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COMMON FIXED POINT THEOREMS IN METRIC SPACES

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ABSTRACT

In this paper we generalize the Theorem [3] and Theorem 1 from [4].

The following common fixed point theorem is proved in [3].

THEOREM A. *Let* (X,d) *be a complete metric space,* S *and* T *one to one continuous mappings from* X *into* X, A *a continuous mapping from* X *into* $SX \cap TX$ *and* A *commute with* S *and* T.

Suppose that the following conditions are satisfied:

1. *For every* $x \in X$ *there exists* $n(x) \in \mathbb{N}$ *so that for every* $y \in X$:

   $$d(A^{n(x)}x, A^{n(x)}y) \leq q \min\{d(Sx,Ty), d(Tx,Sy)\}$$

   where $q \in [0,1)$.

2. *For every* $x \in X$, *one of the sets*

   $\{A^m T^p(x) \mid p \in \mathbb{N},\ m \in \{0,1,\ldots,n(x)-1\}\}$ *and*
   $\{A^m S^p(x) \mid p \in \mathbb{N},\ m \in \{0,1,\ldots,n(x)-1\}\}$
   is bounded.

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Then there exists one and only one element $z \in X$ *such that:*

$$z = Az = Sz = Tz.$$

In the proof of Theorem 1, which is a generalization of Theorem A, we shall use the following Lemma proved in [5].

LEMMA. *Let* $Q_q = \{g \mid g \in \mathbb{R}^+,\ g \le q + a(g)\}$, *for every* $q \in \mathbb{R}^+$, *where* $a : [0,\infty) \to [0,\infty)$ *is a given non-decreasing function such that* $\lim_{n\to\infty} a^n(g) = 0$ *for* $g > 0$ *and* $\lim_{g\to\infty}(g - a(g)) = \infty$. *Then:*

1) $Q_q \ne \emptyset$ *and* $\hat{a}(Q_q) \subseteq Q_q$, *where* $\hat{a}(g) = q + a(g)$, $g \ge 0$.

2) Q_q *is bounded for each* $q > 0$ *and the maximal solution* $m(q) = \sup_{t\in Q_q} t$ *of the inequality* $g \le q + a(g)$ *is a fixed point of* $\hat{a}$.

3) *The maximal solution* $m(0)$ *of the inequality* $g \le a(g)$ *is equal to* 0.

THEOREM 1. *Let* $(X,d), S, T$ *and* A *be as in Theorem A, where instead of* 1. *and* 2. *the following conditions are satisfied:*

1) *For every* $x \in X$ *there exists* $n(x) \in \mathbb{N}$ *so that for every* $y \in X$:

$$d(A^{n(x)}x, A^{n(x)}y) \le \min\{q(d(Sx,Ty))d(Sx,Ty), d(Tx,Sy)\}$$

where $q : [0,\infty) \to [0,1)$ *is a nondecreasing function such that* $\lim_{t\to\infty} t(1-q(t)) = \infty$.

2) *For some* $x_o \in X$ *one of the sets*

$\{A^m T^p(x_o) \mid p \in \mathbb{N},\ m \in \{0,1,\ldots,n(x_o)-1\}\}$ and

$\{A^m S^p(x_o) \mid p \in \mathbb{N},\ m \in \{0,1,\ldots,n(x_o)-1\}\}$

is bounded.

Then there exists one and only one element $y \in X$ *such that:*

$$y = Sy = Ty = Ay$$

P r o o f. Let us suppose that the set:

$$M = \{A^m T^p x_o \mid p \in \mathbb{N},\quad m \in \{0,1,\dots,n(x_o)-1\}\}$$

is bounded. We shall prove that the set:

$$\{d(A^n T^k x_o, Sx_o) \mid n,k \in \mathbb{N}_o\} \qquad (\mathbb{N}_o = \mathbb{N} \cup \{0\})$$

is bounded. Let $n = p\cdot n(x_o) + r$, where $0 \le r < n(x_o)$. Let us prove that:

(1) $d(A^{p\cdot n(x_o)+r} T^k x_o, Sx_o) \le b_p$, for every $p \in \mathbb{N}$ and $k \in \mathbb{N}_o$

where $b_o = \sup\limits_{t \in M \cup \{A^{n(x_o)} x_o\}} d(t, Sx_o)$ and $b_p = b_o + q(b_{p-1})b_{p-1}$, $p \in \mathbb{N}$.

The proof of (1) will be given by induction in respect to $p \in \mathbb{N}$.

For $p = 1$ we have that:

$$d(A^{n(x_o)+r} T^k x_o, Sx_o) \le d(A^{n(x_o)+r} T^k x_o, A^{n(x_o)} x_o) + d(A^{n(x_o)} x_o, Sx_o)$$

$$\le q(d(Sx_o, A^r T^{k+1} x_o))d(Sx_o, A^r T^{k+1} x_o) + d(A^{n(x_o)} x_o, Sx_o)$$

$$\le q(b_o)b_o + b_o = b_1,\quad \text{for every } k \in \mathbb{N}_o.$$

Suppose that (1) is satisfied for some $p \in \mathbb{N}$ and every $k \in \mathbb{N}_o$ and prove that

$$d(A^{(p+1)n(x_o)+r} T^k x_o, Sx_o) \le b_{p+1},\quad \text{for every } k \in \mathbb{N}_o.$$

We have that:

$$d(A^{(p+1)n(x_o)+r} T^k x_o, Sx_o) \le d(A^{(p+1)n(x_o)+r} T^k x_o, A^{n(x_o)} x_o) +$$

$$+ d(A^{n(x_o)} x_o, Sx_o) \le q(d(Sx_o, A^{pn(x_o)+r} T^{k+1} x_o))d(Sx_o,$$

$$A^{pn(x_o)+r} T^{k+1} x_o) + d(A^{n(x_o)} x_o, Sx_o) \le b_o + q(b_p)b_p = b_{p+1}\ .$$

Let $a(t) = t\cdot q(t)$, $t \geq 0$. Then $a^n(t) \leq t(q(t))^n$, for every $t \geq 0$ and since $q(t) < 1$, for every $t \geq 0$ it follows that $\lim_{n\to\infty} a^n(t) = 0$. It is obvious that $\lim_{t\to\infty}[t-a(t)] = \infty$ and so we may apply the Lemma. Let $\hat{a}(t) = b_o + a(t)$ $(t \geq 0)$. From $b_o \leq \hat{a}(b_o)$ we obtain that $b_o \in Q_{b_o}$ and since $\hat{a}(Q_{b_o}) \subseteq Q_{b_o}$ it follows that $\{b_p | p \in \mathbb{N}\} \subseteq Q_{b_o}$ which implies that the sequence $\{b_p\}_{p\in\mathbb{N}}$ is bounded. Thus, the set:

$$\{d(A^nT^kx_o, Sx_o) \,|\, n,k \in \mathbb{N}_o\}$$

is bounded and let $D = \sup_{n,k\in\mathbb{N}_o} d(A^nT^kx_o, Sx_o)$. Since $AX \subseteq SX \cap TX$ it follows that there exists a sequence $\{x_n\}_{n\in\mathbb{N}_o}$ such that $Sx_{2k} = A^{n(x_{2k-1})}x_{2k-1}, Tx_{2k-1} = A^{n(x_{2k-2})}x_{2k-2}, k \in \mathbb{N}$.

Let:

$$y_m = \begin{cases} Tx_{2k-1}, & m = 2k-1 \\ Sx_{2k}, & m = 2k \end{cases} \qquad (k \in \mathbb{N}).$$

We shall prove that $\{y_m\}_{m\in\mathbb{N}}$ is a Cauchy sequence. For every $k \in \mathbb{N}$ we have:

$$d(y_{2k-1}, y_{2k}) = d(Tx_{2k-1}, Sx_{2k}) = d(A^{n(x_{2k-2})}x_{2k-2}, A^{n(x_{2k-1})}x_{2k-1})$$

$$= d(A^{n(x_{2k-2})}x_{2k-2}, A^{n(x_{2k-1})}T^{-1}Tx_{2k-1}) =$$

$$= d(A^{n(x_{2k-2})}x_{2k-2}, A^{n(x_{2k-1})}T^{-1}A^{n(x_{2k-2})}x_{2k-2}) \leq$$

$$\leq q(d(Sx_{2k-2}, A^{n(x_{2k-1})}x_{2k-2}))d(Sx_{2k-2}, A^{n(x_{2k-1})}x_{2k-2}) =$$

$$= q(d(A^{n(x_{2k-3})}x_{2k-3}, A^{n(x_{2k-1})}x_{2k-2}))\cdot d(A^{n(x_{2k-3})}x_{2k-3},$$

$$, A^{n(x_{2k-1})}x_{2k-2})$$

Further:

$$d(A^{n(x_1)}x_1, A^k x_2) = d(A^{n(x_1)}x_1, A^k S^{-1} A^{n(x_1)}x_1) \leq d(Tx_1, A^k x_1) =$$

$$= d(A^{n(x_o)}x_o, A^k x_1) = d(A^{n(x_o)}x_o, A^k T^{-1} A^{n(x_o)}x_o) \leq$$

$$\leq q(d(Sx_o, A^k x_o))d(Sx_o, A^k x_o).$$

Suppose that for some $k > 2$ and every $m \in \mathbb{N}$:

$$(2)\quad d(A^{n(x_{2k-3})}x_{2k-3}, A^m x_{2k-2}) \leq (q(d(Sx_o, A^m x_o)))^{k-1} d(Sx_o, A^m x_o).$$

Then:

$$d(A^{n(x_{2k-1})}x_{2k-1}, A^m x_{2k}) = d(A^{n(x_{2k-1})}x_{2k-1}, A^m S^{-1} Sx_{2k}) =$$

$$= d(A^{n(x_{2k-1})}x_{2k-1}, A^m S^{-1} A^{n(x_{2k-1})}x_{2k-1}) \leq$$

$$\leq d(Tx_{2k-1}, A^m x_{2k-1}) = d(A^{n(x_{2k-2})}x_{2k-2}, A^m x_{2k-1}) =$$

$$= d(A^{n(x_{2k-2})}x_{2k-2}, A^m T^{-1} A^{n(x_{2k-2})}x_{2k-2}) \leq$$

$$\leq q(d(Sx_{2k-2}, A^m x_{2k-2}))d(Sx_{2k-2}, A^m x_{2k-2}) =$$

$$= q(d(A^{n(x_{2k-3})}x_{2k-3}, A^m x_{2k-2}))d(A^{n(x_{2k-3})}x_{2k-3}, A^m x_{2k-2}) \leq$$

$$\leq q(d(Sx_o, A^m x_o))(q(d(Sx_o, A^m x_o)))^{k-1} d(Sx_o, A^m x_o) =$$

$$= (q(d(Sx_o, A^m x_o)))^k d(Sx_o, A^m x_o)$$

and, so (2) is satisfied for every $k \in \mathbb{N}$ and every $m \in \mathbb{N}$.

So we have that:

$$d(y_{2k-1}, y_{2k}) \leq (q(d(Sx_o, A^{n(x_{2k-1})}x_o)))^k d(Sx_o, A^{n(x_{2k-1})}x_o) \leq$$

$$\leq q^k(D)D.$$

Further:

$$d(y_{2k},y_{2k+1}) = d(Sx_{2k},Tx_{2k+1}) = d(A^{n(x_{2k-1})}x_{2k-1},A^{n(x_{2k})}x_{2k}) \leq$$
$$\leq q^k(D)D$$

and so:

$$d(y_m,y_{m+1}) \leq (q(D))^{[\frac{m+1}{2}]} D, \quad m \in \mathbb{N} .$$

This implies that $\{y_m\}_{m\in \mathbb{N}}$ is a Cauchy sequence and let $\lim_{m\to\infty} y_m = y$. Since $\{Sx_{2k}\}_{k\in \mathbb{N}}$ and $\{Tx_{2k-1}\}_{k\in \mathbb{N}}$ are subsequence of the sequence $\{y_n\}_{n\in \mathbb{N}}$ it follows that:

$$\lim_{k\to\infty} Sx_{2k} = \lim_{k\to\infty} Tx_{2k-1} = a$$

We have that:

$$d(Sx_{2k},Ax_{2k}) = d(A^{n(x_{2k-1})}x_{2k-1},Ax_{2k}) \leq (q(d(Sx_o,Ax_o))^k\cdot$$
$$\cdot d(Sx_o,Ax_o) \leq (q(D))^k D$$

for every $k \in \mathbb{N}$ and so $\lim_{k\to\infty} Ax_{2k} = \lim_{k\to\infty} Sx_{2k} = y$. Furthermore,

$$d(Sx_{2k},A^2x_{2k}) = d(A^{n(x_{2k-1})}x_{2k-1},A^2x_{2k}) \leq (q(d(Sx_o,A^2x_o))^k\cdot$$
$\cdot d(Sx_o,A^2x_o) \leq (q(D))^k D$, for every $k \in N$, which implies that:

$$\lim_{k\to\infty} A^2x_{2k} = \lim_{k\to\infty} Sx_{2k} = y .$$

From the continuity of A and S and the commutativity we obtain that:

$$Ay = A(\lim_{k\to\infty} Sx_{2k}) = S(\lim_{k\to\infty} Ax_{2k}) = Sy.$$

The relation $y = Ay$ follows from $y = \lim_{k\to\infty} A^2x_{2k} = A(\lim_{k\to\infty} Ax_{2k}) = Ay$.

Similarly, it follows that $Ay = Ty$ and so we have that y is a common fixed point for the mappings A,S and T.

Let us prove the uniqueness of the common fixed point y.

Suppose that $z \in X$ and $z = Az = Tz = Sz$. Then we have that:

$$d(z,y) = d(A^{n(z)}z, A^{n(z)}y) \leq q(d(Sz,Ty))d(Sz,Ty)$$
$$= q(d(z,y))d(z,y)$$

which implies that $z = y$ and so y is the unique common fixed point for mappings A, S and T.

The following theorem is a generalization of Theorem 1 from [4].

THEOREM 2. *Let* (X,d) *be a complete metric space,* S *and* T *continuous mappings of* X *into* X, $A_j : X \to SX \cap TX$ $(j \in N)$ *so that* A_j *commutes with* S *and* T, $q : [0,\infty) \to [0,1)$ *a nondecreasing continuous function and for every* $x,y \in X$:

$$d(A_i x, A_j y) \leq q(d(Sx,Ty))d(Sx,Ty), \ i \neq j \quad (i,j \in \mathbb{N}) .$$

Then there is a unique common fixed point for $\{A_j\}_{j\in \mathbb{N}}$, S *and* T.

P r o o f. The proof is similar to the proof of Theorem 1 from [4]. As in [4], there exists a sequence $\{x_n\}_{n\in \mathbb{N}}$ such that

$$Tx_{2n+1} = A_{2n+1}x_{2n}, \ n \in \mathbb{N}_o, \ Sx_{2n} = A_{2n}x_{2n-1}, \ n \in \mathbb{N}$$

where x_o is an arbitrary element from X. Let us prove that there exists $w \in X$ so that $\lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} Sx_{2n} = w$. For every $n \in \mathbb{N}$ we have:

$$d(Tx_{2n-1},Sx_{2n}) = d(A_{2n-1}x_{2n-2},A_{2n}x_{2n-1}) \leq q(d(Sx_{2n-2},Tx_{2n-1}))$$

$$d(Sx_{2n-2},Tx_{2n-1}) = q(d(A_{2n-2}x_{2n-3},A_{2n-1}x_{2n-2}))d(A_{2n-2}x_{2n-3},$$
$$,A_{2n-1}x_{2n-2}) \leq$$

$$\leq q(d(A_{2n-2}x_{2n-3},A_{2n-1}x_{2n-2}))q(d(Tx_{2n-3},Sx_{2n-2}))d(Tx_{2n-3},Sx_{2n-2}) =$$

$$= q(d(A_{2n-2}x_{2n-3},A_{2n-1}x_{2n-2}))q(d(A_{2n-3}x_{2n-4},A_{2n-2}x_{2n-3})) \times$$

$$\times d(A_{2n-3}x_{2n-4},A_{2n-2}x_{2n-3}) \leq \prod_{m=2}^{2n-1} q(d(A_{m-1}x_{m-2},A_m x_{m-1}))$$

$$\times d(A_1 x_o,A_2 x_1) = q(d(Sx_{2n-2},Tx_{2n-1}))q(d(Tx_{2n-3},Sx_{2n-2}))\ldots$$

$$\ldots\ q(d(Tx_1,Sx_2))d(Tx_1,Sx_2).$$

This implies that:

$$d(Tx_{2n-1},Sx_{2n}) \leq d(Tx_1,Sx_2)$$

and similarly:

$$d(Sx_{2n},Tx_{2n+1}) \leq d(Tx_1,Sx_2).$$

Since q is nondecreasing we obtain that:

$$d(Tx_{2n-1},Sx_{2n}) \leq (\prod_{m=2}^{2n-1} q(d(Tx_1,Sx_2)))d(Tx_1,Sx_2)$$

$$= (q(d(Tx_1,Sx_2)))^{2n-2}d(Tx_1,Sx_2)$$

and

$$d(Sx_{2n},Tx_{2n+1}) \leq (q(d(Tx_1,Sx_2)))^{2n-1}d(Tx_1,Sx_2).$$

Then we have:

$$d(Tx_{2n-1},Tx_{2n+1}) \leq d(Tx_{2n-1},Sx_{2n}) + d(Sx_{2n},Tx_{2n+1}) \leq$$

$$\leq [(q(d(Tx_1,Sx_2)))^{2n-1} + (q(d(Tx_1,Sx_2)))^{2n-2}]d(Tx_1,Sx_2)$$

and so there exists $\lim_{n\to\infty} Tx_{2n-1} = w$.
From $\lim_{n\to\infty} d(Tx_{2n-1},Sx_{2n}) = 0$ we obtain that $\lim_{n\to\infty} Sx_{2n} = w$.

As in [4] we have that:

$Tw = \lim_{n\to\infty} A_{2n}A_{2n-1}x_{2n-2}$ and $Sw = \lim_{n\to\infty} A_{2n+1}A_{2n}x_{2n-1}$ and so:

(3) $$d(Tw,Sw) = \lim_{n\to\infty} d(A_{2n}A_{2n-1}x_{2n-2},A_{2n+1}A_{2n}x_{2n-1}).$$

Let us prove that the limit in (3) is equal to zero.

We have that:

$d(A_{2n}A_{2n-1}x_{2n-2},A_{2n+1}A_{2n}x_{2n-1}) \leq q(d(SA_{2n-1}x_{2n-2},TA_{2n}x_{2n-1}))$

$d(SA_{2n-1}x_{2n-2},TA_{2n}x_{2n-1}))=q(d(A_{2n-1}A_{2n-2}x_{2n-3},A_{2n}A_{2n-1}x_{2n-2}))$

$d(A_{2n-1}A_{2n-2}x_{2n-3},A_{2n}A_{2n-1}x_{2n-2}) \leq q(d(A_{2n-1}A_{2n-2}x_{2n-3},$

$,A_{2n}A_{2n-1}x_{2n-2}))q(d(TA_{2n-2}x_{2n-3},SA_{2n-1}x_{2n-2}))d(TA_{2n-2}x_{2n-3},$

$,SA_{2n-1}x_{2n-2})) = q(d(A_{2n-1}A_{2n-2}x_{2n-3},A_{2n}A_{2n-1}x_{2n-2}))$

$q(d(A_{2n-2}A_{2n-3}x_{2n-4},A_{2n-1}A_{2n-2}x_{2n-3}))d(A_{2n-2}A_{2n-3}x_{2n-4},$

$,A_{2n-1}A_{2n-2}x_{2n-3}) \leq \dots \leq (\prod_{m=3}^{2n-1} q(d(A_mA_{m-1}x_{m-2},A_{m+1}A_mx_{m-1})))$

$d(A_2A_1x_o,A_3A_2x_1)$.

From this we obtain:

$$d(A_{2n}A_{2n-1}x_{2n-2},A_{2n+1}A_{2n}x_{2n-1}) \leq (\prod_{m=3}^{2n-1} q(d(A_2A_1x_o,A_3A_2x_1))) \times$$

$$\times\, d(A_2A_1x_o,A_3A_2x_1) = (q(d(A_2A_1x_o,A_3A_2x_1)))^{2n-3}d(A_2A_1x_o,A_3A_2x_1)$$

which implies that:

$$\lim_{n\to\infty} d(A_{2n}A_{2n-1}x_{2n-2},A_{2n+1}\,A_{2n}x_{2n-1}) = 0$$

Thus, we have that $Tw = Sw$. Let us prove that $A_nw = Sw$. Suppose that $n \neq 2m$. Then we have that:

$$d(A_{2m}Tx_{2m-1},A_nw) \leq q(d(STx_{2m-1},Tw))d(STx_{2m-1},Tw) \leq$$
$$\leq d(STx_{2m-1},Tw)$$

and since $A_{2m}Tx_{2m-1} = TA_{2m}x_{2m-1} = TSx_{2m}$ we obtain that:

$$d(TSx_{2m},A_nw) \leq d(STx_{2m-1},Tw) \quad (2m \neq n) .$$

This implies that:

$$d(Tw,A_nw) = \lim_{m\to\infty} d(TSx_{2m},A_nw) = \lim_{m\to\infty} d(A_{2m}Tx_{2m-1},A_nw) \le$$

$$\le d(Sw,Tw) \quad (n \in N)$$

and since $Sw = Tw$ we obtain that $Tw = A_nw$. We shall prove that A_nw is the unique fixed point of the mappings $\{A_n\}_{n\in \mathbb{N}}$, S and T.

Let $2m \neq n$. Then from:

$$d(A_nw,A_nA_nw) \le d(A_nw,A_{2m}Tx_{2m-1}) + d(A_{2m}Tx_{2m-1},A_nA_nw) \le$$

$$\le d(A_nw,TA_{2m}x_{2m-1}) + q(d(STx_{2m-1},TA_nw))d(STx_{2m-1},TA_nw) =$$

$$= d(A_nw,TSx_{2m}) + q(d(STx_{2m-1},TA_nw))d(STx_{2m-1},TA_nw)$$

and $A_nTw = TA_nw = A_nA_nw = SA_nw$ we obtain that:

$$d(A_nw,A_nA_nw)\le d(A_nw,TSx_{2m}) + q(d(STx_{2m-1},A_nA_nw))d(STx_{2m-1},A_nA_nw),$$

for every $n \in \mathbb{N}$ and $m \in \mathbb{N}$ such that $2m \neq n$ which implies that:

$$d(A_nw,A_nA_nw) \le \lim_{m\to\infty} d(A_nw,TSx_{2m}) + \lim_{m\to\infty} q(d(STx_{2m-1},A_nA_nw))\cdot$$

$$d(STx_{2m-1},A_nA_nw) = d(A_nw,Tw) + q(d(Sw,A_nA_nw))d(Sw,A_nA_nw) =$$

$$= q(d(A_nw,A_nA_nw))d(A_nw,A_nA_nw).$$

Since $q(d(A_nw,A_nA_nw)) < 1$ it follows that $d(A_nw,A_nA_nw) = 0$ and so A_nw is a common fixed point for $\{A_n\}_{n\in \mathbb{N}}$, S and T. Let us prove that A_nw is the unique fixed point for $\{A_n\}_{n\in \mathbb{N}}$, S and T. Suppose that $u = Tu = A_nu = Su$, for every $n \in \mathbb{N}$ and $v = Tv = A_nv = Sv$, for every $n \in \mathbb{N}$. Then we have that:

$$d(u,v) = d(A_nu,A_mv) \le q(d(Su,Tv))d(Su,Tv) \qquad (n \neq m)$$

and so $d(u,v) \le q(d(u,v))d(u,v)$. From this we obtain that $u = v$.

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REZIME

TEOREME O ZAJEDNIČKOJ NEPOKRETNOJ TAČKI U METRIČKIM PROSTORIMA

U ovom radu su uopštene teorema iz rada [3] i teorema 1 iz rada [4].