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ON ELECTRICAL SELF-DUALITY OF "SMALL" SELF-DUAL MATROIDS

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ABSTRACT

All the non-isomorphic ESD (electrically self-dual, in the sense of [5])matroids on at most 8 elements are determined. The investigation is based on catalogue [4]. It turns out that only ten (out of 266) self-dual matroids on 8 elements are not ESD. We also give all the possible "electrical" permutations for ESD matroids on less than 8 elements. The paper includes a list of all the non-isomorphic self-dual matroids on at most 8 elements, together with the data concerning their "electricity". Some general methods for checking whether an SD matroid is ESD are also presented.

1. PRELIMINARIES

Sets are mostly denoted without brackets and commas. An n-set is a set of cardinality n. Non-defined notions can

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be found in [6].

A self-dual (SD) matroid on a (2n)-set S is ([5]) electrically self-dual (ESD) if there is a permutation p of S which satisfies

a) p (M) = M* (where M* denotes dual of M)

b) p consists of n pairwise disjoint transpositions.

We call the above permutation p - "electrical". An NESD-matroid is a SD matroid, which is not ESD. The word "cpodt" abbreviates "covering product of disjoint transpositions", i.e, a candidate (satisfying b)) for an "electrical" permutation.

A cyclic flat of a matroid is a flat which is also a union of circuits. It is known (cf.[1]) that the complements (w.r.t. the ground-set) of cyclic flats of M are cyclic flats of M*. An essential flat is a cyclic flat, which is different from the ground-set and which cannot be "predicted" (by using semimodularity, as a "minimal" consequence) from the family of cyclic flats of lower ranks. The essential flats, accompanied by their ranks, are sufficient to describe a matroid.

The ground-sets of all the exhibited matroids are subsets of the set W=abcdefgh.

2. DENOTATIONS IN THE LIST OF SELF-DUAL MATROIDS

Our main denotations are of the form "k=SD:E", where

- "SD" denotes a self-dual matroid M on 2n elements ($0 \leq n \leq 4$)
- "E" gives information on the electrical self-duality of M
- "k" denotes the ordinal number of M, within the class of all the non-isomorphic self-dual matroids with n fixed.

The letters "SD" stand for the family of essential flats of M, separated by commas and accompanied by their ranks (in brackets). This family is empty in the uniform case. The denotation of rank is omitted whenever the rank of an essential flat F belongs to the set $\{n-1, |F|-1\}$.

Given n=4, a natural number "j" in the beginning of a list of essential flats (after the sign "=") replaces the entire family of (ranked) essential flats corresponding to the SD matroid with the ordinal number j.

EXAMPLE. Denotations "40=43,dfgh"; "43=45,cegh"; "45=ab,abcd(2),abef(2)" - mean that the ranked essential flats of the self-dual matroid with the ordinal number 40 are ab(1), abcd(2), abef(2), cegh(3), dfgh(3). We point out that the SD matroids 40, 43 and 45 also have the non-essential cyclic flat abcdef(3).

Given again n=4, the denotation "E" stands ,except for the NESD cases, for an "electrical" permutation p of the ground-set abcdefgh. The elements in the same transposition of p are written together, while the transpositions are separated by short lines. We shall write down only three transpositions; they uniquely determine the fourth one. A natural number "i" in the place of "E" means that the permutation p coincides with the one corresponding to the SD matroid with the ordinal number i.

EXAMPLE. The SD matroids 54 and 36 have respectively the denotations "36" and "af-be-cd". It follows that the electrical permutation $p=\begin{pmatrix} a & b & c & d & e & f & g & h \\ f & e & d & c & b & a & h & g \end{pmatrix}$ corresponds to each of those two matroids.

Given $n \in \{0,1,2,3\}$, the denotations "E" are replaced by descriptions of the entire lists of the corresponding "electrical" permutations.

3. A LIST OF SELF-DUAL MATROIDS

0 elements:

The only matroid is trivially ESD.

2 elements:

1 = a: ab
2 = uniform: ab

4 elements:

1 = ab(0): ac-bd, ad-bc
2 = a,abc: ad-bc
3 = ab,cd: all 3 cpodt's
4 = ab,abcd: ac-bd, ad-bc
5 = uniform: all 3 cpodt's

6 elements:

1 = abc(0): those 6 cpodt's, which map abc to def
2 = ab(0), abcd(1): ae-bf-cd, af-be-cd
3 = a,abc(1),ade(1): af-bc-de, af-bd-ce, af-be-cd
4 = a,abc(1),abcde: af-bd-ce, af-be-cd
5 = a,abcde: like number 3
6 = ab,cd,ef: those 7 cpodt's, which include at least one of the transpositions (ab), (cd), (ef)
7 = def(1), abc: like number 1
8 = ab,cd,abef: like number 2
9 = abc(1): like number 1
10 = ef,abcd: ab-cd-ef, ac-bd-ef, ad-bc-ef
11 = ab,abcd,cef: like number 2
12 = ab,abcd: like number 2
13 = abc,ade,bef,cdf: those 7 cpodt's which include at least one of the transpositions (af), (bd), (ce)
14 = abc,ade,cef: ab-cd-ef, ad-be-cf, af-bc-de, af-be-cd
15 = abc,def: like number 1
16 = abc,ade: like number 3
17 = abc: like number 1
18 = uniform: all 15 cpodt's

8 elements:

1 = abcd(0): ah-bg-cf 2 = abc(0),abcde(1): 1 3 = ab(0), abcd(1),abef(1): 1 4 = 5,abcd(1): 1 5 = ab(0), abcdef(2):1 6 = a,abc(1),ade(1),afg(1): ah-bc-de 7 = a,abcd(1),aefg(2):1 8 = a,abc(1),ade(1),abcfg(2): 1 9 = a,abcd(1),abcdefg: 1 10 = a,afg(1),abcde(2): 6 11=a,abc(1),abcde(2),adfg(2): 1 12 = a,abc(1),abcde(2),abcdefg:1 13 = a,abcd(2),abef(2), acfg(2),adeg(2): 1 14 = a,abcd(2),abef(2),adfg(2): 1 15 = 17,aefg(2): 1 16 = 17,abef(2): 1 17 = 18,abcd(2): 1 18 = a,abcdefg: 1 19 = ab,cd,ef,gh: 1 20 = abc(1),de,fgh:1 21 = ab,cd,ef,abgh(2): ah-bg-cd 22 = abcd(1),efgh: 1 23 = abc(1),de(1),abcfgh: 1 24 = abcd(1): 1 25 = ab,cd, efgh(2): ab-cd-eh 26 = ab,cd,abef(2),cdgh(2): 1 27 = 28, egh,cdegh: 21 28 = ab,cd,abef(2),abefgh: 21 29 = 31,dfgh:1 30 = ab,cd,abef,abcdgh: 1 31 = abc(1),abcde(2): 1 32 = 33, efgh: 1 33 = ab,cd,abcdef,abcdgh: 1 34 = 36,cef: af-bd-ce 35 = ab,abgh(2),cdef(2): 1 36 = abc,ade,bdf,gh: af-be-cd 37 = ab,cde,cfg,abdh(2): ag-bf-ch 38 = gh,abc,def: 36 39 = fg,abc,ade,abcdeh: ah-be-cd 40 = 43,dfgh: 1 41 = ab, cef,abcd(2),abegh,dfgh: 1 42 = ab,cde,cfg,abcdeh: 37 43 = 45,cegh: 1 44 = de,abc,adef(2),bdegh: ac-bf-dh 45 = ab,abcd(2),abef(2): 1 46 = de,abc,abcfgh: 1 47 = 49, dfgh: 1 48 = 50,dfgh: 1 49 = 53,cegh: 1 50 = ab,cde, abcdef,abcgh: 1 51 = 53,efgh: 1 52 = ab,cde,abcdef,abfgh: NESD 53 = ab,abcd(2),abcdef: 1 54 = gh,abcdef: 36 55 = 56, efgh: 1 56 = 57,cdgh: 1 57 = ab,abcdef: 1 58 = abcd(2), efgh(2): 1 59 = abc,cde,efg,agh: 1 60 = 59,bdfh: 1 61 = abcd(2),efg,aefgh: 1 62 = abc,ade,fgh,bdfgh,cefgh: NESD 63 = abc,ade,afg: 1 64 = 63,bdfh: 1 65 = 64,begh: 6 66 = 64,cegh: 1 67 = 65,cdgh: 6 68 = 67,cefh: 6 69 = abc, adg,def,abceh: ah-bc-df 70 = 69,bfgh: 69 71 = abcd(2): 1 72 = 71,aefg: 1 73 = 71,efgh: 1 74 = 72,befh: 1 75 = 74, cegh: 1 76 = 75,dfgh: 1 77 = abc,def,abcgh,defgh: ad-be-cf 78 = abc,def,abcgh,adefg: NESD 79 = abc,def,abcdg,adefh: ag-bc-dh 80 = 79,begh: 79 81 = 80,cfgh: 79 82 = abc,ade, abcfg: 1 83 = 82,bdfh: 1 84 = 82,dfgh: 1 85 = 83,cegh: 1 86 = 83,cdgh: 1 87 = 84,begh: ah-bf-cg 88 = 87,cefh:87

89 = 86,begh: 87 90 = 89,cefh: 87 91 = abc,abcde: 1
92 = 91,dfgh: 1 93 = 100,bdfh: 1 94 = 100,befh: 1
95 = 100,efgh: 1 96 = 93,aefh: 102 97 = 93,cdgh: 1
98 = 93,cegh: 1 99 = 93,efgh: 1 100 = 91,adfg: 1 101 = 96, befg: 1 102 = 98,aefh: ag-bh-cf 103 = 100,aefh,begh,cdgh: af-bg-ch 104 = 96,cdgh: 102 105 = 99,cdgh: 1 106 = 104, befg: 1 107 = 94,aegh,bdgh,cdfh: 37 108 = 104,begh cefg: 103 109 = 91,afgh,defg: 1 110 = 91,adfh,bfgh,defg: 102
111 = 91,adfh,begh,cfgh,defg: 37 112 = 91,adfh, bdgh,cfgh, defg: 37 113 = 112,aegh: 103 114 = 113,befh: 37 115 = abc, adefg: 69 116 = 115,bdeh: 69 117 = 116,bfgh: 69 118 = 116, cfgh: 69 119 = 116,cdfh: ah-bc-dg 120 = 117,cdfh: 119
121 = 120,cegh: 119 122 = abc,defgh: NESD 123 = uniform: 1
124 = abcd: 1 125 = 124,abef: 1 126 = 124,efgh: 1 127 = 125, aceg: 1 128 = 125,cdgh: 1 129 = 127,adfg: 1 130 = 125, abgh,cdef: 1 131 = 127,bceh: 1 132 = 127,bdfh: 1 133 = 128, efgh: 1 134 = 129,bceh: 1 135 = 127,abgh,bceh: ad-bf-cg
136 = 130,aceg: 1 137 = 130,cdgh: 1 138 = 132,cdgh: 1
139 = 135,acfh: 135 140 = 127,abgh,acfh,efgh: ad-bc-eg
141 = 134,bdfh: 1 142 = 136,bceh: 1 143 = 136,bdfh: 1
144 = 137,aceg: 1 145 = 137,efgh: 1 146 = 138,efgh: 1
147 = 135,acfh,bdeg: 135 148 = 136,acfh,bdeg: 1 149 = 140, bdeg: ad-bc-ef 150 = 141,cdgh: 1 151 = 144,bdfh: 1
152 = 144,efgh: 1
153 = adeh,adfg,bcfg,bdeg,bdfh,cdef,cdgh,efgh: 135
154 = adeh,adfg,bceh,bcfg,bdeg,bdfh,cdef,cdgh: 149
155 = abgh,acfh,adeh,bcfg,bdeg,cdef, cdgh,efgh: 1
156 = acfh,adeh,adfg,bcfg,bdeg,bdfh,cdgh,efgh: 1
157 = acfh,adeh,adfg,bceh,bcfg,bdeg,cdgh,efgh: 1
158 = acfh,adeh,adfg,bceh,bcfg,bdeg,bdfh,efgh: 1
159 = aceg,acfh,adeh,adfg,bceh,bcfg,bdeg,bdfh: 1
160 = abgh,acfh,adeh,adfg,bceh,bcfg,bdeg,cdef: 1
161 = 155,bdfh: 1 162 = 156,cdef: 135 163 = 157,bdfh: 1
164 = 159,efgh: 1 165 = 160,efgh: 1 166 = 161,bceh: 1
167 = 164,cdgh: 1 168 = 161,adfg: 1 169 = 165,cdgh: 1
170 = 159,abgh,cdef: 1 171 = 169,bdfh: 1 172 = 170,efgh: 1
173 = 172,cdgh: 1 174 = 170,abef,cdgh: 1 175 = 173,abef: 1
176 = 175,abcd: 1 177 = abcd,aefg: 1 178 = 177,befh: 1

179 = 177,abeh,bcfg: ad-bh-ce 180 = 178,cdgh: 1 181 = 180, aceh,bdeg: NESD 182 = 181,bcfg: ab-cf-de 183 = 182,adfh: 182 184 = 178,cegh: 1 185 = 184,dfgh: 1 186 = 184,adfh: 102 187 = 186,bdfg: 102 188 = 186,bdgh: 103 189 = 188, cdfg: 103 190 = 186,bcfg,bdeg: 102 191 = 188,bcfg,cdef: 103 192 = 178,acgh: NESD 193 = 192,bdeg: 34 194 = 192, adeh,bceg: NESD 195 = 192,bdgh,cdef: ab-ch-dg 196 = 195, adeh,bceg: 195 197 = 193,cdfh: ab-ce-dg 198 = 193,adfh: NESD 199 = 198,bcfg: ae-bc-dg 200 = 199,cdeh: 199
201 = 178,bcfg,aceh,adfh: ag-bh-ce 202 = 179,acfh,bdef: 179 203 = 177,abgh,cdef: 1 204 = abcd,befh,abgh,adeh: 103
205 = abcd,cegh,abgh,acfh,adeh,cdef: 102 206 = 177,abgh, adeh,bcfg,cdef: 1 207 = 206,acfh,bdeg: 102 208 = 177,abgh, cegh: af-be-ch 209 = 203,befh: ah-bg-ce 210 = 203,cegh: 208 211 = 208,adeh: 103 212 = 204,dfgh: 103 213 = 211, bcfg: 208 214 = 204,bcfg,cegh: ae-bf-cg 215 = 209,acfh, adeh: 201 216 = 205,dfgh: 201 217 = 209,adeh,bcfg: 201
218 = 213,cdef: 208 219 = 217,acfh: 201 220 = 219,bdeg: 201 221 = 179,acfh: 179 222 = 221, bdeg: 179 223 = 208, bdfh: 208 224 = 209,cdgh: 209 225 = 210,bdfh: 208 226 = 211,bdfh: 103 227 = 212,aceg: 103 228 = 213,bdfh: 208 229 = 214,adfg: 214
230 = 215,cdgh: 201 231 = 216,abef: 201 232 = 217,cdgh: 201 233 = 218,bdfh: 208 234 = 219,cdgh: 201 235 = 234, bdeg: 201 236 = 181,abgh,cdef: NESD 237 = 182,abgh,cdef: 182 238 = 182,adfh, abgh: 44 239 = 183,abgh,cdef: 182
240 = abcd,dfgh, befh,cegh,abgh,cdef: 1 241 = 211,dfgh: 103 242 = 212,aefg,acfh,cdef: 102 243 = 240,bcfg,adeh: 1
244 = 243,acfh,bdeg: 1 245 = 185,abgh: 214 246 = 245, cdef: 214 247 = 245,adeh: 103 248 = 247,bcfg: 214 249 = 247,acfh,cdef: 102 250 = 248,cdef: 214 251 = 249,bcfg: 214 252 = 251,bdeg: 214 253 = 186,abgh,cdef: 102 254 = 253,bdfg: 102 255 = 186,bcfg,bdeg,abgh: 199 256 = 255, cdef: 199 257 = 192,adeh,bcfg: NESD 258 = 193,adeh: 34
259 = 193,bcfg: 34 260 = 258,bcfg: 34 261 = 195,adeh: 195
262 = 261,bcfg: 195 263 = 197,adeh: 197 264 = 263,bcfg: 197 265 = 201,abgh: ad-be-ch 266 = 265,cdef: 201

4. ON CHECKING THE ELECTRICAL SELF-DUALITY OF SELF-DUAL MATROIDS

In this section we shall give a number of methods which were used for checking whether an SD matroid on ≤ 8 elements is ESD. The majority of these methods can also be applied to SD matroids on larger sets.

a) Loops - coloops

Any isomorphism between the matroids M and M* (i.e, a permutation p of the common ground-set, which satisfies p(M)=M*; its existence means that M is SD) maps the loops (coloops) of M to the loops (coloops) of M*, which are at the same time coloops (loops) of M (thus the numbers of loops and coloops in an SD matroid are the same). It follows that

LEMMA 1. Any transposition of an "electrical" permutation (corresponding to an ESD matroid) - either includes a loop and a coloop or none of them.

CONSEQUENCE. There is a natural bijection between all the non-isomorphic ESD matroids on a (2n)-set and all the non-isomorphic ESD matroids with loops on a (2n+2)-set. It is established by the addition of one loop and the corresponding coloop to each matroid of the first class.

EXAMPLE. Since all the SD matroids on 6 elements are ESD, we have that all the SD matroids with loops on 8 elements are ESD.

b) Cyclic chains

A cyclic (=essential,[2]) chain is a matroid M which satisfies that each two cyclic flats of M are comparable.

LEMMA 2. All the SD cyclic chains are ESD.

PROOF. Let $C_o, C_1, \ldots, C_k$ ($C_o \subset C_1 \subset \ldots \subset C_k$) be all the cyclic flats of an SD cyclic chain M on S. We define a corresponding "electrical" permutation p of S as follows:

1) $p(p(x)) = x$ for each $x \in S$
2) If $x \in C_o$, then $p(x) \in S \setminus C_k$
3) If $x \in C_{i+1} \setminus C_i$, where $0 \le i \le \frac{k-2}{2}$, then $p(x) \in (S \setminus C_{k-i}) \setminus (S \setminus C_{k-i-1})$
4) If k is odd and $x \in D = C_{\frac{k+1}{2}} \setminus C_{\frac{k-1}{2}}$, then $p(x) \in D \setminus \{x\}$.

It is routine to check that p maps the sets $C_o, C_1, \ldots, C_k$ to $S \setminus C_k, S \setminus C_{k-1}, \ldots, S \setminus C_o$ respectively. Furthermore, the permutation p has no fixed points (x and p(x) are placed in disjoint sets, except for case 4), where $x \neq p(x)$ is explicitly stated). Observe that the set D must be of an even cardinality, otherwise M would not be SD. Thus p is a cpodt.

CONSEQUENCE. A lower bound for the number of ESD matroids on 2k elements is 2^k (this is the number of non-isomorphic SD cyclic chains ([2])). This bound seems to be very crude (e.g, for n=4 we have that 16<<256).

c) "Standard" permutation

We point out that more than half (exactly 129) of the ESD matroids on 8 elements in our list are represented with the help of the same "electrical" permutation s.p.=ah-bg-cf-de (let us call the permutation s.p. "standard"). We used s.p. whenever possible.

EXAMPLE. The families of cyclic flats, which correspond to SD matroids (on 8 elements) numerated from 123 to 176 inclusive - are subfamilies of

$$F = \left\{ \begin{matrix} abcd, abef, abgh, aceg, acfh, adeh, adfg \\ efgh, cdgh, cdef, bdfh, bdeg, bcfg, bceh \end{matrix} \right\}$$

The "standard" permutation s.p. maps the sets within four complementary pairs of F to each other, while it fixes each of the sets in the complementary pairs P_1={abgh, cdef}, P_2={acfh,bdeg} and P_3={adeh,bcfg}. Since an "electrical" permutation should map a family of cyclic flats to the family of their complements, it immediately follows that s.p. is an "electrical" permutation for 46 of the considered 54 matroids. These 46 matroids satisfy the condition that their families of cyclic flats include either none of both of the sets in the family P_i, for each $i \in \{1,2,3\}$.

d) Suitable matroid representations

When dealing with SD matroids on 8 elements, numerated from 177 to 266 inclusive, we represented them by means of marked D-graphs ([3]):

The vertices of a D-graph associated to an SD matroid M - are the cyclic flats of M, all of cardinality 4. Two vertices X and Y of a D-graph are adjacent iff $|X \cap Y|=1$. If $X \cap Y=\{a\}$, then it is natural to denote the edge $\{X,Y\}$ by "a". A D-graph becomes marked when we introduce the following binary relation r on the set of its edges:

For two edges $x=X_1 \cap X_2$ and $y=Y_1 \cap Y_2$ $r(x,y) \Leftrightarrow x = W \setminus (Y_1 \cup Y_2)$.

The lattice of cyclic flats turned out to be a considerably suitable matroid representation in non-paving cases .

e) Compulsory transpositions

An important shortcut for checking electrical self--duality in many cases is to primarily detect "compulsory" transpositions, which are due to special positions of some elements in the ground-sets of M and M*. Suitable matroid representations help in recognizing them. These transpositions considerably reduce the number of possibilities which should be checked.

EXAMPLE 1. Let M denote the 37^{th} SD matroid on 8 elements. The cyclic flats of M are:

ab,cde,cfg,abdh(2),abcdeh,abcfg,cdefg.

Their complements are the cyclic flats of M*:

fg,deh,abh,cefg(2),cdefgh,abfgh,abdeh.

It is obvious that any permutation mapping M onto M* must map c=cde∩cfg onto h=deh∩abh and ab onto fg. It follows that -ch- and -de- are compulsory transpositions in each "electrical" permutation. We can easily conclude and verify that the only two "electrical" permutations are:

af-bg-ch-de and ag-bf-ch-de.

EXAMPLE 2. Consider the cyclic flats of M and M*, where M denotes the 185^{th} SD matroid on 8 elements:

M: aefg,befh,abcd,cegh,adfh

M*: bcdh,acdg,efgh,abdf,aceg.

Let us denote by 1,2,3,4,5, in this order, the corresponding vertices of the associated D-graphs. It is easy to check that the edges of these graphs are 13,23,34 and 45 in both cases. The 1-element intersections, which correspond to these edges are:

a,b,c,h respectively for M and

h,g,f,b respectively for M*.

Since each of the edges 34 and 45 has a special position in the D-graphs, we immediately have that -cf- and -bh- are compulsory transpositions with any "electrical" permutation. The edges 13 and 23 are "in isomorphic positions" (more precisely, in the same orbit of the automorphism group of the edge-set). Thus it is possible to map edge 13 with M onto edge 23 with M* and conversely. It is now easy to derive that ag-bh-cf-de is the only "electrical" permutation.

f) NESD-matroids

We shall give explicit proofs for the non-existence of an "electrical" permutation for each NESD matroid M in our list. In most cases, the idea was to find a set X of odd cardinality, such that X is invariant under any permutation p, which maps M onto M*. It is then obvious that such a permutation cannot be a cpodt.

The proofs follow the corresponding ordinal numbers of NESD matroids in our list of SD matroids on 8 elements.

52: X = cde. This is the only cyclic 3-flat both in M an M*.

62: X = a (or X = fgh). The cyclic 3-flats are abc,ade,fgh with M and abd,ace,fgh with M*. The element a is, in both cases, the only common element for some two of these flats.

78: X = g. This element belongs (both with M and with M*) to the intersection of two cyclic 5-flats, but not to a cyclic 3-flat.

122: X = abc. The proof is obvious.

181: The D-graphs associated with M, respectively M*, contain a quadrangle with the edges a,b,h,g. In addition, the first of these D-graphs has the isolated edge e, while the second has the isolated edge f. It follows that -ef- and -cd- are the compulsory transpositions. Thus there are three candidates for an "electrical" permutation: p_1, p_2 and p_3, which include the transpositions ag-bh, ah-bg and ab-gh, respectively.

Consider the cyclic flat abcd of M. We have that p_1(abcd)=p_2(abcd)=cdgh and p_3(abcd)=abcd, but none of the sets cdgh and abcd is a cyclic flat of M* □

192: X = h. This is the common first element of the only pair of edges related by r, both in M and in M*. Namely, the marked D-graph, associated with M, respectively M*, is a 3-path which has

- the edges a,b,h with r(h,a), respectively the edges d,g,h with r(h,d).

194: The marked D-graph differs from the 3-path associated with the 192^{th} SD matroid solely in that one isolated edge (e) is added. Such an alteration does not influence the previous proof.

198: X = g. The marked D-graphs associated with M and M* are 5-paths, which (in order) have the edges a,b,h,g,d, respectively c,f,d,g,h. The ordered pairs of edges, related by r, are (d,h), (h,a), (g,b) in the first case and (h,d), (d,c), (g,f) in the second one. The element g is in both cases the first element of the pair, which is disjoint with the other two pairs.

236: The marked D-graph is the same one as with the 181^{th} SD matroid.

257: The marked D-graph is the same one as with the 192^{th} SD matroid.

g) Augmentations

Let two families F_1 and F_2 of cyclic flats be given, which correspond to the matroids M_1 and M_2, respectively. In addition, let us suppose that F_1 is a proper subfamily of F_2 and that M_1 is ESD with an "electrical" permutation p. In many cases, it is possible to show that p maps the family $F_2 \setminus F_1$ onto the family of complementary sets. This implies that p is an "electrical" permutation for M_2: that is, M_2 is also ESD.

EXAMPLE. The 28^{th} SD matroid M_1 on 8 elements is ESD with p=ah-bg-cd-ef. The 27^{th} SD matroid M_2 is obtained from M_1 by the addition of cyclic flats egh and cdegh. Since p({egh,cdegh})={abf,abcdf}={W∖egh,W∖cdegh}, it follows that M_2 is also ESD with p.

h) A shortcut for testing cpodts

Suppose that a paving SD matroid M on 8 elements (the ordinal numbers 123 through 266 in our list) is represented by means of its cyclic 4-flats F and let a cpodt p be given. Let "k(F)" denote the number of transpositions of p, both the elements of which are included in F. We shall suggest the following quick test to determine whether p is "electrical" for M:

If k(F) = 0, then ignore F.
If k(F) = 1, then check whether M contains the cyclic flat W∖p(F).
If k(F) = 2, then check whether M contains the cyclic flat W∖F.

(Note also that K(F)=2 implies p(F)=F.)

The negative answer in any of the last two cases means that p is not "electrical". The affirmative answer means that the flat W∖p(F), respectively W∖F, should be removed from further consideration.

JUSTIFICATION. We should check that p maps the family of cyclic flats onto the family of their complements. If k(F)=0, then the mapping is direct: p(F)=W∖F. If k(F)∈{1,2}, then the cyclic flats F and W∖p(F) may transpose their roles, since

$$W\setminus p(W\setminus p(F)) = W\setminus (W\setminus p(p(F))) = F$$

EXAMPLE. The cyclic 4-flats of the 232th SD matroid M on 8 elements are

abcd,cdgh,adeh,bcfg,aefg,befh,abgh,cdef.

Given p=ag-bh-ce-df, the values of k(F) with the cited eight flats of M are 0,0,0,0,1,1,2,2 respectively. In order to show that p is "electrical", it suffices to check that W∖p(aefg)=W∖acdg=befh is a cyclic 4-flat and to observe that the cyclic 4-flats abgh and cdef are mutually complementary.

REMARK. A general test to show that a cpodt p is "electrical" can be organised as follows:

For each cyclic flat F of a matroid M and for the corresponding cyclic flat p(F) of the dual matroid M* (were $M^*\simeq M$), and also for each transposition -xy- of the permutation p, one of the following four combinations of the conditions (of 16 possibilities in all) should hold:

1) $x\in F$, $y\in F$, $x\in p(F)$, $y\in p(F)$
2) $x\in F$, $y\notin F$, $x\notin p(F)$, $y\in p(F)$
3) $x\notin F$, $y\in F$, $x\in p(F)$, $y\notin p(F)$
4) $x\notin F$, $y\notin F$, $x\notin p(F)$, $y\notin p(F)$

(the elements x and y are placed either in none, or in both, or opposite to the flats F and p(F)).

This is easily derived by the use of

$y = p(x)$, $x = p(y)$ and $p(p(F)) = F$.

i) The number of "electrical" permutations

Except for the ESD matroids on ≤ 6 elements, we have not counted the "electrical" permutations; it was enough to find one (when it exists) for our purposes. However, their enumeration is a question of a particular interest. We shall just make a couple of general remarks:

There are $(2n)!/(2^n n!)$ cpodts on a (2n)-set. We denote this number by "f(n)"; the first four values of f are 1,3,15,105. The number f(n) is the smallest upper bound for the number of "electrical" permutations, which is effectively reached with uniform SD matroids (we conjecture them to be the only examples of this type for $n\geq 3$).

Compulsory submaps play a substantial role in the enumeration.They generalise the idea of compulsory transpositions.

EXAMPLE. Suppose that a k-set X has the property that its image Y=p(X) is uniquely determined, under any permutation p such that p(M)=M*. If $Y\cap X=\emptyset$, then the number of "electrical candidates" reduces to k! f(2n-2k). In the opposite situation, when Y=X (then k must be even, otherwise M M is NESD), the upper bound is f(k)f(2n-k).

REFERENCES

[1] Acketa,D.M., *On the essential flats of geometric lattices*, *Publ.del' Inst.Math. 26(40), 1979, 11-17.*

[2] Acketa,D.M., *On the essential chains and squares, Col. Math.Soc. Janos Bolyai, 37.Finite and Infinite Sets, 1981, 25-33.*

[3] Acketa,D.M., *Another construction of rank 4 paving matroids on 8 elements (II), Rev.of Res., Fac.of Sci., Novi Sad, 12(1981), 277-303.*

[4] Acketa,D.M., *The catalogue of all non-isomorphic matroids on at most 8 elements, Inst.of Math., Novi Sad, spec.iss.1., xviii+157, 1983.*

[5] Recski, A. , *On self-dual matroids with applications, Proc . of 6th Hung.Coll. on Combinatorics, Col.Math. Soc.Janos Bolyai, 37. Finite and Infinite Sets, 1981.*

[6] Welsh,D.J.A., *Matroid Theory, London Math.Soc.Monographs, No.8, Academic Press,1976.*

REZIME

O ELEKTRIČNOJ SAMODUALNOSTI "MALIH" SAMODUALNIH MATROIDA

U ovom radu su, uz pomoć kataloga [4], odredjeni svi neizomorfni električno samodualni matroidi (u smislu rada [5]) na nosačima od najviše 8 elemenata. Medju 266 samodualnih matroida na 8 elemenata samo 10 nisu električno samodualni. Dato je i više metoda za proveru električne samodualnosti samodualnih matroida; većina tih matroida može se primeniti i na većim nosačima.

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