

ON COMMON FIXED POINTS IN METRIC AND PROBABILISTIC
METRIC SPACES WITH CONVEX STRUCTURES

Olga Hadžić

*Prirodno-matematički fakultet. Institut za matematiku
21000 Novi Sad, ul. dr Ilije Djuriđića br.4, Jugoslavija*

ABSTRACT

In this paper we prove some generalizations of Theorems 1 and 2 from [5] in metric and probabilistic metric spaces with convex structures.

W. Takahashi introduced in [14] a notion of convexity in metric spaces and generalized some fixed point theorems in Banach spaces. Some fixed point theorems in metric spaces with convex structures are obtained by S. Itoh [8], S.A. Nainpally, K.L. Singh and J.H.M. Whitfield [10], [11], B.E. Rhoades, K.L. Singh and J.H.M. Whitfield [12] and L.A. Tallman [17] and for metric spaces of the hyperbolic type in [3].

In the first part of this paper we shall prove a generalization of Theorem 1 from [5] where the measure of noncompactness Ψ is, in this paper, the Kuratowski measure of noncompactness α . The second part of this paper contains a generalization of Theorem 2 from [5] on probabilistic metric spaces with a convex structure.

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1. First, we shall give some definitions, where $I = [0, 1]$.

DEFINITION 1. [16] Let X be a metric space. A mapping $W : X \times X \times I \rightarrow X$ is said to be a convex structure if for every $(x, y, \lambda) \in X \times X \times I$:

$$(1) \quad d(u, W(x, y, \lambda)) \leq \lambda d(u, x) + (1-\lambda)d(u, y), \text{ for every } u \in X.$$

A metric space with a convex structure will be called a convex metric space. There are many convex metric spaces which cannot be imbedded in any Banach space [16].

REMARK. From (1) we obtain, for $\lambda = 1$ and $u = x$ that $d(u, W(x, y, 1)) = 0$ which implies that $x = W(x, y, 1)$. Similarly it follows that $y = W(x, y, 0)$. The mapping W is not continuous in general, however if X is compact then W is continuous [16]. If a convex metric space X is such that $co_W(A)$ is compact for every finite subset A of X , where we denote by $co_W(M)$ ($M \subseteq X$) the intersection of all convex sets N such that $M \subseteq N$ (A set $N \subseteq X$ is convex if for every $(x, y, \lambda) \in N \times N \times I : W(x, y, \lambda) \in N$) then the mapping W defines a pseudoconvex structure on X in the sense of the Definition in [7].

DEFINITION 2. Let X be a convex metric space, $x_0 \in X$ and $S : X \rightarrow X$. The mapping S is said to be (W, x_0) -convex if for every $z \in X$ and $\lambda \in I$:

$$W(Sz, x_0, \lambda) = S(W(z, x_0, \lambda)).$$

REMARK. If X is a Banach space and $W(x, y, \lambda) = \lambda x + (1-\lambda)y$, for every $(x, y, \lambda) \in X \times X \times I$, every homogeneous mapping $S : X \rightarrow X$ is $(W, 0)$ -convex.

DEFINITION 3. [12] A convex metric space X satisfies condition (II) if for all $(x, y, z, \lambda) \in X \times X \times X \times I$:

$$d(W(x, z, \lambda), W(y, z, \lambda)) \leq d(x, y).$$

The Kuratowski measure of noncompactness of a set

$D(D \subseteq X)$ is defined by $\alpha(D) = \inf\{\epsilon \mid \epsilon > 0, \text{ there exists } \{A_j\}_{j \in J}, J \text{ is finite, such that } D \subseteq \bigcup_{j \in J} A_j \text{ and } \text{diam } A_j < \epsilon, \text{ for every } j \in J\}$.

A continuous mapping $T : X \rightarrow X$ is said to be α -densifying on $M \subseteq X$ if for any bounded subset D of M , the set $T(D)$ is bounded and:

$$\alpha(D) > 0 \Rightarrow \alpha(T(D)) < \alpha(D).$$

In [2], the following theorem is proved.

THEOREM A. *Let S and T be continuous mappings of a complete metric space (X, d) into itself. Then S and T have a common fixed point in X if and only if there exists a continuous mapping $A : X \rightarrow SX \cap TX$, which commutes with S and T and satisfies the inequality:*

$$d(Ax, Ay) \leq qd(Sx, Ty), \text{ for every } x, y \in X,$$

where $q \in [0, 1)$ and S, T and A then have a unique common fixed point.

We shall use Theorem 1 in the proof of the following theorem.

THEOREM 1. *Let (X, d) be a complete, convex metric space which satisfies condition (II), A, S and T continuous mappings from X into X such that A commutes with S and T , $AX \subseteq SX \cap TX$, AX be bounded and the following conditions are satisfied:*

1. For every $x, y \in X$:

$$d(Ax, Ay) \leq d(Sx, Ty) .$$

2. There exists $m \in \mathbb{N}$ such that A^m is α -densifying on $\{W(z, x_0, \lambda) \mid z \in AX, \lambda \in [0, 1]\}$ for some $x_0 \in X$.

If S and T are (W, x_0) -convex there exists $x \in X$ such that $x = Ax = Sx = Tx$.

P r o o f: Let $\{k_n\}_{n \in \mathbb{N}}$ be a sequence of real numbers from $(0,1)$ such that $\lim_{n \rightarrow \infty} k_n = 1$ and for every $n \in \mathbb{N}$:

$$A_n x = W(Ax, x_0, k_n), \text{ for every } x \in X.$$

We shall prove that for every $n \in \mathbb{N}$ there exists $x_n \in X$, so that

$$x_n = A_n x_n = Sx_n = Tx_n.$$

Since X satisfies condition (II) we have that:

$$\begin{aligned} d(A_n x, A_n y) &= d(W(Ax, x_0, k_n), W(Ay, x_0, k_n)) \leq k_n d(Ax, Ay) \leq \\ &\leq k_n d(Sx, Ty) \end{aligned}$$

for every $x, y \in X$. Further, from $AX \subseteq SX \cap TX$, since S and T are (W, x_0) -convex, it follows that:

$A_n x = W(Ax, x_0, k_n) = W(Sz_x, x_0, k_n) = W(Tw_x, x_0, k_n)$ and so $W(Ax, x_0, k_n) = S(W(z_x, x_0, k_n)) = T(W(w_x, x_0, k_n)) \in SX \cap TX$. Thus $A_n X \subseteq SX \cap TX$ and since A and S are commutative we have: $A_n Sx = W(ASx, x_0, k_n) = W(SAx, x_0, k_n) = S(W(Ax, x_0, k_n)) = SA_n x$, for every $x \in X$ and every $n \in \mathbb{N}$ and similarly $A_n Tx = TA_n x$, for every $n \in \mathbb{N}$ and every $x \in X$. Thus, all the conditions of Theorem A are satisfied and there exists $x_n \in X$ such that $x_n = A_n x_n = Sx_n = Tx_n$. Furthermore:

$$\begin{aligned} d(x_n, Ax_n) &= d(A_n x_n, Ax_n) = d(W(Ax_n, x_0, k_n), Ax_n) \leq \\ &\leq k_n d(Ax_n, Ax_n) + (1-k_n) d(Ax_n, x_0) \end{aligned}$$

and since AX is bounded, it follows that:

$$(2) \quad \lim_{n \rightarrow \infty} d(x_n, Ax_n) = 0.$$

Let us prove that (2) implies:

$$(3) \quad \lim_{n \rightarrow \infty} d(x_n, A_n^m x_n) = 0.$$

Since for every $k \in \mathbb{N}$:

$$\begin{aligned} d(A^k x_n, A^{k+1} x_n) &\leq d(S(A^{k-1} x_n), T(A^k x_n)) = d(A^{k-1} Sx_n, A^k Tx_n) = \\ &= d(A^{k-1} x_n, A^k x_n) \end{aligned}$$

it follows that:

$d(A^k x_n, A^{k+1} x_n) \leq d(x_n, Ax_n)$, for every $k \in \mathbb{N}$ and every $n \in \mathbb{N}$ and so:

$$d(x_n, A^m x_n) \leq \sum_{k=0}^{m-1} d(A^k x_n, A^{k+1} x_n) \leq md(x_n, Ax_n).$$

This implies that (3) is satisfied. Let us prove that the set $\{W(Ax, x_0, \lambda) \mid x \in X, \lambda \in (0, 1)\}$ is bounded. This follows from the inequality:

$$d(u, W(Az, x_0, \lambda)) \leq \lambda d(u, Az) + (1-\lambda)d(u, x_0) \text{ for every } (z, u) \in X \times X$$

since AX is bounded. From the relations $x_n = A_n x_n$ ($n \in \mathbb{N}$) we obtain that $\{x_n \mid n \in \mathbb{N}\} \subseteq \{W(Ax, x_0, \lambda) \mid x \in X, \lambda \in (0, 1)\}$ and so the set $\{x_n \mid n \in \mathbb{N}\}$ is bounded. Furthermore, for every $\varepsilon > 0$ we have from (3) that:

$$\alpha\{x_n \mid n \in \mathbb{N}\} \leq \alpha[B(A^m(\{x_n \mid n \in \mathbb{N}\}), \varepsilon)] \leq \alpha[A^m(\{x_n \mid n \in \mathbb{N}\})] + 2\varepsilon \quad ([10]) \text{ where } B(A, \varepsilon) = \{x \in X \mid d(x, A) < \varepsilon\} (A \subseteq X) \text{ and so:}$$

$$\alpha\{x_n \mid n \in \mathbb{N}\} \leq \alpha[A^m(\{x_n \mid n \in \mathbb{N}\})].$$

This implies that $\alpha\{x_n \mid n \in \mathbb{N}\} = 0$ and there exists a convergent subsequence $\{x_{n_k} \mid k \in \mathbb{N}\}$. Let $\lim_{k \rightarrow \infty} x_{n_k} = y$. Then from:

$$d(y, Ay) \leq d(y, x_{n_k}) + d(x_{n_k}, Ax_{n_k}) + d(Ax_{n_k}, Ay)$$

and (2), since A is continuous, it follows that $y = Ay$. From $x_{n_k} = Sx_{n_k} = Tx_{n_k}$, $k \in \mathbb{N}$, since S and T are continuous, we obtain that $y = Ay = Sy = Ty$.

2. If there exists $m \in \mathbb{N}$ such that $\overline{A^m X}$ is compact, we can prove a generalization of Theorem 2 from [5] for probabilistic metric spaces with a convex structure.

A triplet (S, F, t) is a *Menger space* [14] if and only if S is an arbitrary set, $F : S \times S \rightarrow \Delta$, where Δ denotes the set of all the distribution functions F and t is a T -norm [14] so that the following conditions are satisfied ($F(p, q) = F_{p, q}$ for every $p, q \in S$):

1. $F_{p, q}(x) = 1$, for every $x \in \mathbb{R}^+$ if and only if $p = q$.
2. $F_{p, q}(0) = 0$, for every $p, q \in S$.
3. $F_{p, q} = F_{q, p}$ for every $p, q \in S$ and
4. $F_{p, r}(x+y) \geq t(F_{p, q}(x), F_{q, r}(y))$, for every $p, q, r \in S$ and every $x, y \in \mathbb{R}^+$.

The (ϵ, λ) -topology is introduced by the (ϵ, λ) -neighbourhoods of $v \in S$:

$$U_v(\epsilon, \lambda) = \{u \mid u \in S, F_{u, v}(\epsilon) > 1 - \lambda\}, \quad \epsilon > 0, \lambda \in (0, 1).$$

DEFINITION 4. Let (S, F, t) be a Menger space. A mapping $W : S \times S \times I \rightarrow S$ is said to be a convex structure if for every $(u, x, y, \lambda) \in S \times S \times S \times (0, 1)$:

$$F_{u, W(x, y, \lambda)}(2\epsilon) \geq t(F_{u, x}(\frac{\epsilon}{\lambda}), F_{u, y}(\frac{\epsilon}{1-\lambda})), \quad \text{for every } \epsilon \in \mathbb{R}^+$$

and $W(x, y, 0) = y, W(x, y, 1) = x$.

It is easy to see that a metric space (S, d) with a convex structure is a Menger space with the same convex structure. It is known that (S, F, \min) is a Menger space, where for every $(u, v) \in X \times X$:

$$F_{u, v}(x) = \begin{cases} 0, & d(u, v) \geq x \\ 1, & d(u, v) < x \end{cases} \quad \text{for every } x \in \mathbb{R}.$$

Then we can show that a mapping $W : S \times S \times I \rightarrow S$, which satisfies the inequality from Definition 1., is a convex structure in the sense of Definition 4. Every random normed space (S, F, t) is a probabilistic metric space with a convex structure defined by:

$$W(x, y, \lambda) = \lambda x + (1-\lambda)y, \text{ for } (x, y, \lambda) \in S \times S \times I, \text{ since}$$

$$F_{u, W(x, y, \lambda)}(2\varepsilon) = F_{u - \lambda x - (1-\lambda)y}(2\varepsilon) = F_{\lambda(u-x) + (1-\lambda)(u-y)}(2\varepsilon) \geq$$

$$\geq t(F_{\lambda(u-x)}(\varepsilon), F_{(1-\lambda)(u-y)}(\varepsilon)) = t(F_{u-x}(\frac{\varepsilon}{\lambda}), F_{u-y}(\frac{\varepsilon}{1-\lambda}))$$

for every $\lambda \in (0, 1)$.

DEFINITION 5. A Menger space (S, F, t) with a convex structure $W : S \times S \times I \rightarrow S$ satisfies condition (P II) if for all $(x, y, z, \lambda) \in S \times S \times S \times (0, 1)$:

$$F_{W(x, z, \lambda), W(y, z, \lambda)}(\lambda\varepsilon) \geq F_{x, y}(\varepsilon), \text{ for every } \varepsilon \in \mathbb{R}^+.$$

Every random normed space with the convex structure, which is defined above, satisfies condition (P II) since:

$$F_{\lambda x + (1-\lambda)z - \lambda y - (1-\lambda)z}(\lambda\varepsilon) = F_{\lambda(x-y)}(\lambda\varepsilon) = F_{x-y}(\varepsilon),$$

for every $\lambda > 0$.

In [5] the following theorem is proved.

THEOREM B. Let (X, F, t) be a complete Menger space with continuous T-norm t and let S and T be continuous mappings of X into X . Then S and T have a common fixed point in X if and only if there exists a continuous mapping $A : X \rightarrow SX \cap TX$, which commutes with S and T and satisfies the following conditions:

(1) For every $x, y \in X$:

$$F_{Ax, Ay}(\varepsilon) \geq F_{Sx, Ty}(\frac{\varepsilon}{q}), \text{ for every } \varepsilon > 0,$$

where $q \in (0, 1)$.

(ii) There exists $x_0 \in X$ such that:

$$\sup_{\varepsilon} \inf_{n \in \mathbb{N}} F_{Ax_n, Ax_0}(\varepsilon) = 1$$

where $\{x_n\}_{n \in \mathbb{N}}$ is such that $Ax_{2n-2} = Sx_{2n-1}$ and $Ax_{2n-1} = Tx_{2n}$, $n \in \mathbb{N}$.

Then S, T and A have a unique common fixed point.

REMARK. Condition (ii) is satisfied if AX is a probabilistic bounded subset of X which means that

$\sup_x D_A(x) = 1$ where:

$$D_A(x) = \sup_{t < x} \inf_{p, q \in A} F_{p, q}(t), \quad x \in \mathbb{R}^+$$

If $S : X \rightarrow X$ and (X, F, t) is a Menger space with a convex structure W then S is (W, x_0) -convex ($x_0 \in X$) if, as in Definition 2, $W(Sz, x_0, \lambda) = S(W(z, x_0, \lambda))$, for every $z \in X$, and every $\lambda \in [0, 1]$.

THEOREM 2. Let (X, F, t) be a complete Menger space with a convex structure W which satisfies condition (P II), A, S and T continuous mappings from X into X such that for some $m \in \mathbb{N}$, $A^m X$ is compact, AX be probabilistic bounded, $AX \subseteq SX \cap TX$ and for every $x, y \in X$ and every $\varepsilon \in \mathbb{R}^+$:

$$F_{Ax, Ay}(\varepsilon) \geq F_{Sx, Ty}(\varepsilon).$$

If there exists $x_0 \in X$ so that S and T are (W, x_0) -convex then there exists $x \in X$ such that $x = Sx = Tx = Ax$.

Proof. Similarly as in Theorem 1 it follows that there exists, for every $n \in \mathbb{N}$, $x_n \in X$ such that $x_n = A_n x_n = Sx_n = Tx_n$, where $A_n x = W(Ax, x_0, k_n)$ for every $n \in \mathbb{N}$ and every $x \in X$ and $\lim_{n \rightarrow \infty} k_n = 1$. Furthermore:

$$F_{x_n, Ax_n}(2\varepsilon) = F_{A_n x_n, Ax_n}(2\varepsilon) = F_{W(Ax_n, x_0, k_n), Ax_n}(2\varepsilon) \geq$$

$$\begin{aligned} &\geq t(F_{AX_n, AX_n}(\frac{\epsilon}{k_n}), F_{AX_n, x_0}(\frac{\epsilon}{1-k_n})) = \\ &= t(1, F_{AX_n, x_0}(\frac{\epsilon}{1-k_n})) = F_{AX_n, x_0}(\frac{\epsilon}{1-k_n}). \end{aligned}$$

Since AX is probabilistic bounded, for every $z \in X$ we have:

$$\inf_{n \in \mathbb{N}} F_{AX_n, Az}(x) \geq \sup_{t < x} \inf_{p, q \in AX} F_{p, q}(t) = D_{AX}(x) \text{ and so:}$$

$$(4) \quad \sup_x \inf_{n \in \mathbb{N}} F_{AX_n, Az}(x) = \sup_x D_{AX}(x) = 1$$

$$\text{Let us prove that } \lim_{n \rightarrow \infty} F_{AX_n, x_0}(\frac{\epsilon}{1-k_n}) = 1.$$

Since $F_{AX_n, x_0}(\epsilon) \geq t(F_{AX_n, Az}(\frac{\epsilon}{2}), F_{Az, x_0}(\frac{\epsilon}{2}))$ for every $\epsilon \in \mathbb{R}^+$ we have that:

$$(5) \quad F_{AX_n, x_0}(\frac{\epsilon}{1-k_n}) \geq t(F_{AX_n, Az}(\frac{\epsilon}{2(1-k_n)}), F_{Az, x_0}(\frac{\epsilon}{2(1-k_n)}))$$

for every $n \in \mathbb{N}$. Using the continuity of t , relations (4) and (5) and relation $\lim_{n \rightarrow \infty} k_n = 1$, we obtain that

$$\lim_{n \rightarrow \infty} F_{AX_n, x_0}(\frac{\epsilon}{1-k_n}) = 1 \text{ and so for every } \epsilon > 0,$$

$\lim_{n \rightarrow \infty} F_{x_n, Ax_n}(\epsilon) = 1$. The set $\overline{A^m X}$ is compact and so there exists a subsequence $\{x_{n_k}\}_{k \in \mathbb{N}}$ such that $\lim_{k \rightarrow \infty} A^m x_{n_k} = y$

Similarly as in Theorem 1, we can prove that:

$$F_{A^k x_n, A^{k+1} x_n}(\epsilon) \geq F_{x_n, Ax_n}(\epsilon), \text{ for every } \epsilon \in \mathbb{R}^+, \text{ every } n \in \mathbb{N}$$

and every $k \in \mathbb{N}$.

Since:

$$\begin{aligned} F_{x_n, A^m x_n}(\epsilon) &\geq t(F_{x_n, Ax_n}(\frac{\epsilon}{2}), t(F_{x_n, Ax_n}(\frac{\epsilon}{2^2}), t(F_{x_n, Ax_n}(\frac{\epsilon}{2^3}), \dots, \\ &\quad \dots, F_{x_n, Ax_n}(\frac{\epsilon}{2^{m-1}})) \dots) \end{aligned}$$

from relation $t(1,1) = 1$, the continuity of t and:

$$\lim_{n \rightarrow \infty} F_{x_n, Ax_n} \left(\frac{\epsilon}{2^s} \right) = 1, \quad s \in \{1, 2, \dots, m-1\}$$

we obtain that:

$$(6) \quad \lim_{n \rightarrow \infty} F_{x_n, A^m x_n}(\epsilon) = 1, \quad \text{for every } \epsilon \in \mathbb{R}^+.$$

The continuity of t , relation (6) and the inequality:

$$F_{x_{n_k}, y}(\epsilon) \geq t(F_{x_{n_k}, A^m x_{n_k}} \left(\frac{\epsilon}{2} \right), F_{y, A^m x_{n_k}} \left(\frac{\epsilon}{2} \right))$$

imply that $\lim_{k \rightarrow \infty} x_{n_k} = y$. Since A is continuous from the inequality:

$$F_{y, Ay}(\epsilon) \geq t(F_{y, x_{n_k}} \left(\frac{\epsilon}{2} \right), t(F_{x_{n_k}, Ax_{n_k}} \left(\frac{\epsilon}{4} \right), F_{Ax_{n_k}, Ay} \left(\frac{\epsilon}{4} \right)))$$

it follows that $F_{y, Ay}(\epsilon) = 1$, for every $\epsilon \in \mathbb{R}^+$ and so $y = Ay$. Since S and T are continuous, from $x_{n_k} = Sx_{n_k} = Tx_{n_k}$, $k \in \mathbb{N}$ we obtain that $y = Ay = Sy = Ty$.

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REZIME

O ZAJEDNIČKIM NEPOKRETNIM TAČKAMA U METRIČKIM
I VEROVATNOSNIM METRIČKIM PROSTORIMA
SA KONVEKSNOM STRUKTUROM

U ovom radu dokazana su neka uopštenja Teorema 1 i 2 iz [5] u metričkim i verovatnosnim metričkim prostorima sa konveksnom strukturom.