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Prirodno-matematičkog fakulteta
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THE QUASIASYMPTOTIC OF DISTRIBUTIONS AND THE DISTRIBUTIONAL STIELTJES TRANSFORMATION

D.Nikolić-Despotović and S.Pilipović

University of Novi Sad, Faculty of Science,
Institute of Mathematics, Dr I. Djuričića 4,
21000 Novi Sad, Yugoslavia

ABSTRACT

Using the results for the quasiasymptotic at $\pm\infty$ of distributions ([8],[9]), the paper obtains a final value Abelian theorem and a Tauberian theorem of the Keldyš type for the distributional Stieltjes transformation.

1. NOTIONS AND NOTATION

The quasiasymptotic at ∞ of tempered distributions which have their supports in $[0,\infty)$ was studied by the Soviet mathematicians Vladimirov, Drožinov and Zavialov (see [12] and the references given there). Using this notion, they obtained remarkable results. S.Pilipović [8] extended this notion to the space of Schwartz distributions (on the real line):

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DEFINITION 1. *([8]) It is said that an* $f\in\mathcal{D}'$ *has the quasiasymptotic at* $\pm\infty$ *with respect to some positive continuous function* $c(k)$, $k\in(a,+\infty)$, $a>0$ *if for some* $g\in\mathcal{D}'$, $g\neq 0$,

$$(1.0)\qquad \lim_{k\to\infty}\left\langle \frac{f(kx)}{c(k)},\phi(x)\right\rangle = \langle g(x),\phi(x)\rangle\ ,\quad \phi\in\mathcal{D}\ .$$

In this case we write $f\overset{q}{\sim}g$ *at* $\pm\infty$ *with respect to* $c(k)$.

THEOREM I *([8]) Let* $f\in\mathcal{D}'$ *have the quasiasymptotic at* $\pm\infty$ *with respect to some positive continuous function* $c(k), k>0$. *Then*

(i) $f\in\mathcal{S}'$;

(ii) There are $\nu\in\mathbb{R}$ *and a slowly varying function* $L(k)$, $k>a$, *such that*

$$c(k) = k^{\nu}L(k),\ k>a\ .$$

Moreover, g *is a homogeneous distribution with the order of homogenity* ν;

(iii) If $\nu>-1$, *then (1.0) holds in the sense of convergence in* $\mathcal{S}'$ *(for* $\phi\in\mathcal{S}$*)*;

(iv) If $\nu=-1$ *and* $\frac{1}{L(k)}$, $k>a$, *is bounded, then the assertion in (iii) holds, as well.*

COROLLARY II *([8]) Let* $f\in\mathcal{D}'$ *and* $f\overset{q}{\sim}g$ *at* $\pm\infty$ *with respect to* $k^{\nu}L(k)$, *where* $\nu\in\mathbb{R}\setminus(-\mathbb{N})$. *Then (1.0) holds in the sense of convergence in* $\mathcal{S}'$.

Let us recall that the scale of homogeneous distribution $f_{\nu+1}$, $\nu\in\mathbb{R}$, is defined in the following way ([11]):

$$f_{\nu+1}(x) = \begin{cases} \dfrac{H(x)x^{\nu}}{\Gamma(\nu+1)} & \text{for } \nu>-1 \\ & (x\in\mathbb{R}) \\ f^{(n)}_{\nu+n+1}(x) & \text{for } \nu\le -1,\ n+\nu>-1,\ n\in\mathbb{N}, \end{cases}$$

where H is the Heaviside function.

(We also use the notation $H(x)x^{\nu}=x_+^{\nu}$, $H(-x)|x|^{\nu}=x_-^{\nu}$, $\nu>-1$).

The following two theorems are needed for our investigations.

THEOREM A. *([9]) (i) Let* F *be a locally integrable function and* $\nu \in \mathbb{R}$, $\nu > -1$, *such that*

$$\lim_{\substack{x\to+\infty \\ x\to-\infty}} \frac{F(x)}{|x|^{\nu} L(|x|)} = C_{\pm} \quad \textit{where } (C_+, C_-) \neq (0,0) .$$

Then $F \overset{q}{\sim} g$ *at* $\pm\infty$ *with respect to* $k^{\nu} L(k)$ *where*

$$g(x) = \bar{C}_+ f_{\nu+1}(x) + \bar{C}_- f_{\nu+1}(-x), \quad x \in \mathbb{R},$$

$$(\bar{C}_+, \bar{C}_-) \neq (0,0) .$$

(ii) If $f \overset{q}{\sim} g$ *at* $\pm\infty$ *with respect to* $k^{\nu} L(k)$ *then* $f^{(m)} \overset{q}{\sim} g^{(m)}$ *at* $\pm\infty$ *with respect to* $k^{\nu-m} L(k)$.

(iii) Let $f \in \mathcal{D}'$ *and* $f \overset{q}{\sim} g$ *at* $\pm\infty$ *with respect to* $k^{\nu} L(k)$ *where* $\nu \in \mathbb{R} \setminus (-\mathbb{N})$. *There are* $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ *and a continuous function* F *such that* $m+\nu > -1$,

$$f = F^{(m)} \textit{ and } \lim_{\substack{x\to+\infty \\ x\to-\infty}} \frac{F(x)}{|x|^{\nu+m} L(|x|)} = C_{\pm} ,$$

where $(C_+, C_-) \neq (0,0)$.

Let T be the class of non-decreasing functions defined for $x > 0$ and identically equal to zero in a neighbourhood of the origin. For $\alpha, \beta > 0$ we denote by $T(\alpha,\beta)$ the set of all $\phi \in T$ which are differentiable and satisfy the inequalities

$$\alpha\phi(x) < x\phi'(x) < \beta\phi(x)$$

for $x > a$, $a = \text{const.} > 0$.

THEOREM B *([10]) Let* $\phi(x) \in T(\alpha', \beta')$ *and* $\psi(x) \in T$ *for* $x \geq 0$. *Suppose also that either* $\phi(-x) \in T(\alpha', \beta')$, $\psi(-x) \in T$ *or* $-\phi(-x) \in T(\alpha', \beta')$, $-\psi(-x) \in T$ *when* $x \leq 0$. *Assume that*

$$A_1 \leq \left| \frac{\phi(-x)}{\phi(x)} \right| \leq A_2$$

for large values of x, *where* A_1 *and* A_2 *are positive constants.*

Suppose that the relation $m < \alpha' < \beta'$ *holds for* $m = [\beta']$.
If

$$\int_{-\infty}^{+\infty} (u-z)^{-m-1} d\phi(u) \sim \int_{-\infty}^{+\infty} (u-z)^{-m-1} d\psi(u)$$

when $z \to \infty$ *along a non-real half-ray from the origin, then* $\phi(x) \sim \psi(x)$ *when* $x \to +\infty$ *and* $x \to -\infty$.

We shall follow the definition of the distributional Stieltjes transformation given in [6], [7].

DEFINITION 2. *Let* $f \in S'$. *We say that* $f \in J'(r)$, *if there exist* $m \in \mathbb{N}_o$ *and a locally integrable function* F *such that*

(1.1)
a) $f = F^{(m)}$, supp $F \subset [0,\infty)$;

b) $\int_{-\infty}^{+\infty} |F(x)(x+z)^{-r-m-1}| dx < \infty$ *for* $\operatorname{Im} z \neq 0$:

The Stieltjes transformation S_r of index r, $r \in \mathbb{R} \setminus (-\mathbb{N}_o)$, of a distribution $f \in J'(r)$ with the properties given in (1.1) is a complex valued function given by

$$(S_r f)(z) = (r+1)_m \int_{-\infty}^{+\infty} F(x)(x+z)^{-r-m-1} dx \tag{1.2}$$

$$= (r+1)_m \left< F(x), \frac{1}{(x+z)^{r+m+1}} \right>$$

for $\operatorname{Im} z \neq 0$, where $(r)_k = r(r+1)\dots(r+k-1)$, $k > 0$ and $(r)_o = 1$.

It is easy to see that $(S_r f)(z)$ is a holomorphic function of the complex variable z in the domain $\mathbb{C} \setminus (-\infty,\infty]$.

We shall observe in this paper the function $(S_r f)$ in the upper half plane i.e. when $\operatorname{Im} z > 0$. This is not a restriction because one can easily show that all the assertion which are to follow hold with $\operatorname{Im} z < 0$.

2. ABELIAN AND TAUBERIAN THEOREM

First we shall prove the following lemma which will be used in the proof of the Abelian theorem. Assume that assumptions of Theorem A part (iii) hold for f with $L \equiv 1$. Moreover, assume that $r \in \mathbb{R} \setminus (-\mathbb{N})$ and $\nu < r$.

LEMMA. *The function*

$$z \mapsto z^{r-\nu}(S_r f)(i+z)$$

is uniformly bounded in the domain

$$\Lambda_\varepsilon = \{Re^{i\phi}: R > 0,\ \varepsilon < \phi < \pi-\varepsilon\},\ 0 < \varepsilon < \frac{\pi}{2}\ .$$

Proof. Since

$$\begin{aligned} R^2-2R|t|\cos\varepsilon + t^2 + 1 &> (R^2+t^2)(1+\cos\varepsilon)-(R+|t|)^2\cos\varepsilon + 1 \\ &> (R^2+t^2)(1+\cos\varepsilon)-2(R^2+t^2)\cos\varepsilon + 1 \\ &> (R^2+t^2)(1-\cos\varepsilon) + 1 \end{aligned}$$

we have, for $z \in \Lambda_\varepsilon$

$$\begin{aligned} |i+z+t| &= (R^2+t^2+2Rt\cos\phi + 2R\sin\phi+1)^{1/2} \\ &> (R^2-2R|t|\cos\varepsilon+t^2+1)^{1/2} \\ &> \sqrt{(R^2+t^2+1)(1-\cos\varepsilon)} > (R+|t|+1)\sqrt{\frac{1-\cos\varepsilon}{3}}\ . \end{aligned}$$

Using the fact that for some $C > 0$

$$|F(x)| < C(1+|x|^{\nu+m}) \qquad , x \in \mathbb{R}$$

we have (with suitable C_1 and C_2)

$$|(S_r f)(i+z)| < (r+1)_m \int_{-\infty}^{+\infty} |F(x)(z+x+i)^{-r-m-1}|dx$$

$$< (r+1)_m C\left(\frac{3}{1-\cos\varepsilon}\right)^{(r+m+1)/2} \int_{-\infty}^{+\infty} \frac{1+|x|^{\nu+m}}{(R+|x|+1)^{r+m+1}}\, dx$$

$$= C_1 \left(\int_0^\infty (R+x+1)^{-r-m-1} dx + \int_0^\infty x^{\nu+m} \quad (R+x+1)^{-r-m-1} dx\right)$$

$$= C_1 \left[\frac{1}{(r+m)(R+1)^{r+m}} + \frac{\Gamma(r-\nu)\Gamma(\nu+m+1)}{\Gamma(r+m+1)} \frac{1}{(R+1)^{r-\nu}} \right]$$

$$< C_2 (R+1)^{\nu-r} \quad \text{(see [5, p. 233])}.$$

This completes the proof of the lemma.

ABELIAN THEOREM. *(i) Let* $f \in S'$ *and* $f \overset{q}{\sim} g$ *at* $\pm\infty$ *with respect to* $k^\nu L(k)$, $\nu \in \mathbb{R} \setminus (-\mathbb{N})$. *Let* $r \in \mathbb{R} \setminus (-\mathbb{N})$ *and* $r > \nu$. *Then, there is* $(C_+, C_-) \neq (0,0)$ *such that for any* $z \in \mathbb{C} \setminus \mathbb{R}$

$$\frac{(S_r f)(kz)}{k^{\nu-r} L(k)} \to \frac{\Gamma(r-\nu)}{\Gamma(r+1)} z^{\nu-r} (C_+ + C_- e^{-\pi i(\nu-2r-m-1)}) \text{ as } k \to \infty$$

(ii) If $L \equiv 1$ *this holds uniformly in any angle of the form*

$$\Lambda_\varepsilon = \{Re^{i\phi},\ R > 0,\ \varepsilon < \phi < \pi - \varepsilon\} \quad (0 < \varepsilon < \tfrac{\pi}{2}).$$

Proof. First, we shall prove (i) for $r > \nu+1$.

According to Theorem A (iii) we have that $f = F^{(m)}$ for some $m \in \mathbb{N}_0$ and some continuous function F such that $m+\nu > -1$ and that for every $\phi \in S$

$$\lim_{k\to\infty} \left\langle \frac{F(kx)}{k^{m+\nu} L(k)}, \phi(x) \right\rangle = \langle C_+ f_{\nu+m+1}(x) + C_- f_{\nu+m+1}(-x), \phi(x)\rangle, \tag{2.1}$$

where $(C_+, C_-) \neq (0,0)$.

Moreover, we have that $f \in J'(r)$ for $r > \nu$ because

$$F(x) \sim \frac{C_+ x_+^{\nu+m} L(x)}{\Gamma(\nu+m+1)}, \quad x \to \infty$$

$$F(x) \sim \frac{C_- x_-^{\nu+m} L(|x|)}{\Gamma(\nu+m+1)}, \quad x \to -\infty .$$

If C_+ or C_- is equal to 0, then we have to substitute the symbol $\sim$ with the small "o".

The behaviour of F implies that (2.1) holds in the sense of convergence in $\mathcal{S}'_p$ for $p > \nu+m+1$ (see [11]).

Since the function $x \mapsto (x+z)^{-r-m-1}$ belongs to $\mathcal{S}_{r+m}$ ($\operatorname{Im} z > 0$), the assumption $\nu < r-1$ implies that for $p = r+m$, (2.1) holds with $\phi(x) = (x+z)^{-r-m-1}$, $x \in \mathbb{R}$. This implies

$$\lim_{k\to\infty} \frac{(S_r f)(kz)}{k^{\nu-r} L(k)} = (r+1)_m \lim_{k\to\infty} \left\langle \frac{F(x)}{k^{\nu-r} L(k)}, \frac{1}{(kz+x)^{r+m+1}} \right\rangle$$

$$= \langle (r+1)_m \lim_{k\to\infty} \left\langle \frac{F(kx)}{k^{\nu+m} L(k)}, \frac{1}{(z+x)^{r+m+1}} \right\rangle$$

$$= (r+1)_m \Big(\left\langle \frac{C_+}{\Gamma(\nu+m+1)} x_+^{\nu+m}, (z+x)^{-r-m-1} \right\rangle$$

$$+ \left\langle \frac{C_-}{\Gamma(\nu+m+1)} x_-^{\nu+m}, (x+z)^{-r-m-1} \right\rangle \Big)$$

$$= (r+1)_m \Big(\frac{C_+ \Gamma(\nu+m+1)\Gamma(r-\nu)}{\Gamma(\nu+m+1)\Gamma(r+m+1)} z^{\nu-r}$$

$$+ \frac{C_- \Gamma(\nu+m+1)\Gamma(r-\nu)\ (-z)^{\nu-r}}{\Gamma(\nu+m+1)\Gamma(r+m+1)(-1)^{r+m+1}} \Big) \quad ([5,\ p.\ 233]).$$

Thus we have

$$\lim_{k\to\infty} \frac{(S_r f)(kz)}{k^{\nu-r} L(k)} = A(z) ,$$

where

$$\text{(2.2)} \quad A(z) = \frac{C_+ \Gamma(r-\nu)}{\Gamma(r+1)} z^{\nu-r} + \frac{C_- \Gamma(r-\nu)}{\Gamma(r+1)} e^{-\pi i(\nu-2r-m-1)} z^{\nu-r}$$

Now we shall prove (i) in case $r > \nu \geq r-1$.

Lebesgue's theorem implies

$$(S_r f)(kz) = (r+1)_m \left\langle F(x), \frac{1}{(kz+x)^{r+m+1}} \right\rangle \to 0 \text{ as } k \to \infty$$

Because of

$$\frac{d}{dk}((S_r f)(kz)) = -z(r+1)(S_{r+1}f)(kz)$$

we have ($k > 0$, $\operatorname{Im} z > 0$)

$$(S_r f)(kz) = -\int_k^\infty ((S_r f)(tz))' dt = z(r+1)\int_k^\infty (S_{r+1}f)(tz)dt.$$

From the proof of the first part of the Abelian theorem, it follows that

$$\frac{(S_{r+1}f)(kz)}{k^{\nu-(r+1)}L(k)} \to (C_+ + C_-(e^{-\pi i})^{\nu-2r-m-3}) \frac{\Gamma(r+1-\nu)}{\Gamma(r+2)} z^{\nu-r-1}$$

as $k \to \infty$.

Also, as $k \to \infty$

$$(S_r f)(kz) \sim z(r+1) \frac{\Gamma(r+1-\nu)}{\Gamma(r+2)} z^{\nu-r-1}$$

$$\cdot (C_+ + C_-(e^{-\pi i})^{\nu-2r-m-3}) \int_k^\infty t^{\nu-r-1}L(t)dt .$$

Since (see [1])

$$\int_k^\infty t^{\nu-r-1}L(t)dt \sim k^{\nu-r}L(k) \int_1^\infty u^{\nu-r-1}du = \frac{k^{\nu-r}L(k)}{r-\nu}, \quad k \to \infty,$$

we obtain

$$(S_r f)(kz) \sim k^{\nu-r}L(k)A(z), \quad k \to \infty .$$

where $A(z)$ is given by (2.2).

Proof of (ii). Let $L \equiv 1$. Since we have proved in (i) that

$$(2.3) \quad (S_r f)(ki) \sim \frac{\Gamma(r-\nu)}{\Gamma(\nu+1)} (ki)^{\nu-r}(C_+ + C_-(e^{-\pi i})^{\nu-2r-m-1})$$

as $k \to \infty$,

we have

$$(2.4)\quad (S_r f)(ki+i) \sim \frac{\Gamma(r-\nu)}{\Gamma(\nu+1)}(ki)^{\nu-r}(C_+ + C_-(e^{-\pi i})^{\nu-2r-m-1})$$

as $k \to \infty$

The Lemma implies that for $z \to (S_r f)(z+i)$, $z \in \Lambda_\varepsilon$ the conditions of Montel's Theorem ([2], p.5) are satisfied. So, we have that (2.4) (with z instead of ki) holds uniformly in Λ_ε.

Obviously, this implies that (2.3) holds (with z instead ki) uniformly in $\Lambda_{\varepsilon'}$ for any $\varepsilon' < \varepsilon$.

This completes the proof of (ii).

Let F and F_1 be continuous functions such that $\mathrm{supp}(F-F_1)$ is a compact set and

$$\lim_{\substack{x \to +\infty \\ x \to -\infty}} \frac{F(x)}{|x|^\nu L(|x|)} = \lim_{\substack{x \to +\infty \\ x \to -\infty}} \frac{F_1(x)}{|x|^\nu L(|x|)} = C_\pm$$

where $(C_+, C_-) \neq (0,0)$.

Then for $r > \nu$ and any z (with $\mathrm{Im}\, z > 0$) there holds

$$\lim_{k \to \infty} \frac{(S_r(F-F_1))(kz)}{k^{\nu-r}L(k)} = 0.$$

This means

$$(S_r F)(kz) \sim (S_r F_1)(kz) \quad \text{as } k \to \infty,\ \mathrm{Im}\, z > 0.$$

This fact implies that in Theorem B we can assume that the function ϕ is differentiable for $|x| > a > 0$.

TAUBERIAN THEOREM. *Let* $\nu = r-1+2\varepsilon$, $0 < \varepsilon < 1/3$.

(i) Let ψ *be a non-decreasing function such that*

$$\int_{-\infty}^{+\infty}(x+nz)^{-r-k}d\psi(x) = (r+k)\int_{-\infty}^{+\infty}(x+nz)^{-r-k-1}\psi(x)dx$$

$$\sim n^{\nu-r}L(n)z^{\nu-r}\Gamma(\nu+k+1)\,\frac{\Gamma(r-\nu)}{\Gamma(r+k)}\,(C_1 - C_2 e^{-\pi i(\nu-2r-k-1)})$$

as $n \to \infty$ *where* $C_1, C_2 > 0$.

Then

$$\lim_{\substack{x\to+\infty \\ x\to-\infty}} \frac{\psi(x)}{|x|^{\nu+k}L(|x|)} = \begin{cases} C_1 \\ -C_2 \end{cases}$$

(ii) Let $f = \psi^{(k)}$, $k+\nu > -1$, *where* ψ *is a non-decreasing function. If* $\frac{(S_r f)(nz)}{(r+1)_{k-1}} \sim \frac{n^{\nu-r}L(n)\Gamma(\nu+k+1)\Gamma(r-\nu)}{\Gamma(r+k)} z^{\nu-r}(C_1 - C_2e^{-\pi i(\nu-2r-k-1)})$, $n\to\infty$ $(C_1, C_2 > 0)$, *then* f *has the quasi-asymptotic at* $\pm\infty$ *with respect to* $n^{\nu}L(n)$ *and with the limit* $(C_1x_+^{\nu+k} - C_2x_-^{\nu+k})^{(k)}$.

Proof. (i) Let us put $\alpha' = r+k-1+\varepsilon$, $\beta' = r+k-1+3\varepsilon$ and

$$\phi(x) = \begin{cases} C_1x^{\nu+k}L(x) & , x > a \\ 0 & , -a \le x \le a \\ -C_2|x|^{\nu+k}L(|x|), & x < -a \end{cases}$$

Then functions ϕ and ψ satisfy the conditions of Theorem B. By the Abelian theorem we have

$$\int_{-\infty}^{+\infty} (x+nz)^{-r-k}d\phi(x) = (r+k)\int_{-\infty}^{+\infty} (x+nz)^{-r-k-1}\phi(x)dx$$

$$\sim n^{\nu-r}L(n)z^{\nu-r}\Gamma(\nu+k+1)\frac{\Gamma(r-\nu)}{\Gamma(r+k)}(C_1 - C_2e^{-\pi i(\nu-2r-k-1)}) \text{ as } n\to\infty.$$

So, Theorem B implies the assertion.

(ii) We have

$$\frac{(S_r f)(nz)}{(r+1)_{k-1}} = (r+k)\int_{-\infty}^{+\infty} (x+nz)^{-r-k-1}\psi(x)dx =$$

$$= \int_{-\infty}^{+\infty} (x+nz)^{-r-k}d\psi(x)$$

$$\sim n^{\nu-r}L(n)\Gamma(\nu+k+1)\frac{\Gamma(r-\nu)}{\Gamma(r+k)} z^{\nu-r}(C_1 - C_2e^{-\pi i(\nu-2r-k-1)})$$

as $n\to\infty$.

From the first part of this theorem and Theorem A (i) it follows

$$\psi \overset{q}{\sim} C_1 x_+^{\nu+k} - C_2 x_-^{\nu+k} \quad \text{with respect to } n^{\nu+k}L(n) \;,\; n \to \infty .$$

Now Theorem A (ii) implies the assertion.

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REZIME

KVASIASIMPTOTIKA DISTRIBUCIJA I
DISTRIBUCIONA STIELTJESOVA TRANSFORMACIJA

Koristeći rezultate [8],[9] za kvaziasimptotiku u $\pm\infty$ temperiranih distribucija dobivene su dve teoreme jedna Abelova a druga Tauberova tipa Keldiša za distribucionu Stieltjesovu transformaciju.

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