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ON n-FINITE FORCING COMPANIONS

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ABSTRACT

The main result of this paper is that for Peano arithmetic (T_{PA}) holds that for each n there exists k > n such that $T_{PA}^{f_n} \neq T_{PA}^{f_k}$ (3.3).

INTRODUCTION. This paper is directly attached to [5] and [6]. The generalization of (most of) the results from [8], as well as of some of the results from [10] (§ 1), which is, by the way, routine and easy job (like the one in [5]), is done here mainly because of the possible use of the new versions of these results in some coming papers. We are really interested in the relation between a given theory and n-finite forcing companion corresponding to it. This time we shall present some of the first observations on this subject (§ 2, § 3). Numerous open questions impose themselves, and we think there is no need to state them explicitly.

§0. We assume a familiarity with [5] and [6]. All the assumptions (and a main part of the notation) are as in [6], § 0. The following theorem, which, together with some other results from

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§ 1, should have been given in [5], will be used in § 2.

THEOREM 0.1. *(a) If a language* L *of a theory* T *is countable and* $T = T^{fn}$, *then for each sentence* ϕ *of the language* L *consistent with* T *there exists a model* M *of* $T \cup \{\phi\}$, *such that* $T \cup D_n(M)$ *is a complete theory (we shall also say that* M *n-completes* T*).*

(b) Let T *be a theory (defined in a language* L *of arbitrary cardinality), such that for each sentence* ϕ *of the language* L *consistent with* T, *there exists a model of* $T \cup \{\phi\}$ *which n-completes* T. *Then* $T = T^{fn}$ *and, in general,* $T = T^{fk}$ *for each* $k \geq n$.

§ 1. In this paragraph we follow [8], more or less consistently. The point is in the "translation for n" of the results of this paper. The proofs are, because of the same reasons as in [5], mainly omitted. We refer those interested in more details to the original [8].

For a given set of formulas Ψ $fv(\Psi) = \{fv(\psi) | \psi \in \Psi\}$. A complete m-type Γ is a maximal set of formulas consistent with T such that $|fv(\Gamma)| = m$. (We can really demand: $fv(\Gamma) = \{v_0, \ldots, v_{m-1}\}$). An (n+1)-existential type is defined analogously, but this time elements are Σ_{n+1} formulas. We shall generally denote complete (m)-types by $\Gamma(\tilde{v})$ (or just Γ), (n+1)-existential (m)-types by $E(\tilde{v})$ (or just E), their elements by, $\phi(\tilde{v})$ i.e. $\sigma(\tilde{v})$ (or just ϕ, σ), respectively, the set of all the complete m-types by T_m and the set of (n+1)-existential m-types by E_m. We shall consider T_m and E_m topological spaces with bases, elements of which are in a succession $A\phi(\tilde{v}) = \{\Gamma | \phi(\tilde{v}) \in \Gamma\}$ and $A\sigma(\tilde{v}) = \{E | \sigma(\tilde{v}) \in E\}$. For the space T_m it holds obviously that $A\phi_1(\tilde{v}) = A\phi_2(\tilde{v})$ iff $T \vdash \phi_1(\tilde{v}) \leftrightarrow \phi_2(\tilde{v})$; $A\phi_1(\tilde{v}) \subseteq A\phi_2(\tilde{v})$ iff $T \vdash \phi_1(v) \rightarrow \phi_2(v)$. Of course directions ($\leftarrow$) hold for E_m, as well.

In further considerations of this paragraph, the language L is countable. We just note that this condition is superfluous in many of the propositions.

LEMMA 1.1. *(a) A model* M *from* $\mu(T \cap \Pi_{n+1})$ *is n-existentially complete (of T) if and only any* m *elements of* M *realize an (n+1)-existential m-type.*

(b) E *is an (n+1)-existential m-type if and only if it is realized by some m elements of some n-existentially complete model.*

DEFINITION 1.2. *A complete (m-) type is normal if it is an extension of an (n+1)-existential (m-) type.*

A complete (m-) type is a complete (n+1)-existential m-type if it is the unique extension of an (n+1)-existential (m-) type.

LEMMA 1.3. *(a) A model* $M \in \mu(T \cap \Pi_{n+1})$ *is an n-existentially complete model of* T *if and only if for each* Π_{n+1} *formula* $\neg\sigma_0(\tilde{v})$ *it omits a type (here, of course, the word type does not assume the maximality of the set of formulas)* $\{\neg\sigma_0(\tilde{v})\} \cup \{\neg\sigma(\tilde{v}) | fv(\sigma) \subseteq fv(\sigma_0)$ *and* $T \vdash \sigma(\tilde{v}) \to \neg\sigma_0(\tilde{v})\}$;

(b) A model $M \in \mu(T)$ n-completes T *if and only if it omits all types of the form* $\{\phi(\tilde{v})\} \cup \{\neg\sigma(\tilde{v}) | fv(\sigma) \subseteq fv(\phi)$ *and* $T \vdash \sigma(\tilde{v}) \to \phi(\tilde{v})\}$.

THEOREM 1.4. *The following are equivalent:*

(a) If $\phi(\tilde{v})$ *is consistent with* T *then* $T \cup \{\phi\}$ *has a model which is n-existentially complete;*

(b) For each $m \in \omega$ *the set of all normal m-types is dense in* T_m.

COROLLARY 1.5. *A complete theory* T *has a model which is n-existentially complete if and only if the set of normal m-types is dense in* T_m *for each* $m \in \omega$.

COROLLARY 1.6. *If* T *is a complete theory such that* $T = T \cap \Pi_{n+2}$ *then for each* $m \in \omega$ *the set of normal m-types is dense in* T_m.

PROOF. By 2.5 and 1.5.

THEOREM 1.7. *(a) The class* E_T^n *(of all n-existentially complete models) is axiomatizable by a single sentence of the language* $L_{\omega_1\omega}$;

(b) The class F_T^n *(of all n-finitely generic models) is axiomatizable by a single sentence of the language* $L_{\omega_1\omega}$.

PROOF. (a) From 1.3.(a) it follows directly that $\bigwedge(T \cap \Pi_{n+1}) \wedge \bigwedge\{\psi_\phi | \phi$ is a Π_{n+1} formula of L} where $\psi_\phi \equiv \neg\exists\tilde{v}(\phi(\tilde{v}) \wedge \bigwedge\{\neg\sigma(\tilde{v}) | T \vdash \sigma \to \phi\})$ is the axiom for E_T^n.

(b) Let us just recall that a model of T^{fn} is n-finitely generic iff it n-completes T^{fn}. Hence (and by 1.3.(b)) $\bigwedge T^{fn} \wedge \bigwedge\{\psi_\phi | \phi(\tilde{v})$ is a formula of L} where $\psi_\phi \equiv \neg\exists\tilde{v}(\phi(\tilde{v}) \wedge \bigwedge\{\neg\sigma(\tilde{v}) | T^{fn} \vdash \sigma \to \phi\})$ is the axiom for F_T^n.

(REMARK. In case the language L of T is of cardinality λ, the axioms of the classes E_T^n and F_T^n are sentences of $L_{\lambda^+,\omega}$).

THEOREM 1.8. *The following are equivalent:*

(a) T *is n-model complete;*

(b) $\{A\sigma(\tilde{v}) | |fv(\sigma)| < m\}$ *is a base of* T_m *for each* $m \in \omega$, *where, now, of course,* $A\sigma(\tilde{v}) = \{\Gamma \in T_m | \sigma(\tilde{v}) \in \Gamma\}$

(c) *Every complete type is a complete (n+1)-existential type.*

(d) *Every complete type is normal.*

THEOREM 1.9. *The following are equivalent:*

(a) $T = T^{fn}$

(b) For each $m \in \omega$ and *for each open set* 0 *in* T_m *there exists a* (Σ_{n+1}) formula $\sigma(\tilde{v})$ *such that* $A\sigma(\tilde{v}) \subseteq 0$;

(c) For each $m \in \omega$ *the set of complete (n+1)-existential m-types is dense in* T_m.

PROOF. Let us show, firstly, that (b) is equivalent to (b´): T is an f_n-complete theory (for each formula ϕ consistent with T there exists a (Σ_{n+1}) formula $\sigma(\tilde{v})$ such that $fv(\sigma) \subseteq fv(\phi)$ and $T \vdash \sigma \to \phi$).

(b) $\to$ (b′). Let $\phi(\tilde{v})$ be consistent with T. Then $A\phi(\tilde{v}) \neq \emptyset$, thus for some Σ_{n+1} formula $\sigma(\tilde{v})$ $A\sigma(\tilde{v}) \subseteq A\phi(\tilde{v})$, consequently also $T \vdash \sigma(\tilde{v}) \to \phi(\tilde{v})$.

(b′) $\to$ (b). If O is an empty set then $A(\exists v(v \neq v)) = O$. In case O is nonempty and $O = \bigcup_{i \in I} A\phi_i(\tilde{v})$ then for some $i \in I$ $A\phi_i(\tilde{v}) \neq \emptyset$ and so for Σ_{n+1} formula $\sigma(\tilde{v})$ consistent with T and such that $T \vdash \sigma(\tilde{v}) \to \phi_i(\tilde{v})$ $A\sigma(\tilde{v}) \subseteq A\phi_i(\tilde{v})$ $(\subseteq O)$.

Since we have already been given (a) <-> (b′) (see [5]), we shall prove only (a) $\to$ (c) and (c) $\to$ (b′).

(a) $\to$ (c). Let $A\phi(\tilde{v})$ be a nonempty set. By 0.1.(a) there exists a model M of $T \cup \{\exists \tilde{v}\phi(\tilde{v})\}$ such that $T \cup D_n(M)$ is a complete theory. If $M \models \phi[\tilde{a}]$, $\Gamma = \{\psi(\tilde{v}) \mid M \models \psi[\tilde{a}]\}$ and $E = \{\sigma(\tilde{v}) \mid M \models \sigma[\tilde{a}]\}$ then E is an (n+1)-existential type (1.1.(b)) and because of the completeness of $T \cup D_n(M)$ Γ is its unique extension, hence, a complete (n+1)-existential type and $\Gamma \in A\phi(\tilde{v})$. It follows that the set of complete (n+1)-existential types has a nonempty intersection with any (nonempty) open set, in other words, it is dense in T_m.

(c) $\to$ (b′). Let $\phi(\tilde{v})$ be a formula consistent with T, $\Gamma(\tilde{v})$ a complete (n+1)-existential type from $A\phi(\tilde{v})$ and $E(\tilde{v})$ the (n+1)-existential type contained in Γ. Then for some formula $\sigma(\tilde{v}) \in E$ $T \vdash \sigma \to \phi$(in the opposite $T \cup E \cup \{\neg\phi(\tilde{v})\}$ would be consistent and the type Γ would not be the unique extension of E).

THEOREM 1.10. *The following are equivalent:*

(a) T *has n-model companion;*

(b) *For each* $m \in \omega$ E_m *is a compact (topological) space.*

THEOREM 1.11. *If* $T = T \cap \Pi_{n+2}$ *and* $T = T^{fn}$ *then the following conditions are equivalent:*

(a) *Every model of* T *is an n-elementary submodel of some n-finitely generic model;*

(b) *Every n-existentially complete model is n-finitely generic;*

(c) $T \cup E(c_0,\dots,c_{m-1})$, *where* $c_0,\dots,c_{m-1}$ *are new*

constants, is a complete theory for each (n+1)-existential type E *(of course, we shall use* $E(\tilde{c})$ *instead of* $E(c_0,\dots,c_{m-1})$*).*

PROOF. (a) <=> (b). Already known.

(b) $\to$ (c). Let $E(\tilde{v})$ be an (n+1)-existential type and M an n-existentially complete model in which $E(\tilde{c})$ is realized. By (b), $T \cup D_n(M)$ is complete, hence also $T\cup E(\tilde{c})$. (For any sentence $\psi(\tilde{c})$ either $T \cup D_n(M) \vdash \psi(\tilde{c})$ or $T \cup D_n(M) \vdash \neg\psi(\tilde{c})$, let us say, $T \cup D_n(M) \vdash \psi(\tilde{c})$. Then for some sentence $\sigma(\tilde{c},\tilde{d}) \in D_n(M)$ $T \vdash \sigma(\tilde{c},\tilde{d}) \to \psi(\tilde{c})$ whence $T \vdash \exists\tilde{v}\sigma(\tilde{c},\tilde{v}) \to \psi(\tilde{c})$ while $\exists\tilde{v}\sigma(\tilde{c},\tilde{v}) \in E(\tilde{c})$).

(c) $\to$ (b). Let M be an n-existentially complete model, $\psi(\tilde{c})$ a sentence of the language $L(M)$ and $E(\tilde{v})$ the (n+1)-existential type which is realized in M by (images of constants) $c_0,\dots,c_{m-1}$. By (c), $T \cup E(\tilde{c})$ is a complete theory, thus either $T \cup E(\tilde{c}) \vdash \psi(\tilde{c})$ or $T \cup E(\tilde{c}) \vdash \neg\psi(\tilde{c})$, let us suppose $T \cup E(\tilde{c}) \vdash$ $\vdash \psi(\tilde{c})$. But then also $T \cup D_n(M) \vdash \psi(\tilde{c})$ for if for $\exists\tilde{v}\sigma(\tilde{c},\tilde{v}) \in E(\tilde{c})$ $T \vdash \exists\tilde{v}\sigma(\tilde{c},\tilde{v}) \to \psi(c)$ the hypothesis of consistency of $T \cup D_n(M) \cup$ $\cup \{\neg\psi(\tilde{c})\}$ would imply the consistency of $T \cup \{\exists\tilde{v}\sigma(\tilde{c},\tilde{v})\} \cup \{\neg\psi(\tilde{c})\}$.

LEMMA 1.12. *If* $T = T \cap \Pi_{n+2}$ *and* $T^{fn} = T^{fn} \cap \Pi_{n+2}$, *and if every (n+1)-existential type of* T *completes* T *(condition (c) from previous theorem) then every model of* T *can be n-elementary embedded in some n-finitely generic model.*

PROOF. By the condition of the lemma $T \subseteq T^{fn}$, while every n-existentially complete model is n-finitely generic too.

COROLLARY 1.13. *If* $T^{fn} = T^{fn} \cap \Pi_{n+2}$ *the following are equivalent:*

(a) Every model of T *can be n-elementary embedded in some n-finitely generic model;*

(b) Every (n+1)-existential type completes T^{fn}.

PROOF. A direct consequence of Theorem 1.11 (T and

T^{fn} have the same (n+1)-existential types, T^{fn} is n-forcing complete and every model of T^{fn} is an n-elementary substructure of some model of T).

THEOREM 1.14. *Let* $I_n(T) = \{\phi \mid \phi$ *is* Π_{n+2} *sentence such that* $(T \cap \Pi_{n+1}) \cup \{\phi\}$ *and* T *have a common* Π_{n+1} *segment*$\}$ *and let* T *be the theory generated by* $I_n(T)$. *Then:*

(a) $(-)^0$ *is an n-companion operator, and, that, the smallest one (in the sense of inclusion);*

(b) If $T^* = T^* \cap \Pi_{n+2}$ *and* $T \cap \Pi_{n+1} = T^* \cap \Pi_{n+1}$ *then* $T^* \subseteq T^0$. *(In case* $n = 0$ T^0 *is the so-called inductive hull of* T*).*

THEOREM 1.15. $I_n(T) = \{\forall\tilde{v}\sigma(\tilde{v}) \mid A\sigma(\tilde{v}) = E_m,\ m = 0,1,\ldots\}$.

THEOREM 1.16. *If* $T^{fn} = T^{fn} \cap \Pi_{n+2}$ *then* T^{fn} *is generated by* $\{\forall\tilde{v}\sigma(\tilde{v}) \mid A\sigma(\tilde{v}) = E_m,\ m = 0,1,\ldots\}$.

§ 2. The next three corollaries are the immediate consequences of 0.1.

COROLLARY 2.1. *For a complete theory* T *to be n-forcing complete, the sufficient condition is that there exists a model* $M \in \mu(T)$ *which n-completes* T. *If the language* L *of* T *is countable this condition is necessary too.*

COROLLARY 2.2. *If the language* L *of* T *is countable and for some n* $T = T^{fn}$ *then also* $T = T^{fk}$ *for each* $k > n$.

COROLLARY 2.3. *If* T *is n-model complete, then* $T = T^{fk}$ *for each* $k \geq n$.

One can also give the proof of 2.3 using the following facts: $T \cap \Pi_{n+2} \subseteq T^{fn} \cap \Pi_{n+2}$; an n-model complete theory is equal to its Π_{n+2} segment and two n-model complete theories with the common Π_{n+1} segment coincide.

LEMMA 2.4. (a) *If* T *is complete and* $T = T \cap \Pi_{n+2}$, *then* $T = T^{fk}$ *for each* $k > n$;

(b) *If for some n* $T^{fn} = T^{fk}$ *for each* $k > n$ *then also* $T = T^{fn}$.

COROLLARY 2.5. *If* T *is a complete theory of the countable language* L *and* $T = T \cap \Pi_{n+2}$, *there exists a model* $M \in \mu(T)$ *which n-completes* T.

LEMMA 2.6. *If for some* $n(> 0)$ T^{fn} *is a complete theory then for each* $k < n$ T^{fk} *is complete as well.*

PROOF. *Clearly,* T^{fn} *is a complete theory if and only if* T *has the n-joint embedding property.*

LEMMA 2.7. *For each theory* T *of a language* L *there exists an extension* T_1 *defined in a suitable expansion of* L *such that* $T_1 = T_1^{fn}$ *for each* $n > 0$.

PROOF. Let M be a model of T and Γ_M its elementary diagram (the set of all sentences of $L(M)$ which hold in M). M, clearly, completes (the complete) theory Γ_M, whence, according to 2.1., $\Gamma_M = \Gamma_M^{fn}$ for each $n > 0$ (compare this assertion with 3.8 from [6]).

§ 3. The set $\{T^{fn} | n \in \omega\}$ is not linearly ordered. In truth, this holds for the set of the sets of conditions ($C_m \subseteq C_n$ for $m < n$) but from $p \Vdash_m$ ($p \in C_m$) it does not necessarily follow that $p \Vdash_n$ whence $T^{fn} \subseteq T^{fm}$ does not have to hold.

THEOREM 3.1. *Let* T_{DLOM} *be the theory of dense linear order with maximal and minimal element defined in the language* $L = \{R\}$. *Then* $T_{DLOM} = T_{DLOM}^{fk}$ *for* $k > 1$ (T_{DLOM}^{f} *is the theory of dense linear order without endpoints).*

PROOF. T_{DLOM} is a complete theory and none of its models completes it (moreover, none of its models is existentially complete). (By 2.1 and 2.5 $T_{DLOM} \neq T_{DLOM} \cap \Pi_2$ and $T^f_{DLOM} \neq T_{DLOM}$). Clearly, $T_{DLOM} = T_{DLOM} \cap \Pi_3$ and so $T_{DLOM} = T^{fk}_{DLOM}$ for each $k \geq 1$ (2.4.(a)).

The proof can also be based on the fact that T_{DLOM} is 1-model complete. For, let M be its countable model, $M \prec_1 N \models T_{DLOM}$ and K a countable elementary submodel of N such that $M \subseteq K$. Of course, $M \prec_1 K$, so also $M \prec K$ (using Cantor's argument, one can easily show that if $a_1,\ldots,a_m$ are (arbitrarily chosen) elements of M and b any element of K, then there exists an isomorphic mapping from K onto K, which element b maps onto some element from M while it leaves $a_1,\ldots,a_m$ fixed - for the proof of 1-model completeness, it is sufficient to consider just countable models). Hence $M \prec N$.

REMARK. If we defined the theory T_{DLOM} in a language which, besides a binary relation symbol, also contains two constants (the notions for the minimal and maximal element), we would obtain a complete theory which coincides with its Π_2 segment and therefore coincides with n-finite forcing for each $n \in \omega$ (such a theory would be model complete too, and this case is (in some way) analogous to the example of the theory of dense order without endpoints).

THEOREM 3.2. Let $T_{\mathbf{N}}$ be the complete arithmetic (of the first order) (the set of all sentences of the language $L = \{+,\times,=,0,1,\}$ which hold in the model $\mathbf{N} = \langle N,+,\times,=0,1\rangle$, where N is the set of natural numbers). Then $T_{\mathbf{N}} = T^{fk}_{\mathbf{N}}$ for each $k > 0$. ($\mathbf{N}$ is the only generic model).

PROOF. Since all elements of N are definable, we can consider $T_{\mathbf{N}}$ to be the elementary diagram of $\mathbf{N}$. $\mathbf{N}$, clearly, completes $T_{\mathbf{N}}$ (at the same time it is the only model which completes $T_{\mathbf{N}}$) and so $T_{\mathbf{N}} = T^{fk}_{\mathbf{N}}$ for each $k > 0$.

THEOREM 3.3. *Let* T_{PA} *be Peano arithmetic (defined in the language* L = { +,x, = ,0,1 }). *Then the following hold:*

(a) For each n *there exists* k > n *such that* $T_{PA}^{f_n} \neq T_{PA}^{f_k}$;

(b) None of the theories $T_{PA}^{f_k}$, k > 0 *is complete;*

(c) N *is not finitely generic model;*

(d) None of the theories $T_{PA}^{f_k}$, k > 0 *is model complete; in particular,* T_{PA}^{f} *is not a model companion of* T_{PA}.

PROOF. (a) Let us suppose that for some n $T_{PA}^{f_n} = T_{PA}^{f_k}$ for each k > n. Then, by 2.4.(b) $T_{PA} = T_{PA}^{f_n} = T\,(F^{n}_{T_{PA}})$. Therefore there exists a nonstandard model of the Peano arithmetic which n-completes it. But this is in contradiction with theorem 5 from [3].

(b) T_{PA} does not have the joint embedding property (one can construct two existential sentences, both consistent with T_{PA} but whose conjuction is not consistent with T_{PA}). Thus T_{PA}^{f}, and hence, of course, no theory $T_{PA}^{f_k}$, k > 0, is complete.

(c) s a direct consequence of Gödel's result that for each subtheory T of T_N with an effectively given recursively enumerable set of axioms, there exists a universal sentence ϕ such that $N \models \phi$ (i.e. $\phi \in T_N$) but $\phi \notin T$.

And (d) follows from the result from [4]: no theory T of the language L) such that $T_{PA} \cap \Pi_2 \subseteq T$ is model complete.

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REZIME

O n-KONAČNIM FORSING PRIDRUŽENJIMA

Ovaj rad se neposredno nadovezuje na [5] i [6]. Generalizacija(većine) rezultata iz [8] i nekih iz [10] (§1), uzgred rutinski i lak posao (kakav je već obavljen u [5] sa rezultatima iz [1]) ovde je data više zbog eventualne primene u nekim budućim radovima. Zapravo, više nas interesuje odnos date teorije i njoj korespodentnog n-konačnog forsing pridruženja ($n \in \omega$). Ovom prilikom iznosimo neka od prvih zapažanja na tu temu (§2, §3). Medju njima izdvajamo 3.1: za teoriju gustog linearnog uredjenja bez krajnjih tačaka (T_{DLOM}) važi: $T_{DLOM} = T_{DLOM}^{f_k}$ za svako

$k \geq 1$; i 3.3 - za Peanovu aritmetiku T_{PA} važi: za svako n postoji $k > n$ takvo da $T_{PA}^{f_n} \neq T_{PA}^{f_k}$.