Zbornik radova Prirodno-matematičkog fakulteta-Universitet u Novom Sadu
knjiga 14,1 (1984)
Review of Research Faculty of Science-University of Novi Sad, Volume 14,1(1984)

ACCURACY INCREASE FOR SOME SPLINE SOLUTIONS OF TWO-POINT BOUNDARY VALUE PROBLEMS

Katarina Surla
Prirodno-matematički fakultet. Institut za matematiku
21000 Novi Sad, ul.dr Ilije Duričića br.4, Jugoslavija

ABSTRACT

In [3] a method which gives the solution to problem (1), (2) in the form of a spline function has been described. In [6] the fitting factor of the form $\sigma_i = (h_i p_i/2)\mathrm{cth}(h_i p_i/2)$ has been introduced by which the conditions from [3] are relaxed. In that way a numerically more stable scheme has been obtained. Here is presented a better error estimation for the case $q \neq 0$ without the barrier function. Also, the convergence order for Dirichlet's boundary conditions is increased.

Let us consider the boundary value problem

$$(1)\quad Ly = y'' + p(x)y' + q(x)y = f(x) \quad x \in (a,b) \quad a,b \in R$$

$$(2)\quad \begin{cases} \ell_1 y = \alpha_a y + \beta_a y' = \gamma_a, & x = a, \quad |\alpha_a| + |\beta_a| \neq 0 \\ \ell_2 y = \alpha_b y + \beta_b y' = \gamma_b, & x = b, \quad |\alpha_b| + |\beta_b| \neq 0, \end{cases}$$

where the functions p,q and f are sufficiently smooth.

AMS Mathematics subject classification (1980): 65N35
Key words and phrases: Spline functions, two-point boundary value problem, Dirichlet's boundary conditions, inverse monotone, fitting factor.

On $[a,b]$ let the grid be given

$$a = x_o < x_1 \dots < x_{n+1} = b \; ; \quad \max_i \frac{h_i}{h_{i\pm 1}} \le M \; , \quad h_i = x_{i+1} - x_i$$

M denotes different constants independent on h, $h = \max_i h_i$. We want to obtain the approximative solution to problem (1), (2) in the form of the cubic spline $v(x)$ with the following properties:

a) $v(x) \in C^2[a,b]$

b) $v(x)$ on each interval $[x_i, x_{i+1}]$ has the form

$$(3) \quad v_i(x) = v_i^{(0)} + v_i^{(1)}(x-x_i) + \frac{1}{2}v_i^{(2)}(x-x_i)^2 + \frac{1}{6}v_i^{(3)}(x-x_i)^3$$
$$(i = 0,1,\dots,n)$$

where $v_i^{(K)}$ are constants determined from the equations

$$(4) \quad \sigma_i v_i^{(2)} + p_i v_i^{(1)} + q_i v_i^{(0)} = f_i \qquad (i = 0,1,\dots,n)$$

$$(5) \quad \sigma_{n+1}(v_n^{(2)} + h_n v_n^{(3)}) + p_{n+1}(v_n^{(1)} + h_n v_n^{(2)} + h_n^2 \frac{v_n^{(3)}}{2}) +$$
$$+ q_{n+1}(v_n^{(0)} + v_n^{(1)} h_n + \frac{h_n^2}{2} v_n^{(2)} + \frac{h_n^{(3)}}{6} v_n^{(3)}) = f_{n+1}$$

$$\sigma_i = \frac{h_i p_i}{2} \operatorname{cth} \frac{h_i p_i}{2} \quad \text{for } p_i \ne 0, \; \sigma_i = 1 \quad \text{for } p_i = 0,$$

$$h_i = x_{i+1} - x_i, \; h_{n+1} = h_n, \quad p(x_i) = p_i, \; q(x_i) = q_i, \; f(x_i) = f_i$$

$$(6) \quad v_i^{(0)} = v_{i-1}^{(0)} + h_{i-1} v_{i-1}^{(1)} + \frac{h_{i-1}^2}{2} v_{i-1}^{(2)} + \frac{h_{i-1}^3}{6} v_{i-1}^{(3)}$$

$$(7) \quad v_i^{(1)} = v_{i-1}^{(1)} + h_{i-1} v_{i-1}^{(2)} + \frac{h_{i-1}^2}{2} v_{i-1}^{(3)}$$

$$(8) \quad v_i^{(2)} = v_{i-1}^{(2)} + h_{i-1} v_{i-1}^{(3)} \qquad (i = 1,2,\dots,n)$$

Equalities (6), (7) and (8) result from property a). Thus, we have a system of 4n+2 equations with 4n+4 unknowns $v_i^{(K)}$ ($i = 0,1,\dots,n$; $K = 0,1,2,3$). The other two equations are obtained from the boundary conditions.

By the elimination of $v_i^{(1)}$, $v_i^{(2)}$ and $v_i^{(3)}$ from the above equations we form the system (see [3])

(9) $-k_i v_{i-1}^{(0)} + \ell_i v_i^{(0)} - m_i v_{i+1}^{(0)} = R_i \quad (i = 1,2,\dots,n-1)$

where

$$k_i = \frac{\beta_i b_i}{\gamma_i} + \frac{h_{i-1}q_{i-1}}{2\sigma_{i-1}}, \quad \ell_i = \frac{a_i\beta_{i+1}}{\gamma_{i+1}} + \frac{b_i\alpha_i}{\gamma_i} - \frac{h_{i-1}q_i}{2\sigma_i},$$

$$m_i = \frac{a_i\alpha_{i+1}}{\gamma_{i+1}}, \quad R_i = \frac{b_i s_i}{\gamma_i} - \frac{a_i s_{i+1}}{\gamma_{i+1}} - r_i, \quad a_i = 1 + \frac{h_{i-1}p_i}{2\sigma_i},$$

$$b_i = 1 - \frac{h_{i-1}p_{i-1}}{2\sigma_{i-1}}, \quad r_i = \frac{h_{i-1}}{2}\left(\frac{f_{i-1}}{\sigma_{i-1}} + \frac{f_i}{\sigma_i}\right),$$

$$\alpha_i = 1 + \frac{h_{i-1}^2 q_i}{6\sigma_i a_i}, \quad \beta_i = 1 - \frac{4h_{i-1}^2 q_{i-1} a_i \sigma_i - h_{i-1}^3 p_i q_{i-1}}{12 a_i \sigma_i \sigma_{i-1}}$$

$$\gamma_i = h_{i-1}\left(1 - \frac{h_{i-1}p_{i-1}}{3\sigma_{i-1}} - \frac{h_{i-1}b_i p_i}{6\sigma_i a_i}\right)$$

$$s_i = \frac{h_{i-1}^2}{6}\left(2\frac{f_{i-1}}{\sigma_{i-1}} - \frac{p_i r_i}{\sigma_i a_i} + \frac{f_i}{\sigma_i}\right)$$

From (2) the following two equations are obtained:

(10) $(\beta_1\beta_a - \gamma_1\alpha_a)\gamma_1^{-1}v_0^{(0)} - \alpha_1\beta_a\gamma_1^{-1}v_1^{(0)} = -\gamma_a - s_1\beta_a\gamma_1^{-1}$

(11) $-k_n v_{n-1}^{(0)} + \ell_n v_n^{(0)} = R_n$, where

$$k_n = \frac{b_n\beta_n}{\gamma_n} + \frac{h_{n-1}q_{n-1}}{2\sigma_{n-1}}$$

$$\ell_n = \frac{b_n\alpha_n}{\gamma_n} - \frac{h_{n-1}q_n}{2\sigma_n} + \frac{a_n}{F}\{q_{n+1} + A(\frac{q_nC}{\sigma n} - \frac{6\alpha_b}{h_nB}) - \frac{q_n}{\sigma_n}E\}$$

$$R_n = \frac{b_ns_n}{\gamma_n} - r_n + \frac{a_n}{F}(f_{n+1} - \frac{f_nE}{\sigma_n} + AC\frac{f_n}{\sigma_n} - \frac{6A\gamma_b}{h_nB})$$

$$A = \sigma_{n+1} + \frac{p_{n+1}h_n}{2} + \frac{q_{n+1}h_n^2}{6}, \quad B = \alpha_bh_n + 3\beta_b$$

$$C = \frac{3\alpha_bh_n + 6\beta_b}{\alpha_bh_n + 3\beta_b}, \quad D = \frac{6\alpha_bh_n + 6\beta_b}{h_nB}$$

$$E = \sigma_{n+1} + p_{n+1}h_n + q_{n+1}\frac{h_n^2}{2}$$

$$F = p_{n+1} + h_nq_{n+1} + AC\frac{p_n}{\sigma_n} - AD - \frac{p_n}{\sigma_n}E$$

By solving the system (9), (10) and (11) we obtain $v_i^{(0)}$ $(i = 1,2,\dots,n)$ and then determine $v_i^{(1)}$ from the relations:

$$(12)\quad \begin{cases} (\alpha_iv_i^{(0)} - \beta_iv_{i-1}^{(0)} - s_i)\gamma_i^{-1} = v_{i-1}^{(1)} & (i = 1,2,\dots,n) \\ a_nv_n^{(1)} = b_nv_{n-1}^{(1)} + r_n - h_{n-1}(q_{n-1}v_n^{(0)}\sigma_{n-1}^{-1} + q_nv_n^{(0)}\sigma_n^{-1})/2 \end{cases}$$

After that, $v_i^{(2)}$ we get from (4), $v_i^{(3)}$ $(i = 1,2,\dots,n-1)$ from (8) and $v_n^{(3)}$ from (5).

THEOREM 1. *The matrix of the system determined by (9), (10) and (11) is an inverse monotone if* $\alpha_a\beta_a \le 0$, $\alpha_b\beta_b \ge 0$ *and if one of the following conditions is fulfilled:*

(i) $q = 0$, $\alpha_a \ne 0$ or $\alpha_b \ne 0$, $h_{i-1} < h_i|2\operatorname{cth} h_ip_i|$ for $p_i < 0$

(ii) $q \le 0$, $\xi_i \ge 0$, $\mu_i \ge 0$ $(i = 1,\dots,n)$, *where*

$$\xi_i = 6\sigma_i + 3h_{i-1}p_i + h_{i-1}^2q_i$$

$$\mu_i = 6\sigma_{i-1} - 3h_{i-1}p_{i-1} + h_{i-1}^2q_{i-1}$$

and at least one of q_{i-1}, q_i, q_{i+1} $(i = 2,\dots,n-1)$ *and* q_{i-1} *or* q_i $(i = 1,n)$ *is different from zero.*

P r o o f.

(i) For $q = 0$, it holds that

$$k_i > 0, \quad \ell_i > 0, \quad m_i > 0, \quad -k_i + \ell_i - m_i = 0$$

$(i = 1,2,\dots,n-1)$

$$-k_n + \ell_n > \frac{3\alpha_b}{2h_n FB} > 0 \qquad \text{for} \quad \alpha_b \neq 0$$

$$\ell_o - m_o = -\alpha_a > 0 \quad \text{for} \quad \alpha_a \neq 0 \quad (\alpha_a < 0)$$

(ii) Since $\gamma_i > 0$, $k_i\gamma_i = \dfrac{\mu_i}{6\sigma_{i-1}} \geq 0$

$$\alpha_i = \frac{\xi_i}{3\sigma_i(2\sigma_i + h_{i-1}p_i)} \geq 0 \text{ we have } m_i \geq 0 \text{ and } k_i \geq 0.$$

Further

$$\Delta_i = -k_i + \ell_i - m_i = \overline{C}_i + \overline{D}_i \geq Mh > 0 \ (i = 1,2,\dots,n-1)$$

$$\overline{C}_i = -h_{i-1}[(2\sigma_i + h_{i-1}p_i)(\sigma_i q_{i-1} + \sigma_{i-1}q_i) + q_i\sigma_i(2\sigma_{i-1} - h_{i-1}p_{i-1})]/G_i$$

$$G_i = \sigma_i[12\sigma_i\sigma_{i-1} + 4\sigma_{i-1}h_{i-1}p_i - 4\sigma_i h_{i-1}p_{i-1} - h_{i-1}^2 p_{i-1}p_i]$$

$$\overline{D}_i = -\frac{2\sigma_i + h_{i-1}p_i}{2\sigma_i G_{i+1}}\Big[\sigma_{i+1}h_i q_i(4\sigma_{i+1} + h_i p_{i+1}) +$$

$$+ h_i q_{i+1}\sigma_i(2\sigma_{i+1} + h_i p_{i+1})\Big]$$

$$\Delta_o = \frac{\beta_a(\beta_1 - \alpha_1)}{\gamma_1} - \alpha_a \geq Mh > 0$$

$$\Delta_n = \frac{b_n}{\gamma_n}(\beta_n - \alpha_n) - \frac{h_{n-1}}{2}\left(\frac{q_n}{\sigma_n} + \frac{q_{n-1}}{\sigma_{n-1}}\right) + \frac{a_n}{F}\cdot V \geq Mh > 0, \text{ where}$$

$$V = q_{n+1} + A\frac{q_n h_n C - 6\alpha_b\sigma_n}{\sigma_n h_n B} - \frac{q_n}{\sigma_n}E$$

Thus from [4] we have that in both cases, the matrix is an inverse monotone.

In order to obtain the error estimation, we shall consider the differences $z_i^{(K)} = y_i^{(K)} - v_i^{(K)}$ where $y_i^{(K)}$ is the value of the K-th derivative of the function in point x_i. Similar to [3], it can be shown that $z_i^{(0)}$ satisfies the equations

$$(14)\quad -k_i z_{i-1}^{(0)} + \ell_i z_i^{(0)} - m_i z_{i+1}^{(0)} = \psi_i \qquad (i = 0,1,\dots,n)$$

where $k_o = 0$, $m_n = 0$,

$$\psi_i = \frac{a_i \phi_{i+1}^{(2)}}{\gamma_{i+1}} - \frac{b_i \phi_i^{(2)}}{\gamma_i} + \phi_i^{(1)} \qquad (i = 1,\dots,n-1)$$

$$\psi_o = -\phi_1^{(2)} \beta_a \gamma_1^{-1}$$

$$\psi_n = -\frac{b_n \phi_n^{(2)}}{\gamma_n} + \phi_n^{(1)} - \frac{a_n}{F}[\eta_{n+1} - \psi - \frac{6\psi_b A}{h_n B} - \frac{\eta_n}{\sigma_n}(E - AC)]$$

$$\eta_i = y_i^{(2)}(\sigma_i - 1)$$

$$\phi_i^{(2)} = \psi_i^{(0)} + h_{i-1}^2 \left(\frac{\eta_{i-1}}{3\sigma_{i-1}} + \frac{\eta_i}{6\sigma_i} - \frac{\psi_i^{(2)}}{6} - \frac{p_i \phi_i^{(1)}}{6 a_i \sigma_i}\right)$$

$$\phi_i^{(1)} = \psi_i^{(1)} - \frac{h_{i-1}\psi_i^{(2)}}{2} + \frac{h_{i-1}}{2}\left(\frac{\eta_{i-1}}{\sigma_{i-1}} + \frac{\eta_i}{\sigma_i}\right)$$

$$\psi = \sigma_{n+1}\psi_{n+1}^{(2)} + p_{n+1}\psi_{n+1}^{(1)} + q_{n+1}\psi_{n+1}^{(0)}$$

$$\psi_b = -\alpha_b \psi_{n+1}^{(2)} - \beta_b \psi_{n+1}^{(1)}$$

$$\psi_i^{(0)} = \frac{y^{(IV)}(\theta_{1i})h_{i-1}^4}{24}, \qquad \psi_i^{(1)} = \frac{y^{IV}(\theta_{2i})h_{i-1}^3}{6}$$

$$\psi_i^{(2)} = \frac{y^{(IV)}(\theta_{3i})h_{i-1}^2}{2}, \quad x_{i-1} \le \theta_{mi} \le x_i \qquad (m = 1,2,3).$$

By taking into account the assumption on smoothness of function $y(x)$ and the estimations

$$|\sigma_i - 1| \le Mh_i^2 p_i^2 \quad ([1]), \quad |\frac{a_n}{F}| \le Mh ,$$

we obtain $|\psi_i| \le Mh^3$ $(i = 0,1,\dots,n)$

THEOREM 2. *Let the boundary-value problem (1), (2) have a unique solution* $y(x) \in C^4[a,b]$. *Let condition (ii) of Theorem 1 be fulfilled. Then*

$$(15) \qquad \begin{cases} |y_i^{(K)} - v_i^{(K)}| \le Mh^2 & (K = 0,1,2) \\ |y_i''' - v_i^{(3)}| \le Mh & , \text{ for } \beta_a \ne 0 \end{cases}$$

and

$$|y_i^{(0)} - v_i^{(0)}| \le Mh^2$$

$$|y_i^{(K)} - v_i^{(K)}| \le Mh \qquad \text{for } K = 1,2$$

$$|y_i^{(3)} - v_i^{(3)}| = 0(1) \quad \text{for } \beta_a = 0, \ (i = 0,1,\dots,n+1)$$

P r o o f. The equations (14) can be written in the matrix form $Az^{(0)} = \psi$.
On the basis of Theorem 1 it holds that

$$|z_i^{(0)}| \le \|A^{-1}\|_\infty \|\psi\|_\infty \le M\|A^{-1}\|_\infty h^3$$

Since $h^{-1}\Delta_i \ge M > 0$ $(i = 0,1,\dots,n)$ and $h\|A^{-1}\|_\infty \le M$ we have $|z_i^{(0)}| \le Mh^2$ $(i = 0,1,\dots,n)$

Similar to [3], it can be shown that

$$(17) \quad \alpha_a z_o^{(0)} + \beta_a z_o^{(1)} = 0$$

$$(18) \quad a_i z_i^{(1)} = b_i z_{i-1}^{(1)} - \frac{h_{i-1}}{2}\left(\frac{q_{i-1} z_{i-1}^{(0)}}{\sigma_{i-1}} + \frac{q_i z_i^{(0)}}{\sigma_i}\right) + \phi_i^{(1)},$$

$(i = 1,\dots,n)$

$$(19)\quad \alpha_i z_i^{(0)} = \beta_i z_{i-1}^{(0)} + \gamma_i z_{i-1}^{(1)} + \phi_i^{(2)} \quad , \qquad (i = 1,\dots,n)$$

$$(20)\quad z_i^{(2)} = (\eta_i - p_i z_i^{(1)} - q_i z_i^{(0)})/\sigma_i \qquad (i = 0,\dots,n)$$

$$(21)\quad z_{i-1}^{(3)} = (z_i^{(2)} - z_{i-1}^{(2)} - \psi_i^{(2)})/h_{i-1} \qquad (i = 1,\dots,n)$$

$$(22)\quad \sigma_{n+1}(z_n^{(2)} + h_n z_n^{(3)}) + p_{n+1}(z_n^{(1)} + h_n z_n^{(2)} + \frac{h_n^2}{2} z_n^{(3)}) +$$

$$+ q_{n+1}(z_n^{(0)} + h_n z_n^{(1)} + \frac{h_n^2 z_n^{(2)}}{2} + \frac{h_n^3}{6} z_n^{(3)}) = \eta_{n+1} - \psi$$

For $\beta_a \neq 0$ from (17), we obtain $|z_o^{(1)}| \leq Mh^2$ and then from (18), we have $|z_i^{(1)}| \leq Mh^2$ $(i = 1,\dots,n)$. Since $|\eta_i| \leq Mh^2$ from (20), we have $|z_i^{(2)}| \leq Mh^2$ $(i = 0,\dots,n)$. The estimation for $z_i^{(3)}$ $(i = 0,\dots,n-1)$ is obtained from (21), for $z_n^{(3)}$ from (22), and for $z_{n+1}^{(K)}$ from the following relations:

$$(23)\quad \begin{cases} z_{n+1}^{(0)} = z_n^{(0)} + h_n z_n^{(1)} + \frac{h_n^2}{2} z_n^{(2)} + \frac{h_n^3}{6} z_n^{(3)} + \psi_{n+1}^{(0)} \\ z_{n+1}^{(1)} = z_n^{(1)} + h_n z_n^{(2)} + \frac{h_n^2}{2} z_n^{(3)} + \psi_{n+1}^{(1)} \\ z_{n+1}^{(2)} = z_n^{(2)} + h_n z_n^{(3)} + \psi_{n+1}^{(2)} \\ z_{n+1}^{(3)} = z_n^{(3)} + h_n y^{IV}(\xi) \quad , \qquad x_n \leq \xi \leq x_{n+1} \end{cases}$$

For $\beta_a = 0$, from (19) we obtain $|z_i^{(1)}| \leq Mh$ $(i = 0,\dots,n-1)$ because $\gamma_i = 0(h)$. From (18) we obtain $|z_n^{(1)}| \leq Mh$ and then the other estimations from (20), (21) and (22).

THEOREM 3. *Let* $p(x) \equiv 0$, $\beta_a = \beta_b = 0$, $h_i = h$, $\mu_i \geq 0$ $\xi_i \geq 0$, $(i = 0,\dots,n+1)$. *Then*

$$|z_i^{(0)}| \leq Mh^3,\ |z_i^{(K)}| \leq Mh^2\ (k = 1,2),\ |z_i'''| \leq Mh .$$

P r o o f. Now, the system analogous to the system (14) has the form

$$\bar{A}\, z^{(0)} = \psi$$

and $\bar{A} \geq h^{-1}B$, B is tridiagonal matrix with the elements b_{ij} $(i,j = 1,2,\dots,n)$ $b_{ii} = 2$, $b_{i-1,i} = -1$ $(i = 2,\dots,n)$ $b_{i+1,i} = -1$ $(i = 1,\dots,n-1)$. The solution of system $Bu = \omega$, $\omega = \max_i |\psi_i| h$ has the form $u_i = i(n+1-i)\omega/2$.
Since $|z_i^{(0)}| \leq u_i$ we have $z_1^{(0)} = 0(h^3)$, $z_n^{(0)} = 0(h^3)$.
From the first equation of the system, $\ell_1 z_1^{(0)} - m_1 z_2^{(0)} = \psi_1$, we have $z_2^{(0)} = 0(h^3)$ and then from other equations we obtain $z_i^{(0)} = 0(h^3)$ $(i = 2,\dots,n-1)$. Estimation of the derivatives we obtain from (18) - (23). This scheme is identical to scheme [8] p.286.

REMARK 1. When the above procedure is applied on to equation

$$\varepsilon y'' + p(x)y' + q(x) = f(x)\ ,\quad p(x) > 0\ ,\ \varepsilon << 1$$

with the boundary conditions (2) a system uniformly stable with respect to ε is obtained.

REMARK 2. For the proof of Theorem 2 unlike to [3] and [6] it is not necesary a barrier function. This simplifies its application.

REMARK 3. In [3] was obtained the same order of convergency for the general and Dirichlet's conditions. In the part of the proof relating to Dirichlet's conditions the inequality $L_h^{-1} \leq \tilde{L}_h^{-1}$ was used (for L_h, $\tilde{L}_h$ see [3]), which for our scheme is not satisfied (Example: $n = 3$, $p = 1$, $q = 0$, $\beta_a = \beta_b = 0$, $h_i = 0{,}25$). The lost of accuracy is made up for in the special case when $p(x) = 0$.
The proof of theorem 4 [7] has not been given. It was based on the theorem [3], which is not clear comlitly.

REMARK 4. When cth x is approximated by 1/x our scheme is reduced to the Il'in's scheme. In that case our conditions on the functions p and q are reduced to those in [8], which are simpler then Il'in's. In [8] q = 0 is not allowed and convergence of derivatives has not been obtained.

REFERENCES

[1] Doolan E.P., Miller J.J.H., Schilders W.H.A., *Uniform Numerical Methods for Problems with Initial and Boundary Layers*, BOOLE Press, Dublin, 1980.

[2] Il'in A.M., *Raznostnaja shema dlja differencial'nogo uravnenija s malym parametrom pri staršej proizvodnoj*, Matem. zametki, 1969, 6 vyp.2., 237-248.

[3] Il'in V.P., *O splajnovyh rešenijah obyknovenyh differencial'nyh uravnenij*, Žurnal vyčis.mat. i mat. fiziki, 1978, No.3, 621-627.

[4] Stojaković Z., Herceg D., *Numeričke metode linearne algebre*, Beograd 1982.

[5] Surla K., *On the convergence of some finite difference schemes for a singular perturbation problem*, Rev. of Research, Fac. of Sci.Univ. of Novi Sad, Volume 12, 1982, 191-203.

[6] Surla K., Kulpinski M., *O splajn rešenjima običnih diferencijalnih jednačina*, V znanstveni skup, Pror. i proj. pomoću računala, Stubičke Toplice 1983, Zbornik radova, 137-141.

[7] Surla K., *On the spline solution of boundary value problems of the second order*, Numer.Math. and Approx. theory, Niš, 1984, (ed. G. Milovanović) 131-136

[8] Zavjalov Ju.S., Kvasov B.I., Mirošničenko Z.L., *Metodi splajn funkcii*, Moskva 1980.

Received by the editors December 16, 1983.

REZIME

POVEĆANJE TAČNOSTI ZA NEKA SPLAJN REŠENJA DVOTAČKASTIH KONTURNIH PROBLEMA

U [3] je dat postupak za nalaženje približnog rešenja konturnog problema (1), (2) u obliku kubnog splajna. U [6] je uvodjenjem "fiting faktora" dobijeno uopštenje rezultata Iljina pod oslabljenim pretpostavkama na funkcije p i q. Pri tome je dobijen niži red konvergencije za Dirichletove uslove. Ovde je dobijen veći red tačnosti u specijalnom slučaju. Oslabljeni su uslovi nekih teorema iz [2] i [6] čime je pojednostavljena njihova primena.