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σ-INVERSE SEMIGROUPS

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ABSTRACT

In this short note we introduce the concept of a σ-inverse semigroup which is a generalization of the concept of Cliffords's semigroup. A characterization of a σ-inverse semigroup is given.

S is a *Clifford semigroup* if S is regular and the idempotents of S are central, [4]. By Z^+ we shall denote the set of all positive integers. A semigroup S is *completely π-regular* if for every $a \in S$ there exist $b \in S$ and $n \in Z^+$ such that $a^n = a^n ba^n$ and $a^n b = ba^n$. By $E(S)$ we denote the set of all idempotents of a semigroup S.

For undefined notions and notations refer to [4].

DEFINITION 1. *An element* b *of a semigroup* S *is σ-inverse of* $a \in S$ *if* $a = aba$, $b = bab$. *A semigroup* S *is σ-inverse if for every element* $a \in S$ *there exists a unique σ-inverse element* $b \in S$.

The semigroup S given by the Table

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	0	e	f	a	b
0	0	0	0	0	0
e	0	e	0	a	0
f	0	0	f	0	b
a	0	0	a	0	e
b	0	b	0	f	0

is a σ-inverse semigroup. But, $ae \neq ea$, and, so, S is not a Clifford semigroup.

LEMMA 1. *A semigroup* S *is inverse if and only if* S *is regular and for every* $e,f \in E(S)$ *there exists* $m \in Z^+$ *such that* $(ef)^m = (fe)^m$.

PROOF: Let S be a regular semigroup in which for every $e,f \in E(S)$ there exists $m \in Z^+$ such that $(ef)^m = (fe)^m$, then by Theorem 3.1. [1] we have that $a = axa = aya$ implies $xax = yay$. Thus S is an inverse semigroup.

The converse follows immediately.

By the following theorem a characterization of a σ-inverse semigroup will be given.

THEOREM 1. *The following conditions are equivalent on a semigroup* S*:*

(i) S *is inverse and completely* π*-regular;*

(ii) S *is regular and for every* $a \in S$, $e \in E(S)$ *there exists* $m \in Z^+$ *such that* $(ae)^m = (ea)^m$*;*

(iii) S *is* σ*-inverse.*

PROOF: (i) => (ii). If S is inverse and completely π-regular, then by Theorem 1.7. [3] we have that for every $a \in S$, $e \in E(S)$ there exists $m \in Z^+$ such that $(ae)^m = (ea)^m$.

(ii) => (i). By the hypothesis we have that for every $e,f \in E(S)$ there is an $m \in Z^+$ such that $(ef)^m = (fe)^m$ and so by Lemma 1 we have that S is an inverse semigroup. S is completely π-regular by Theorem 1.7. [3].

(i) => (iii). Let S be an inverse completely π-regular semigroup and let $x \in S$ be an inverse element of $a \in S$. Then, by Theorem 1.7. [3], there exist $r,s \in Z^+$ such that

$$(a \cdot ax)^r (axa)^r = a^r, \quad (xa \cdot a)^s = (axa)^s = a^s .$$

Then

$$(a^2x)^{rs} = a^{rs} = (xa^2)^{rs} .$$

Let us put $t = rs$. Then

$$\begin{aligned}(a^2x)^t &= a(axa)^{t-1}x = a(axa)^{t-1}axax \\ &= a(axa)^t x = a(xa^2)^t x = (axa)^t ax \\ &= a^t ax = a^{t+1}x\end{aligned}$$

and

$$(xa^2)^t = xa^{t+1}.$$

Thus

$$a^{t+1}x = xa^{t+1} .$$

Therefore, S is a σ-inverse semigroup.

(iii) => (i). Follows immediately.

THEOREM 2. *S is a simple σ-inverse semigroup if and only if S is a group.*

PROOF: Follows by Theorem 1 [5] (see also Theorem 2 [2]).

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REZIME

σ-INVERZNE POLUGRUPE

U ovoj kratkoj noti je uveden pojam σ -inverzne polugrupe koja je generalizacija koncepta Cliffordove polugrupe . Data je jedna karakterizacija σ -inverzne polugrupe .