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RANDOM FIXED POINT THEOREMS FOR RANDOM UPPER SEMICONTINUOUS MULTIVALUED OPERATORS

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ABSTRACT

A general random fixed point theorem for random upper semicontinuous multivalued operators with a stochastic domain in Fréchet spaces is proved. Using this theorem it is possible to obtain special random fixed point theorems for mappings of condensing type. The results generalize theorems by Engl [4], Itoh [10] and Schleinkofer [16].

1. INTRODUCTION

Recently many results from fixed point theory for random operators have been proved (cf. for instance [4], [10], [16] and their references). If F is a random continuous multivalued operator, then F has a random fixed point, if the corresponding deterministic fixed point problem is solvable (s. Engl. [4, Theorem 6].

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Many special random fixed point theorems are contained in this excellent result. However this theorem is unknown for the important case of random upper semicontinuous multivalued operators.

Therefore special random fixed point theorems for such operators have been given only (cf. [4], [8], [10], [16]). In this paper the main theorem assures the existance of a random fixed point of a random upper semicontinuous multivalued operator provided that the corresponding deterministic fixed point problem is solvable "in the strong sense" (cf. Definition 6). This theorem includes as special cases a number of known, as well as some new, random fixed point theorems.

Throughout this paper let E be a real separable Fréchet space and let (Ω,γ,μ) be a σ-finite complete measure space. We shall denote by $L(E)$, $\gamma\otimes L(E)$ the σ-algebra of Borel sets of E and the smallest σ-algebra containing $\{S\times B:S\in\gamma,\ B\in L(E)\}$. Let $K\subseteq E$. We shall denote by $\overline{\mathrm{conv}}\ K$, $\bar{K}$, int K, ∂K the closed convex hull, the closed hull, the interior and the boundary of K, respectively.

We define $N(K):=\{M\subseteq K:M\neq\emptyset\}$, $Cl(K):=\{M\in N(K):M$ is closed in $K\}$ and $k(K):=\{M\in N(K):M$ is convex and compact in $K\}$. Let X be a set and $F:X\to N(E)$ a mapping. We define for such "multivalued" (on X in E) mappings $\mathrm{Gr}\ F:=\{(x,y)\in X\times E:y\in F(x)\}$ (the graph of F), $F^{-1}(B):=\{x\in X:F(x)\cap B\neq\emptyset\}$ $(B\subseteq E)$, $F(A):=\cup\{F(x):x\in A\}$ $(A\subseteq X)$.

2. QUASICOMPACT MAPPINGS

Let $M\subseteq E$ and $F:M\to N(E)$. $x\in M$ is called a fixed point of F, iff $x\in F(x)$. F is called upper semicontinuous (usc) iff for all $x\in M$ and for all open $G\subseteq E$ with $F(x)\subseteq G$, there exists a neighbourhood U of x with $F(U\cap M)\subseteq G$.

$F:M\to N(R)$ is usc iff $F^{-1}(B)$ is a closed subset of M for all closed $B\subseteq E$. If F is usc and assume that F(x) is compact for all $x\in M$, then F(K) is compact for all compact $K\subseteq M$. If $F:M\to Cl(E)$ is usc, then Gr F is a closed subset of $M\times E$.

$F:M\to N(E)$ will be called compact, iff F is usc and $\overline{F(M)}$ is compact.

DEFINITION 1. *If C is a cone in a vector space, which defines the partial ordering* $\leq$, *then a mapping* $\psi : 2^E \to C$ *is called a measure of noncompactness of* E *provided that the following conditions hold for any* M, N *in* 2^E:

(1) $\psi(\overline{\text{conv}}\ M) = \psi(M)$

(2) $|t|\psi(M) = \psi(tM)$

(3) $\psi(M \cup N) = \max\{\psi(M), \psi(N)\}$

(4) $\psi(M) = 0$ iff M is compact.

This known notation is a generalization of the "set-measure" and the "ball-measure" of noncompactness (Kuratowski- and Hausdorff measure of noncompactness).It follows that if $M \subseteq N$, then $\psi(M) \leq \psi(N)$ and $\psi(M \cup \{a\}) = \psi(M)$ for any $a \in E$.

DEFINITION 2. *Let* ψ *be a measure of noncompactness of* E *and* $M \subseteq E$. *A usc mapping* $F : M \to k(E)$ *is called condensing, provided that if* $N \subsetneq M$ *and* $\psi(F(N)) \geq \psi(N)$, *then* F(N) *is relatively compact.* $F : M \to k(E)$ *is called 1-condensing, iff* $\psi(F(N)) \leq \psi(N)$ $(N \subseteq M)$.

We introduced in [5] the following notion.

DEFINITION 3. *Let* $M \subseteq E$ *with* $M \neq \emptyset$. *A usc mapping* $F : M \to k(E)$ *will be called quasicompact, iff there exists a closed, convex subset* $S \subseteq E$ *such that the following conditions hold:*

(1) $M \cap S \neq \emptyset$,

(2) $F(M \cap S) \subseteq S$,

(3) $F(M \cap S)$ *is relatively compact* .

REMARK 1. Let $M \subseteq E$ with $M \neq \emptyset$ and let $F : M \to k(E)$ be a usc mapping. Then F is quasicompact, if any of the following conditions is satisfied:

(1) F is compact

(2) F is condensing.

(3) $F(M) \subseteq M$, M is convex and closed and F is generalized condensing [2], i.e. if $A \subseteq M$, $F(A) \subseteq A$ and $A = \overline{\text{conv}}\, F(A)$ or $A \setminus F(A) \subseteq \{a\}$ for some $a \in E$ then $\overline{F(A)}$ is compact.

(4) F is a C_1-mapping ([15]), i.e. if $S = \overline{\text{conv}}\,(\{a\} \cup F(M \cap S))$ $(a \in E,\ S \subseteq E)$, then $\overline{F(M \cap S)}$ is compact.

(5) F is ultimately compact ([14]) and for the limit set F_∞ of F it holds that $F_\infty \neq \emptyset$.

PROOF. Cf. [5, 1.3] or the proof of Theorem 2 of [6].

We shall apply some deterministic fixed point theorems to obtain random fixed point theorems.

PROPOSITION 1 ([6, Theorem 1]). *Let* K *be a nonvoid, closed, convex subset of* E. *Suppose* $F: K \to k(K)$ *is quasicompact. Then* F *has a fixed point.*

PROPOSITION 2. *Let* $U \subseteq E$ *be open,* $K \subseteq E$ *be closed and convex such that* $U \cap K$ *is nonvoid and convex. Suppose* $F: \bar{U} \cap K \to k(K)$ *is quasicompact. If* $F(x) \cap U \neq \emptyset$ *for each* $x \in \partial U \cap K$ *then* F *has a fixed point.*

PROOF. This result is known for compact mappings ([1, Theorem 1.2.45]). There exists a closed, convex subset $S \subseteq E$ with $\bar{U} \cap K \cap S \neq \emptyset$, $F(\bar{U} \cap K \cap S) \subseteq K \cap S$ and $F_o := F|\bar{U} \cap K \cap S$ is a compact mapping. We define $R := K \cap S$. If $\partial U \cap R = \emptyset$, then we can apply Proposition 1, and F_o has a fixed point. If $\partial U \cap R \neq \emptyset$, then $U \cap R \neq \emptyset$, because $F(x) \cap U \cap R \neq \emptyset$, for each $x \in U \cap R$. Now, we shall use the result [1] for the compact mapping $F_o: \bar{U} \cap R \to k(R)$.

PREPOSITION 3. *Let* $U \subseteq E$ *be an open symmetric neighbourhood of the origin,* $K \subseteq E$ *be a closed, absolute convex subset and* $F: \bar{U} \cap K \to k(K)$ *be a mapping with* $x \notin F(x)$ $(x \in \partial U \cap K)$ *and* $F(-x) = -F(x)$ $(x \in \partial U \cap K)$. *Suppose that* F *satisfies any of the following two conditions:*

(1) *F is condensing.*

(2) *F is 1-condensing,* $F(\bar{U}\cap K)$ *is bounded and F is demicompact in 0 (i.e. if for bounded sequences* (x_n), (z_n) *with* $x_n \in \bar{U}\cap K$, $z_n \in F(x_n)$ $(n\in N)$ *we have* $x_n - z_n \to 0$, *then there exists* $x\in E$ *and a subsequence* (x_{n_k}) *with* $x_{n_k} \to x$).

Then, in either case, F has a fixed point in $U\cap K$.

PROOF. This result is a simple consequence of theorems 4.2.3 and 4.2.4 in [5].

3. RANDOM OPERATORS

The mapping $A:\Omega \to N(E)$ is called measurable (weakly measurable in [9]), iff we have $A^{-1}(G)\in\gamma$ for each open $G\subseteq E$.

REMARK 2 ([9, Theorem 3.5.]) Let $A:\Omega\to Cl(E)$. Then the following statements are equivalent:

(1) A is measurable.

(2) $A^{-1}(B)\in\gamma$ for each $B\in L(E)$.

(3) $A^{-1}(M)\in\gamma$ for each closed $M\subseteq E$.

(4) $\mathrm{Gr}\ A\in\gamma\otimes L(E)$.

DEFINITION 4. *Let* $A:\Omega\to Cl(E)$ *be a measurable mapping and* $F:\mathrm{Gr}\ A\to N(E)$ *a mapping. F will be called a random usc (multivalued) operator iff*

(1) $\{w\in\Omega: x\in A(w),\ F(w,x)\cap G\neq\emptyset\}\in\gamma$ *for each* $x\in E$ *and each open* $G\subseteq E$ *and*

(2) $F(w,\cdot)$ *is usc for each* $w\in\Omega$.

This notion ([3]) considers the general case, that the domain of F is stochastic. For $A(w)=A_o\in Cl(E)$ $(w\in\Omega)$, we obtain the special case of the "deterministic domain" $\Omega\times A_o$ as in [10]. Then F is a random usc mapping, iff $F(\cdot,x)$, is measurable for each $x\in A_o$ and $F(W,\cdot)$ is usc for all $w\in\Omega$.

DEFINITION 5. *Let* $A:\Omega\to Cl(E)$ *be a measurable mapping and* $F:Gr\,A\to N(E)$ *a random usc operator. The function* $x:\Omega\to E$ *will be called a random fixed point of* F *iff*

(1) $x(w)\in F(w,x(w))$ *for each* $w\in\Omega$ *and*

(2) x *is a measurable function.*

The following result is a fundamental lemma for the proof of random fixed point theorems.

REMARK 3 ([12]). If $P:\Omega\to Cl(E)$ is measurable, then there exists a measurable function $x:\Omega\to E$ with $x(w)\in P(w)$ for each $w\in\Omega$.

DEFINITION 6. *Let* $A:\Omega\to Cl(E)$ *be a measurable mapping.* A *is called separable, iff there exists a countable set* $Z\subseteq E$ *with* $A(w)=\overline{Z\cap A(w)}$ *for all* $w\in\Omega$.

If $A(w)=A_o\in Cl(E)$ for each $w\in\Omega$, then A is separable (we supposed that E is separable). If $A:\Omega\to Cl(E)$ is measurable and $A(w)=\overline{\text{int}\,A(w)}$ $(w\in\Omega)$, then A is separable ([4, p.70], the proof for Banach spaces holds for our general case too). Therefore, if $O:\Omega\to N(E)$ is measurable, $O(w)$ is open for each $w\in\Omega$ and $A(w)=\overline{O(w)}$ $(w\in\Omega)$, then A is separable (cf. [9, Prop. 2.6]).

4. RANDOM FIXED POINT THEOREMS

Following [3], [10], [16] we shall prove:

LEMMA. *Let* E *be a separable Fréchet space with a metric* d *and* $A:\Omega\to Cl(E)$ *a separable mapping. Suppose* $F:Gr\,A\to k(E)$ *is a random usc operator. Let* Z *be a countable set as it appears in Definition 6. We shall define for all* $(w,x)\in Gr\,A$:

$$F_n(w,x):=\bigcup\{F(w,z):z\in A(w)\cap Z,\ d(z,x)<\tfrac{1}{n}\}\quad (n\in N)$$

$$H(w,x):=\bigcap_{n=1}^{\infty}\overline{\text{conv}}\,F_n(w,x).$$

Then $H:Gr\,A\to N(E)$ *has the following properties:*

(1) $\emptyset \neq H(w,x) \subseteq F(w,x) \quad ((w,x) \in \mathrm{Gr}\ A)$.

(2) $H(w,\cdot): A(w) \to k(E)$ *is usc for each* $w \in \Omega$.

(3) *Let* $T(w,x) := x - H(w,x) \quad ((w,x) \in \mathrm{Gr}\ A)$. *Then* $\{(w,x) \in \mathrm{Gr}\ A : T(w,x) \cap D \neq \emptyset\} \in \gamma \otimes L(E)$ *for each compact* $D \subseteq E$.

PROOF. (Similar to [3], [10], [16], [8] for the special case of Banach spaces). In the proof of (1) and (2), we choose w fixed (but arbitrary). Therefore, we do not write the argument $w \in \Omega$.

(1) Let $x \in A(w)$. For all $n \in N$, we choose $y_n \in F_n(x)$. Then, there exists a $z_n \in A(w) \cap Z$ with $d(z_n,x) < \frac{1}{n}$ and $y_n \in F(z_n) \subseteq F(\bigcup_{n=1}^{\infty} \{z_n\})$ $(n \in N)$. Since $\{z_n, n \in N\} \cup \{x\}$ is compact and $F(w,\cdot): A(w) \to k(E)$ is usc the set $\{y_n, n \in N\}$ is relatively compact. Therefore, we can take without loss of generality $y_n \to y \in E$. Assume $y \notin H(x)$. Then there exists $n_o \in N$ with $y \notin \overline{\mathrm{conv}\ F_{n_o}(x)}$, and we can find $n \geq n_o$ with $y_n \notin \overline{\mathrm{conv}\ F_{n_o}(x)}$. However, $y_n \in \overline{\mathrm{conv}\ F_n(x)} \subseteq \overline{\mathrm{conv}\ F_{n_o}(x)}$. Consequently, $H(x) \neq \emptyset$. Now, we shall prove that for all $x \in A(w)$, there exists $n \in N$ with $\overline{\mathrm{conv}}\ F_n(x) \subseteq F(x)$. This implies $H(x) \subseteq F(x)$.
Assume that we can find a $y \in \overline{\mathrm{conv}}\ F_n(x)$ $(n \in N)$ but $y \notin F(x)$ for some $x \in A(w)$. Since $F(x)$ is closed and E is locally convex, there exists an absolute convex open neighbourhood V of zero with $(y+\bar{V}) \cap F(x) = \emptyset$. Because F is usc, there exists $n \in N$ such that we have $F(z) \subseteq F(x)+V$ for each $z \in A(w)$ with $d(z,c) < \frac{1}{n}$. Consequently $F_n(x) \subseteq F(x)+\bar{V}$. Since $F(x) \in k(E)$, $F(x)+\bar{V}$ is closed and convex, and therefore, $y \in F(x)+\bar{V}$. This is a contradiction.

(2) Clearly, using (1), $H(x) \in k(E)$ for each $x \in A(w)$. We prove that $H^{-1}(B)$ is closed for each closed $B \subseteq E$. Let $B \subseteq E$ be closed and (x_j) be a sequence in $\{x \in A(w) : H(w,x) \cap B \neq \emptyset\}$ with $x_j \to x$. For each n we choose x_j with $d(x_j,x) < \frac{1}{2n}$. Since $d(z_k,x) \leq d(z_k,x_j) + d(x_j,x)$ for all $z_k \in A(w) \cap Z$, it follows $F_{2n}(x_j) \subseteq F_n(x)$. Therefore $F_n(x) \cap B \supseteq F_{2n}(x_j) \cap B \supseteq H(x_j) \cap B \neq \emptyset$ $(n \in N)$. Since $\overline{\mathrm{conv}}\ F_n(x) \subseteq F(x)$ $(n \geq n_o)$ (cf. the second part of the

proof of (1)), $\overline{\text{conv}}\, F_n(x)$ is compact for $n \geq n_o$. Because $F_n(x) \supseteq F_{n+1}(x)$ $(n \in N)$ and B is closed, we have

$$\emptyset \neq \bigcap_{n=1}^{\infty} [\overline{\text{conv}}\, F_n(x) \cap B] = H(x) \cap B.$$

(3) Let $G \subseteq E$ be open and $n \in N$. Then (cf. [3])
$\{(w,x) \in \Omega \times E : x \in A(w),\ F_n(w,x) \cap G \neq \emptyset\}$

$$= \bigcup_{z \in Z}^{\infty} ([\Omega \times \{x \in E : d(x,z) < \tfrac{1}{n}\}] \cap \mathrm{Gr}\, A$$

$\cap [w \in \Omega : z \in A(w),\ F(w,z) \cap G \neq \emptyset\} \times E]) \in \gamma \otimes \mathcal{L}(E)$ (cf. Remark 2 and Definition 4). Then F_n is measurable on $(\mathrm{Gr}\, A,\ (\gamma \otimes \mathcal{L}(E)) \cap \mathrm{Gr}\, A)$. Now, we shall apply the results by Himmelberg ([9, Theorem 9.1., Prop 2.6.]) and therefore $\overline{\text{conv}}\, F_n$ is measurable on $\mathrm{Gr}\, A$ $(n \in N)$. We define $T_n(w,x) := x - \overline{\text{conv}}\, F_n(w,x)$ $((w,x) \in \mathrm{Gr}\, A)$ $(n \in N)$. Then T_n is measurable on Gr A too $(n \in N)$. Since $T(w,x) =$

$$= \bigcap_{n=1}^{\infty} T_n(w,x),$$ using [9, Corollary 4.3. and Theorem 3.2], we obtain

$\{(w,x) \in \mathrm{Gr}\, A : T(w,x) \cap D \neq \emptyset\} \in \gamma \otimes \mathcal{L}(E)$ for each compact $D \subseteq E$.
Now, we shall introduce the following notation.

DEFINITION 6. Let $A \subseteq E$ be closed and $F : A \to k(E)$ a usc mapping. Let $X \subset A$ be closed with $\emptyset \subseteq X$ and $x \notin F(x)$ for each $x \in X$. We define that F has a fixed point in the strong sense on $A \setminus X$, iff all the usc mappings $\tilde{F} : A \to k(E)$ with $\tilde{F}(x) \subseteq \tilde{F}(x)$ $(x \in A)$ and $\tilde{F}(x) = F(x)$ $(x \in X)$ have a fixed point in $A \setminus X$.

For instance, let W be an open subset of E, $A = \bar{W}$, $F : \bar{W} \to k(E)$ a compact mapping with $x \notin F(x)$ $(x \in W)$ and deg $(I-F, W, o) \neq 0$ (the Leray-Schauder-degree for multivalued compact mappings [13]). Then, F has a fixed point in the strong sence on $W = \bar{W} \setminus \partial W$, because each usc mapping $\tilde{F} : \bar{W} \to k(E)$ with $\tilde{F}(x) \subseteq \tilde{F}(x)$ $(x \in A)$ is compact and $\tilde{F}(x) = F(x)$ $(x \in W)$ implies deg $(I-F, \dot{W}, o) \neq 0$. Another case, if $A = K$ is closed and convex, then each condensing mapping $F : K \to k(K)$ has a fixed point in the strong sense on K (we can choose $X = \emptyset$).
Now, we can prove our main theorem.

THEOREM 1. *Let* E *be a separable Fréchet space,* $A:\Omega\to Cl(E)$ *separable and* $F:gr\ A\to k(E)$ *a random usc operator. Suppose that* $F(w,\cdot)$ *has for all* $w\in\Omega$ *a fixed point in the strong sense on* $A(w)\setminus X(w)$ *for some closed* $X(w)\subseteq A(w)$. *Then* F *has a random fixed point.*

PROOF. We define $H:Gr\ A\to k(E)$ as in Lemma. Let $w\in\Omega$ be fixed, but arbitrary. Let $A_o(w):=\{y\in A(w):y\in F(w,y)\}$. Since $F(w,\cdot)$ is usc, $A_o(w)$ is a closed subset of E. Let $X(w)$ be a closed subset of E with $X(w)\subseteq A(w)$ and $x\notin F(w,x)$ $(x\in X(w))$. Therefore, $X(w)\cap A_o(w)=\emptyset$. Since E is normal, we can find a continuous function $f_w:E\to[0,1]$ with $f_w(x)=0\,(x\in A_o(x))$ and $f_w(x)=1$ $(x\in X(w))$. We define $\tilde{F}(w,x):=f_w(x)F(w,x)+(1-f_w(x))\times H(w,x)$ $(x\in A(w))$.

Since by our Lemma $H(w,\cdot)$ is usc and $\emptyset\neq H(w,x)\subseteq F(w,x)\,(x\in A(w))$, $\tilde{F}(w,\cdot):A(w)\to k(E)$ is usc with $\tilde{F}(w,x)\subseteq F(w,x)$ (we apply that $F(w,x)$ is convex) for each $x\in A(w)$. If $x\in X(w)$, then $f_w(x)=1$, and therefore $\tilde{F}(w,x)=F(w,x)$. Since we supposed that F has a fixed point in the strong sense on $A(w)\setminus X(w)$, there exists a $x_o\in A(w)\setminus X(w)$ with $x_o\in\tilde{F}(w,x_o)\subseteq F(w,x_o)$. Therefore $x_o\in A_o(w)$, $f(x_o)=0$ and $\tilde{F}(w,x_o)=H(w,x_o)$, $x_o\in H(w,x_o)$. We have proved that $P(w):=\{x\in A(w):x\in H(w,x)\}\neq\emptyset$ for each $w\in\Omega$. Because by Lemma $H(w,\cdot)$ is usc, the sets $P(w)$ are closed $(w\in\Omega)$. Now, we shall prove that the mapping $P:\Omega\to Cl(E)$ is measurable. We define $T(w,x)=x-H(w,x)$ $((w,x)\in Gr\ A)$, and obtain

$Gr\ P=\{(w,x)\in\Omega\times E:x\in P(w)\}=\{(w,x)\in Gr\ A:x\in H(w,x)\}=T^{-1}(\{o\})$.

Using our Lemma (3), $T^{-1}(\{o\})\in\gamma\otimes L(E)$.

Hence, applying Remark 2, P is measurable. By Remark 3, there exists a measurable function $x:\Omega\to E$ with $x(w)\in P(w)$, also $x(w)\in H(w,x(w))$ $(w\in\Omega)$. Since $H(w,x(w))\subseteq F(w,x(w))$, $x:\Omega\to E$ is a random fixed point for F.

Now, we shall apply this general Theorem 1 to the derivation of random fixed point theorems for various special classes of mappings. We remark, that these theorems are valid for the special case, that the domains are "deterministic", i.e. $A(w)=A_o\in Cl(E)$ $(w\in\Omega)$, $Gr\ A=\Omega\times A_o$ (Then A is separable).

THEOREM 2. *Let* $A:\Omega\to Cl(E)$ *be separable and* $F:Gr\ A\to k(E)$ *a random usc operator. We suppose for each* $w\in\Omega$:

(1) $A(w)=W(w)\cap K(w)$ *such that* $K(w)$ *is a finite intersection of closed convex subsets of* E *and* W *a closed neighbourhood of an* $u(w)\in K(w)$ *and* $K(w)$ *is starshaped relative* $u(w)$.

(2) $F(w,x)\subseteq K(w)\quad (x\in A(w))$.

(3) $F(w,\cdot)$ *is condensing.*

(4) $\beta x+(1-\beta)u(w)\notin F(w,x)\quad (x\in\partial W(w)\cap K(w)\ \beta>1)$.

Then, F *has a random fixed point.*

PROOF. F has a fixed point in the strong sense on $A(w)=W(w)\cap K(w)$ (we choose that $X(w)=\emptyset$) for each $w\in\Omega$, because each usc mapping $\tilde{F}(w,\cdot):A(w)\to k(E)$ with $\tilde{F}(w,x)\subseteq F(w,x)$ $(x\in A(w))$ is condensing, $\tilde{F}(w,x)\subseteq K(w)\quad(x\in A(w))$ and the Leray--Schauder condition (4) holds for $\tilde{F}(w,\cdot)$, too. Therefore, we can apply for $\tilde{F}(w,\cdot)$ a fixed point theorem by Jerofsky [11] or the theorem in [7] (for $c=1$).
Then $\tilde{F}(w,\cdot)$ has, in fact, a fixed point in $A(w)$.
Hence, by Theorem 1, F has a random fixed point.

Theorem 2 generalizes for the special case $K=E$ Theorem 24 in [16]. For the special case K is convex, we have proved Theorem 2 for in o demicompact 1-condensing mapping (with the set-measure of noncompactness) in [8]. We can deduce this result from Theorem 1, too. We omit the details.

COROLLARY 1. *Let* $A:\Omega\to Cl(E)$ *be separable and* $F:Gr\ A\to k(E)$ *a random usc operator. We suppose for each* $w\in\Omega$:

(1) $A(w)=\overline{W(w)}\cap K(w)$ *such that* $K(w)$ *is closed and convex and* $W(w)$ *is open and convex.*

(2) $F(w,\cdot)$ *is condensing.*

(3) $F(w,x)\subseteq K(w)\quad (x\in A(w))$ *and* $F(w,x)\subseteq\bar{W}\quad (x\in\partial W(w)\cap K(w))$.

Then, F *has a random fixed point.*

PROOF. Since W is convex, condition $F(w,x) \subseteq \bar{W}$ $(x \in \partial W(w) \cap K(w))$ implies the Leray-Schauder condition (4) from theorem 2.

If we assume in (3) that $F(w,x) \subseteq W$ $(x \in \partial W(w) \cap K(w))$, then it suffices to suppose that $W(w)$ is starshaped relative to some $u(w) \in W(w) \cap K(w)$ $(w \in \Omega)$.

COROLLARY 2. *Let* $A:\Omega \to Cl(E)$ *be separable and* $F:Gr\, A \to k(E)$ *a random usc operator. We suppose for each* $w \in \Omega$:

(1) $A(w)$ *is convex and* $F(w,x) \subseteq A(w)$ $(x \in \partial A(w))$.

(2) $F(w,\cdot)$ *is condensing.*

Then F *has a random fixed point.*

PROOF. Let $w \in \Omega$ be with $\operatorname{int} A(w) = \emptyset$. Then $\partial A(w) = A(w)$ and conditions (1) and (3) from Corollary 1 hold with $W(w) = E$. Let $w \in \Omega$ be with $\operatorname{int} A(w) \neq \emptyset$. Then, we choose in (1) and (3) of Corollary 1 $K(w) = E$. Therefore, we can apply Corollary 1.

Corollary 2 generalizes the Rothe-type results by Itoh [10] and Schleinkofer [16, Theorem 23] for condensing and Engl [4, Theorem 16] for compact mappings.

THEOREM 3. *Let* $A:\Omega \to Cl(E)$ *be separable and* $F:Gr\, A \to k(E)$ *a random usc operator. We suppose for each* $w \in \Omega$:

(1) $A(w) = \overline{U(w)} \cap K(w)$ *such that* $U(w)$ *is open,* $K(w)$ *is closed convex and* $U(w) \cap K(w)$ *is nonvoid and convex.*

(2) $F(w,\cdot)$ *is quasicompact with* $F(w,x) \subseteq K(w)$ $(x \in A(w))$.

(3) *For all* $x \in \partial U(w) \cap K(w)$, *we have* $F(w,x) \cap U(w) \neq \emptyset$ *and* $x \notin F(w,x)$.

Then, F *has a random fixed point.*

PROOF. We shall apply Theorem 1 with $X(w) = \partial U(w) \cap K(w)$ and Proposition 2. Let $w \in \Omega$.

$F(w,\cdot)$ is quasicompact. It is easy to show that each

$\tilde{F}(w,\cdot):A(w)\to k(E)$ with $\tilde{F}(w,x)\subseteq F(w,x)$ $(x\in A(w))$ and $\tilde{F}(w,\cdot)$ is usc must be quasicompact and we have $\tilde{F}(w,x)\subseteq K(w)$ $(x\in A(w))$. If $\tilde{F}(w,x)=F(w,x)$ for all $x\in X(w)=\partial U(w)\cap K(w)$ then $\tilde{F}(w,\cdot)$ satisfies the conditions of Proposition 2.
Therefore, $F(w,\cdot)$ has a fixed point in the strong sense on $A(w)\setminus X(w)$. By Theorem 1, F has a random fixed point.

COROLLARY 3. *Let* $A:\Omega\to Cl(E)$ *be measurable and* $F:Gr\ A\to k(E)$ *a random usc operator. We suppose for each* $w\in\Omega$:

(1) $A(w)=\overline{U(w)}$ *and* $U(w)$ *is open.*

(2) $F(w,\cdot)$ *is quasicompact with* $F(w,x)\subseteq U$ $(x\in\partial U)$.

Then F has a random fixed point.

Using Remark 1, we can see that such a Rothe-type-theorem holds for ultimately compact operators F with a nonvoid limit set $F_\infty\neq\emptyset$, too.
For quasicompact mappings, we obtain from Proposition 1 and Theorem 1 with $X(w)=\emptyset$ $(w\in\Omega)$:

THEOREM 4. *Let* $A:\Omega\to Cl(E)$ *separable and* $F:Gr\ A\to k(E)$ *a random usc operator. We suppose for each* $w\in\Omega$.

(1) $A(w)$ *is closed and convex.*

(2) $F(w,\cdot)$ *is quasicompact with* $F(w,x)\subseteq A(w)$ $(x\in A(w))$.

Then F has a random fixed point.

Theorem 4, for the special case when $F(w,\cdot)$ is ultimately compact with $F_\infty\neq\emptyset$, contains Theorem 19 in [16].
Our Theorem 1 implies easy special random fixed point theorems for such usc mappings, for which a degree theory is known.
For instance, we obtain Theorem 17 in [16]:

THEOREM 5. *Let* $A:\Omega\to Cl(E)$ *be measurable,* $F:Gr\ A\to k(E)$ *a random usc operator and* $A(w)=O(w)$ *with open* $O(w)$ $(w\in\Omega)$. *Suppose that for each* $w\in\Omega$:

(1) $F(w,\cdot)$

(2) $x\notin F(w,x)$ $(x\in\partial O(w))$

(3) $\deg\,(I-F(w,\cdot),\ O(w),o)\neq 0$ (s. [14]).

Then, F *has a random fixed point.*

PROOF. If $\tilde{F}(w,x) \subseteq F(w,x)$, $(x \in A(w))$ $\tilde{F}(w,\cdot)$ is usc and $F(w,\cdot)$ is ultimately compact, then $\tilde{F}(w,\cdot)$ is ultimately compact too. Suppose $\tilde{F}(w,x) = F(w,x)$ $(x \in \partial O(w))$, then deg $(I-\tilde{F}(w,\cdot), O(w), o) \neq 0$ and F has a fixed point (s. [14]) in $O(w)$. Therefore, $F(w,\cdot)$ has on $O(w) = A(w) \setminus \partial O(w)$ a fixed point in the strong sense.

Finally, we shall prove a random fixed point theorem for mappings, which are odd on subsets of the domain.

THEOREM 6. *Let* $A:\Omega \to Cl(E)$ *be separable and* $F:Gr\ A \to k(E)$ *a random usc operator.*
We suppose for each $w \in \Omega$.

(1) $A(w) = \overline{U(w)} \cap K(w)$ *such that* $U(w)$ *is an open symmetric neighbourhood of* $o \in E$ *and* K *is a closed, absolute convex subset of* E.

(2) $F(w,x) \subseteq K(w)$ $(x \in A(w))$, $w \notin F(w,x)$ $(x \in \partial U(w) \cap K(w))$.

(3) $F(w,x) = -F(w,-x)$ *for each* $x \in \partial U(w) \cap K(w)$.

(4) $F(w,\cdot)$ *is condensing or*

(4') $F(w,\cdot)$ *is* 1*-condensing and demicompact in* o *and* $F(w,\bar{U} \cap K)$ *is bounded.*

Then, F *has a random fixed point.*

PROOF. We shall apply Theorem 1 again and prove, that F has a fixed point in the strong sense on $A(w \setminus X(w)$ with $X(w) = \partial U(w) \cap K(w)$ $(w \in \Omega)$. Let $w \in \Omega$ fixed, but arbitrary. Now, we do not write this fixed argument w. Let $\tilde{F}:A \to k(E)$ be a usc mapping with $\tilde{F}(x) \subseteq F(x)$ $(x \in A)$ and $\tilde{F}(x) = F(x)$ $(x \in X)$. Then $\tilde{F}(A) \subseteq K$. We denote by ψ the measure of noncompactness. We obtain $\psi(\tilde{F}(N)) \leq \psi(F(N))$ $(N \subseteq A)$. If F is condensing, then it implies $\psi(N) \leq \psi(\tilde{F}(N)) \leq \psi(F(N))$ $(N \subseteq A)$ that $F(N)$ is relatively compact and therefore $\tilde{F}(N) \subseteq F(N)$, too. Hence, $\tilde{F}$ is condensing and by $\tilde{F}(x) = F(x)$ $(x \in \partial U \cap K)$ and condition (3), we obtain $\tilde{F}(-x) = -\tilde{F}(x)$ $(x \in \partial U \cap K)$. Then, by Proposition 3, $\tilde{F}$ has a fixed point in $U \cap K$. In the other case, if F is 1-condensing and demicompact in o, we must show, that $\tilde{F}$ is demicompact in o.

$\tilde{F}$ is clearly 1-condensing and $\tilde{F}(U\cap K)$ is bounded. Let (x_n), (z_n) be bounded sequences with $w_n\in A$, $z_n\in\tilde{F}(x_n)$ $(n\in N)$ and $x_n - z_n \to o$. Since $\tilde{F}(w_n) \subseteq F(x_n)$ $(n\in N))$ we have $z_n\in F(x_n)$ $(n\in N)$. Since F is demicompact in o, there exists a subsequence (x_{n_k}) of (x_n) with $x_{n_k} \to x'\in A$. This implies $z_{n_k} \to x'$. Because $z_{n_k}\in\tilde{F}(x_{n_k})$ and Gr $\tilde{F}$ is closed, we obtain in the fact $x'\in\tilde{F}(x')$. By Proposition 3 $\tilde{F}$ has a fixed point. Therefore, F has in either case a fixed point in the strong sense on $A\setminus X$, and our result follows from Theorem 1.

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REZIME

STOHASTIČKE TEOREME O NEPOKRETNOJ TAČKI ZA STOHASTIČKE ODGORE POLUNEPREKIDNE VIŠEZNAČNE OPERATORE

Dokazana je jedna opšta teorema o nepokretnoj tački za stohastičke poluneprekidne višeznačne operatore sa stohastičkim domenom u Frešeovim prostorima. Koristeći ovu teoremu dobijene su stohastičke teoreme o nepokretnoj tački za preslikavanja kondenzujućeg tipa. Ovi rezultati uopštavaju teoreme koje su dokazali Engl [4], Itoh [10] i Šlajnkofer [16].

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