty,$$

and

$$\mu(t) := \phi'(t)\exp(-\phi(t)) \in C^2\ [0,q] .$$

Let $A(t) = \int_t^q \mu(s)ds$, $t \in [0,q]$. We have

$\phi(t) = -\ln A(t)$, $t \in [0,q)$

and

(7) $\phi^{(i)}(t) \leq MA(t)^{-i}$, $i = 1,2,3$, $t \in [0,q)$.

The examples for such a function are:

$\phi_o(t) = -\ln(1 - (t/q)^p)$, for $p = 1,2$,
or $p \in [3,\infty)$;

and

$\phi_1(t) = (q/(q-t))^p - 1$, for $p > 0$.

Let $\psi(t) = a\epsilon\phi(t)$, $t \in [0,q)$, where $a\beta \geq 2$ and suppose $a\epsilon_o\phi'(0) < 1$. Then $\psi'(0) < 1$ and there exists a unique point $\alpha \in (0,q)$ at which $\psi(t)$ contacts its tangent line from $(1/2,1/2)$.

Let

$$\psi'(\alpha_1) = 1/(1-2q), \quad \psi'(\alpha_2) = 1 .$$

The points α_1 and α_2 exist uniquely and we have $0 < \alpha_2 < \alpha < \alpha_1 < q$.

Take

$$\lambda(t) = \begin{cases} \psi(t), & t \in [0,\alpha] \\ \psi(\alpha) + \psi'(\alpha)(t-\alpha), & t \in [\alpha,1/2] \\ 1 - \lambda(1-t), & t \in [1/2,1] \end{cases}$$

We construct the mesh points x_i by

(8) $x_i = \lambda(t)$, $t_i = i/n$, $i = 0,1,\dots,n$,
$n = 2n_o$, $n_o \in \mathbb{N}$.

To use $\lambda(t)$ we have to know α. It is the solution to the equation

$$(9)\qquad \psi(\alpha) + \psi'(\alpha)(1/2 - \alpha) = 1/2$$

which can be solved by successive approximations as in [1]. Note that for ϕ_1 with $p = 1$ (9) reduces to a quadratic equation and α can be easily evaluated.

For $p = 1$ ϕ_o is the function from [1]. The function ϕ_1 for $p \in \mathbb{N}$ is more convenient for practical use because it is a simple rational function.

4. DISCRETIZATION OF (1) AND THE CONVERGENCE THEOREM

Let $h_i = x_i - x_{i-1}$, $i = 1,2,\dots,n$. We form the discretization of problem (1):

$$u_o = U_o$$

$$(10)\quad T_h u_i := -\varepsilon^2 D_h u_i + b(x_i,u_i) = 0, \quad i = 1,2,\dots,n-1,$$

where
$$u_n = U_1,$$

$$D_h u_i = \frac{2}{(h_i + h_{i+1})h_i h_{i+1}}(h_{i+1}u_{i-1} - (h_i + h_{i+1})u_i + h_i u_{i+1}).$$

The solution $u_h = [u_o,u_1,\dots,u_n]^T \in \mathbb{R}^{n+1}$ to the non-linear system (10) exists uniquely and it can be evaluated by the Newton method, see [7] for instance. Note that the perturbation parameter causes no truoble in the convergence of this method.

The system (10) can be written in the form:

$$A_h u_h + B_h u_h = f_h \quad ,$$

where $f_h = [U_o,0,\dots,0,U_n]^T \in \mathbb{R}^{n+1}$; $A_h = [a_{ij}] \in \mathbb{R}^{n+1,n+1}$ is a tridiagonal matrix with elements:

$$a_{oo} = a_{nn} = 1$$

and for $i = 1,2,\dots,n-1$

$$a_{i,i-1} = -2\varepsilon^2/((h_i + h_{i+1})h_i) \;;$$

$$a_{ii} = 2\varepsilon^2/(h_i h_{i+1}) \ ,$$
$$a_{i,i+1} = -\ 2\varepsilon^2/((h_i + h_{i+1})h_{i+1}) \ ;$$

and $B_h u_h = \mathrm{diag}(0, b(x_1,u_1),\dots,b(x_{n-1},u_{n-1}),0) \in \mathbb{R}^{n+1,n+1}$. Putting $u_\varepsilon^h = [\,u_\varepsilon(x_o), u_\varepsilon(x_1),\dots,u_\varepsilon(x_n)]^T \in \mathbb{R}^{n+1}$ and $r_h = [\,0, r_1, r_2,\dots,r_{n-1},0]^T \in \mathbb{R}^{n+1}$, where

$$r_i = r_i(u_\varepsilon) = (Tu_\varepsilon)(x_i) - T_h u_\varepsilon(x_i) =$$
$$= \varepsilon^2(D_h u_\varepsilon(x_i) - u_\varepsilon''(x_i)), \quad i = 1,2,\dots,n-1 \ ,$$

we can easily get, see [6]:

(11) $$\|u^h - u_h\| \le \frac{1}{\beta}\|r_h\| \ .$$

Here $\|\cdot\|$ denotes the maximum norm: $\|z_h\| = \max_{0\le i\le n} |z_i|$ for $z_h = [\,z_o, z_1,\dots,z_n]^T \in \mathbb{R}^{n+1}$.

Thus, for our discretization (10) we have a stability uniform in ε, (11).

In the next section we shall prove the following theorem (a second order consistency, uniform in ε):

THEOREM 2. *Let the mesh points be given by* (8) *and let* $a\beta \ge 2$, $a\varepsilon_o\phi'(0) < 1$, $n > 3/q$ *and* $k = 4$. *Then we have*

$$\|r_h\| \le M/n^2 \ .$$

From this and (11) we get immediately

THEOREM 3. *Under the assumption of the previous theorem we have*

$$\|u_\varepsilon^h - u_h\| \le M/n^2 \ .$$

5. PROOF OF THE CONSISTENCY THEOREM

To prove Theorem 2 we use the same technique as in Theorem 3 from [1].

First we have $r_i(u_\varepsilon) = r_i(m) + r_i(y_\varepsilon)$, $i = 1,2,\dots,n-1$, and since $|r_i(m)| \le M/n^2$, we only have to prove

(12) $$|r_i(v_\varepsilon)| \le M/n^2, \quad i = 1,2,\dots,n_o-1 ,$$

because for $i = n_o, n_o+1,\dots,n-1$ and w_ε the proof of (12) is analogous.

Now let $r_i = r_i(v_\varepsilon)$. We have

(13) $$|r_i| \le \varepsilon^2 \tfrac{1}{3}(h_{i+1} - h_i)|v_\varepsilon'''(x_i)| + \varepsilon^2 \tfrac{1}{6}h_{i+1}^2|v_\varepsilon^{iv}(\theta_i)|$$

and

(14) $$|r_i| \le \varepsilon^2 \cdot 2|v_\varepsilon''(y_i)|,$$

with $\theta_i, \eta_i \in (x_{i-1}, x_{i+1})$. Using the definition of mesh points and the estimates from Theorem 1 we get from (13)

(15a) $$|r_i| \le M(P_i + Q_i)/n^2 ,$$

(15b) $$P_i = \lambda''(t_{i+1}) \tfrac{1}{\varepsilon} v_\varepsilon(x_i) ,$$

(15c) $$Q_i = (\lambda'(t_{i+1}))^2 \varepsilon^{-2} v_\varepsilon(x_{i-1}) ;$$

and from (14)

(16) $$|r_i| \le M v_\varepsilon(x_{i-1}) .$$

For the function $\lambda(t)$, $t \in [0,1]$, we have

(17) $$\lambda'(t) \le 1/(1-2q) ,$$

$$|\lambda''(t)| \le a\varepsilon\phi''(\alpha_1)$$

and because of (7)

(18) $$|\lambda''(t)| \le M\varepsilon A(\alpha_1)^{-2} = M\varepsilon(\phi'(\alpha_1)/\mu(\alpha_1))^2 \le M/\varepsilon .$$

1° Let $t_{i-1} \geq \alpha_2$. Then

$$v_\varepsilon(x_{i-1}) \leq v(\lambda(\alpha_2)) = \exp(-a\beta\phi(\alpha_2)) \leq \exp(-2\phi(\alpha_2)) =$$
$$= (\mu(\alpha_2)/\phi'(\alpha_2))^2 \leq M\varepsilon^2 .$$

Using this inequality and (18) from (15b) we get $P_i \leq M$. From (15c) and (17) we get $Q_i \leq M$ in this case. Thus (15) gives us (12).

2° Now let $t_{i-1} < \alpha_2$ and $t_{i-1} \leq q - 3/n$. Then $t_{i+1} \leq q - 1/n < q$ and

(19) $$q - t_{i+1} \geq \frac{1}{3}(q - t_{i-1}) \quad .$$

Because of

$$\lambda''(t_{i+1}) \leq \psi''(t_{i+1}) \quad ,$$

from (15b) we get

$$P_i \leq M\phi''(t_{i+1})\exp(-2\phi(t_{i-1})) \leq$$
$$\leq M(A(t_{i-1})/A(t_{i+1}))^2$$

and because of (19) $P_i \leq M$.

In the same way we use $\lambda'(t_{i+1}) \leq \psi'(t_{i+1})$ to get $Q_i \leq M$ from (15c). Then from (15) we have (12) in this case.

3° The last case is $q - 3/n < t_{i-1} < \alpha_2$. Note that $q - 3/n > 0$. Now it follows

$$\exp(-2\phi(t_{i-1})) < \exp(-2\phi(q - \frac{3}{n})) =$$
$$= A(q - \frac{3}{n})^2 \leq M/n^2$$

and from (16) we conclude (12) in this case and the theorem is proved.

6. LINEAR INTERPOLATION

For any $[z_o,z_1,\dots,z_n]^T \in \mathbb{R}^{n+1}$ and $x \in [x_i,x_{i+1}]$, $i = 0,1,\dots,n-1$, let

$$\ell(z_i,x) = z_i + \frac{1}{h_{i+1}}(z_{i+1} - z_i)(x - x_i) \quad .$$

We aproximate $u_\varepsilon(x)$, $x \in [x_i,x_{i+1}]$, by $\ell(u_i,x)$, where, as before, u_i denotes the solution to the discrete problem (10) on the mesh (8).

THEOREM 4. *Under the assumptions of Theorem 2 we have*

$$|u_\varepsilon(x) - \ell(u_i,x)| \le M/n^2 \ , \quad x \in [x_i,x_{i+1}] \ .$$

P r o o f. Let $x \in [x_i,x_{i+1}]$. Because of Theorem 2 we have

$$|\ell(u_\varepsilon(x_i),x) - \ell(u_i,x)| \le M/n^2 \ .$$

Now we shall prove

$$|u_\varepsilon(x) - \ell(u_\varepsilon(x_i),x)| \le M/n^2 \ .$$

Again, it is sufficient to show that

$$(20) \qquad |R_i| \le M/n^2 \ , \quad i = 0,1,\dots,n_o-1 \ ,$$

where $R_i = v_\varepsilon(x) - \ell(v_\varepsilon(x_i),x)$. For other i's the proof of (20) is analogous. We have

$$|R_i| \le M(\lambda'(t_{i+1}))^2\varepsilon^{-2}v_\varepsilon(x_i)/n^2$$

and we get (20) in both cases 1^o and 2^o of the proof of Theorem 2. In case 3^o we use

$$|R_i| \le Mv_\varepsilon(x_i)$$

to get (20) again. □

7. NUMERICAL RESULTS

We shall test our method on the following linear problem from [8]:

$$(21)\qquad \begin{aligned} &-\varepsilon^2 u''(x) + u(x) = -(\cos^2\pi x + 2(\varepsilon\pi)^2\cos 2\pi x) ,\\ &x \in I, \quad u(0) = u(1) = 0 , \end{aligned}$$

with the exact solution:

$$u_\varepsilon(x) = (\exp(-x/\varepsilon) + \exp(-(1-x)/\varepsilon))/(1 + \exp(-1/\varepsilon)) - \cos^2\pi x .$$

Since $u_\varepsilon(1/2 + x) = u_\varepsilon(1/2 - x)$, $x \in [0,1/2]$, it is sufficient to solve (21) on the interval $[0,1/2]$.

We use the mesh given via ϕ_1 with $p = 1$, because it is the simplest function and the results for ϕ_o and ϕ_1 with $p \neq 1$ are very similar. Note that here $\alpha_2 = q - \sqrt{aq\varepsilon}$, and we do not need the condition $a\beta \geq 2$ in Theorem 2. So, a is such a constant that $0 < a\varepsilon_o/q < 1$.

In our numerical experiments we shall vary ε, a, q and n_o. The width of the boundary layer is of order ε. We shall be interested in a number n_1 of mesh points in $(0,\varepsilon]$. For a,q and n_o fixed, this number is invariable to the change of ε. Let

$$E = \max_{n_1 < i < n_o} |u_\varepsilon(x_i) - u_i| ,$$

$$E_1 = \max_{0 < i \leq n_1} |u_\varepsilon(x_i) - u_i|$$

and let P and P_1 be the corresponding maximal percentage errors.

Tables 1-4 contain the results for u_i. In Table 5 we give the results of linear interpolation. We interpolate the numerical results of the first row of Table 4.

TABLE 1. $a = 1,\ q = 0.4,\ n_o = 10 \Rightarrow n_1 = 4$

ε	E_1	E	P_1	P
0.1	$7.22\cdot 10^{-3}$	$3.14\cdot 10^{-3}$	1.3	4.4
$10^{-2}-10^{-16}$ *)	$1.35\cdot 10^{-2}$	$1.72\cdot 10^{-2}$	2.2	2.1

*) ε was changed as $\varepsilon = 10^{-2s}$, $s = 1,2,\dots,8$.

TABLE 2. $a = 0.5,\ q = 0.48,\ n_o = 10 \Rightarrow n_1 = 6$

ε	E_1	E	P_1	P
0.1	$1.62\cdot 10^{-2}$	$2.21\cdot 10^{-2}$	3.3	3.9
$10^{-2}-10^{-16}$	$1.74\cdot 10^{-2}$	$3.31\cdot 10^{-2}$	3.1	3.6

TABLE 3. $a = 0.5,\ q = 0.48,\ n_o = 20 \Rightarrow n_1 = 12$

ε	E_1	E	P_1	P
0.1	$4.15\cdot 10^{-3}$	$5.70\cdot 10^{-3}$	0.83	3.7
$10^{-2}-10^{-16}$	$4.20\cdot 10^{-3}$	$7.24\cdot 10^{-3}$	0.74	0.87

TABLE 4. $n_o = 100$, $\varepsilon = 10^{-6}$

	n_1	E_1	E	P_1	P
$a = 1$ $q = 0.4$	40	$1.32\cdot 10^{-4}$	$1.71\cdot 10^{-4}$	0.021	0.021
$a = 0.3$ $q = 0.49$	75	$3.78\cdot 10^{-4}$	$6.70\cdot 10^{-4}$	0.061	0.074

TABLE 5. $a = 1$, $q = 0.4$, $n_o = 100$, $\varepsilon = 10^{-6}$

x	$E_2 = \|\ell(u_i, x) - u_\varepsilon(x)\|$	$(E_2/\|u_\varepsilon(x)\|)\cdot 100$
10^{-9}	$5.62\cdot 10^{-6}$	0.56
10^{-7}	$3.17\cdot 10^{-6}$	0.0033
10^{-3}	$1.65\cdot 10^{-5}$	0.0017
0.1	$4.19\cdot 10^{-4}$	0.046
0.2	$1.36\cdot 10^{-4}$	0.021
0.3	$6.96\cdot 10^{-5}$	0.020
0.4	$1.05\cdot 10^{-4}$	0.11
0.45	$6.38\cdot 10^{-5}$	0.26

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REZIME

O NUMERIČKOM REŠAVANJU JEDNOG TIPA SINGULARNO PERTURBIRANOG PROBLEMA KORIŠĆENJEM SPECIJALNE MREŽE DISKRETIZACIJE

U radu se daje uopštenje konstrukcije mreže iz [1] za diskretizaciju singularno preturbiranog problema (1) metodom konačnih razlika. Nalazi se klasa funkcija koje generišu tačke mreže, omogućujući kvadratnu konvergenciju, uniformnu po malom perturbacionom parametru ε. Takodje su ispitane mogućnosti linearne interpolacije numeričkih rezultata i za ovaj metod je pokazana uniformnost po ε i očuvanje reda tačnosti numeričkih rezultata.