Zbornik radova Prirodno-matematičkog fakulteta-Universitet u Novom Sadu
knjiga 14,1 (1984)
Review of Research Faculty of Science-University of Novi Sad, Volume 14,1(1984)

A NOTE ON THE AXIOMS OF A GENERALIZED EQUIVALENCE RELATION

Mališa R. Žižović
Pedagoško-tehnički fakultet,
Čačak, Jugoslavija

ABSTRACT

A generalized equivalence relation is introduced in [1] and its properties are carefully studied in [2], [3], [4] and [5]. In this note an equivalent definition of a generalized equivalence relation is given.

Let A be a set with at least n elements.

DEFINITION. [1] *A generalized equivalence relation, designated as* E_n, *on the set* A *is any* $(n+1)$*-ary relation* E_n *on* A, *which satisfies the following conditions:*

E1 : *For all* $x_1,\dots,x_n$ *from* A, $(x_1,\dots,x_n,x_1)$ *belongs to* E_n.

E2 : *If* $(x_1,\dots,x_{n+1}) \in A^{n+1}$ *belongs to* A, *then for each* s *from the set* S_{n+1} *of all the permutations on the set* N_{n+1} *one has* $(x_{s(1)},\dots,x_{s(n+1)}) \in E_n$.

E3 : *If for all* $x_o,\dots,x_{n+1}$ *from* A, *with* $x_i \neq x_j$ *for* $i \neq j$, $i,j = 1,\dots,n$, $(x_o,\dots,x_n)$ *and* $(x_1,\dots,x_{n+1})$ *belong to* E_n *then* $(x_o,\dots,x_{n-1},x_{n+1})$ *also belongs to* E_n.

AMS Mathematics subject classification (1980): 08A25
Key words and phrases: Generalized equivalence relations.

Let us designate by $D(a_o,\dots,a_n) = k\{a_o,\dots,a_n\}$ and $P((a_o,\dots,a_n), (b_o,\dots,b_n)) = k(\{a_o,\dots,a_n\} \cap \{b_o,\dots,b_n\})$.

THEOREM. *An* $(n+1)$*-ary relation* R *defined on set* A $(kA > n)$*, is a generalized equivalence relation, if and only if the following conditions are satisfied:*

K1 : $D(a_o,\dots,a_n) \leq n$ $(a_o,\dots,a_n) \in R$, *for all* $a_o,\dots,a_n \in A$.

K2 : *For all* $(a_o,\dots,a_n), (b_o,\dots,b_n) \in R$ *such that* $P((a_o,\dots,a_n), (b_o,\dots,b_n)) \geq n$ *it follows* $(c_o,\dots,c_n) \in R$ $(c_i = c_j$ *or* $c_i = b_k$, $i,j,k \in \{0,\dots,n\})$.

P r o o f. If R is a generalized equivalence relation, condition K1 is obviously satisfied because of E1 and E2. Let us prove that condition K2 is satisfied, too. Let us suppose that $(a_o,\dots,a_n),(b_o,\dots,b_n) \in R$ such that $P((a_o,\dots,a_n),(b_o,\dots,b_n)) = n$ (If $P((a_o,\dots,a_n),(b_o,\dots,b_n)) = n+1$ then condition K2 is satisfied with E2). Without less generalization we can suppose (by E2), that $a_1 = b_1,\dots,a_n = b_n$, and that only a_o and b_o do not belong to the common intersection. Since $(a_o,\dots,a_n) \in R$, and because of E2 $(a_1,\dots,a_n,b_o) \in R$ it follows that $(a_o,\dots,a_{n-1},b_o) \in R$. Similarly, as we omitted the element $a_n \in \{a_1,\dots,a_n\}$, we can omit any other one, where because of E2 it follows that K2 is valid.

Conversely, if K1 and K2 are satisfied, let us prove that E1, E2 and E3 are satisfied.

E1 : For all $x_1,\dots,x_n \in A$, $D(x_1,\dots,x_n,x_1) \leq n$ and by K1 $(x_1,\dots,x_n,x_1) \in R$.

E2 : If $(x_1,\dots,x_{n+1}) \in R$ then it may be either $D(x_1,\dots,x_{n+1}) \leq n$ or $D(x_1,\dots,x_{n+1}) = n+1$. If $D(x_1,\dots,x_{n+1}) \leq n$ then so that $D(x_{s(1)},\dots,x_{s(n+1)}) \leq n$ so that $(x_{s(1)},\dots,x_{s(n+1)}) \in R$. If $D(x_1,\dots,x_{n+1}) = n+1$ then and $(x_1,\dots,x_n,x_1) \in R$ since E1 is valid as $P((x_1,\dots,x_{n+1}),(x_1,\dots,x_n,x_1)) = n$ then $(x_{s(1)},\dots,x_{s(n+1)}) \in R$ by K2.

E3 : Let for arbitrary $x_o,\dots,x_{n+1} \in A$ ($x_i \neq x_j$, $i \neq j$, $i,j \in N_n$) $(x_o,\dots,x_n) \in R$ and $(x_1,\dots,x_{n+1}) \in R$, then it is either

(1) $P((x_o,\dots,x_n),(x_1,\dots,x_{n+1})) = n$ or

(2) $P((x_o,\dots,x_n),(x_1,\dots,x_{n+1})) = n+1$.

In case (1) according to K2, using $c_o = x_o,\dots,c_{n-1} = x_{n-1}$ and $c_n = x_{n+1}$, we have $(x_o,\dots,x_{n-1},x_{n+1}) \in R$. In case (2) $x_o = x_{n+1}$, then according to E1 $(x_o,\dots,x_{n-1},x_o) \in R$, namely $(x_o,\dots,x_{n-1},x_{n+1}) \in R$.

REFERENCES

[1] Picket, H.E., *A Note on Generalized Equivalence Relations, Amer.Math. Monthly, 1966, 73, No.8, 860-861.*

[2] Ušan, J., Šešelja, B., *Transitive n-ary Relations and Characterizations of Generalized Equivalences, Zbornik radova PMF u Novom Sadu, 11(1981), 231-245.*

[3] Šešelja, B., Ušan, J., *On one representation of generalized equivalences, Algebraic conference, Novi Sad 1981 (1982), 155-162.*

[4] Ušan, J., Šešelja, B., *The Structure of Generalized Equivalences Contained in $(2,nA_1)$-RT Relations, Zbornik radova PMF u Novom Sadu, 11 (1981), 275-286.*

[5] Ušan, J., Šešelja, B., *On Some Operations on the Set $P(S^{n+1})$, Macedonian Academy of Sciences and Arts, Contributions, IV 1-Section of Mathematical and Technical Sciences, 1983, 77-84.*

Received by the editors September 10, 1984.

NOTA O AKSIOMIMA UOPŠTENE
RELACIJE EKVIVALENCIJE

Data je jedna ekvivalentna definicija uopštene relacije ekvivalencije.