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THE SPACE OF THE GENERALIZED DIRICHLET SERIES OF SEVERAL COMPLEX VARIABLES ON A POLYDISC

Endre Pap

*Institute of Mathematics University of Novi Sad,
Dr Ilije Djuričića 4, 21000 Novi Sad, Yugoslavia*

ABSTRACT

In this paper a version of the results, from the previous author's paper [8] for the space $X_D(f_1 f_2)$ of the functions representable by the absolute and uniform convergent generalized Dirichlet series on a polydisc $D=D(R_1,R_2)$, is obtained. A representation of the continuous linear functional on $X_D(f_1,f_2)$ is constructed.

1. INTRODUCTION

We have investigated in paper [8] the space

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$X(f_1,f_2)$ of the absolute convergent generalized series (in the whole C^2)

$$(1.1) \qquad \sum_{m,n=0}^{\infty} a_{m,n} f_1(\lambda_m s_1) f_2(\mu_n s_2)$$

where f_1 and f_2 were entire functions of the given corresponding orders ρ_1 and types σ_1 and σ_2, $\lambda_0=\mu_0=0$. $(\lambda_m)_{m\geq 1}$, $(\mu_n)_{n\geq 1}$ are two sequences of complex numbers such that

$$0<|\lambda_1|<|\lambda_2|< \dots \to \infty$$

and

$$0<|\mu_1|<|\mu_2|< \dots \to \infty$$

and further

$$(1.2) \qquad \lim_{m+n\to\infty} \sup \frac{\ln(m+n)}{|\lambda_m|^{\rho_1}+|\mu_n|^{\rho_2}} = D<\infty.$$

In this way we have generalized some results by S.Daoud [1]-[3].

In this paper we shall investigate the generalized Dirichlet series (1.1) on a polydisc $D(R_1,R_2)=\{s_1:|s_1|<R_1\}\times\{s_2:|s_2|<R_2\}$ with a more restrictive condition on the sequences (λ_m) and (μ_n) than (1.2). Nemely, we shall suppose that

$$\lim_{m+n\to\infty} \frac{\ln(m+n)}{|\lambda_m|^{\rho_1} + |\mu_n|^{\rho_2}} = 0 \quad \text{holds.}$$

In this paper, for simplicity, we consider series and functions of only two variables, but all the results can easily be extended to any finite number of variables.
We shall construct a representation of the continuous linear functional on the space $X_D(f_1,f_2)$ of functions which have a representation with the generalized Dirichlet series.

2. Generalized Dirichlet series on a polydisc

The entire function

$$f(z) = \sum_{k=0}^{\infty} a_k z^k$$

is of order ρ, $0<\rho<\infty$ and type $\sigma\neq 0,\infty$ if

$$\sup_{z\in\mathbb{C}} |f(z)|\exp(-(\sigma+\varepsilon)|z|^{\rho})<+\infty$$

for all ε, $0<\varepsilon<1$.

Let $E_{\rho,\sigma}$ denote the set of all entire functions of order ρ and type σ such that $a_k\neq 0\,(k=0,1,2,\ldots)$.

Let us consider the double generalized Dirichlet series

$$(2.1)\qquad F(s_1,s_2) = \sum_{m,n=0}^{\infty} a_{m,n} f_1(\lambda_m s_1) f_2(\mu_n s_2)$$

of complex variables s_1 and s_2, where $f_i\in E_{\rho_i,\sigma_i}\,(i=1,2)$. coefficients $a_{m,n}$ are complex numbers, $\lambda_0=\mu_0=0$, $(\lambda_m)_{m\geq 1}$, $(\mu_n)_{n\geq 1}$ are two sequences of complex numbers such that

$$0<|\lambda_1|<|\lambda_2|< \ldots \to \infty$$

and

$$0<|\mu_1|\leq|\mu_2|\leq \dots \to \infty$$

and further

(2.2) $$\lim_{m+n\to\infty} \frac{\ln(m+n)}{\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2}} = 0 ,$$

where R_1 and R_2 are positive real numbers.

Theorem 2.1. Let

(2.3) $$\lim_{m+n\to\infty} \sup \frac{\ln|a_{m,n}|}{\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2}} \leq -1.$$

Then, the series (2.1) converges absolutely and uniformly on the polydisc

$$D(R_1,R_2)=\{(s_1,s_2):|s_1|<R_1,|s_2|<R_2\}.$$

Proof. We take the real numbers r_o', r_1, r_o'' and r_2 such that $r_o' < r_1 < R_1$ and $r_o'' < r_2 < R_2$. By (2.3) there exist $N\in\mathbb{N}$, such that

$$\frac{\ln|a_{m,n}|}{\sigma_1 r_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 r_2^{\rho_2}|\mu_n|^{\rho_2}} < -1$$

for $m+n\geq N$. Hence,

(2.4) $$|a_{m,n}|<\exp\{-\sigma_1 r_1^{\rho_1}|\lambda_m|^{\rho_1}-\sigma_2 r_2^{\rho_2}|\mu_n|^{\rho_2}\}$$

for $m+n\geq N$.

Since $f_i \in E_{\rho_i,\sigma_i}$, $(i=1,2)$, there exist natural numbers $N_1=N_1(\varepsilon)$ and $N_2=N_2(\varepsilon)$ for arbitrary ε, $0<\varepsilon<1$, such that

$$|f_1(\lambda_m s_1)|<k_1\exp\{|\lambda_m|^{\rho_1} r_o'^{\rho_1}(\sigma_1+\varepsilon)\}, \quad (|s_1|<r_o', m\geq N_1)$$

(2.5)
$$|f_2(\mu_n s_2)|<k_2 \exp\{|\mu_n|^{\rho_2} r_o''^{\rho_2}(\sigma_2+\varepsilon), \quad (|s_2|<r_o'', n\geq N_2)$$

for some

$$k_1>0, k_2>0.$$

Then, we obtain by (2.4) and (2.5)

$$(2.6) \quad |a_{m,n}||f_1(\lambda_m s_1)||f_2(\mu_n s_2)|<k_1k_2 \exp\{-|\lambda_m|^{\rho_1}\cdot$$

$$\cdot[\sigma_1 r_1^{\rho_1}-(\sigma_1+\varepsilon)r_o'^{\rho_1}]-|\mu_n|^{\rho_2}[\sigma_2 r_2^{\rho_2}-(\sigma_2+\varepsilon)r_o''^{\rho_2}]\}$$

for $m+n\geq N_o=\max\{N,N_1+N_2\}$.

There exists such a positive number ε that the number u defined by

$$u := \min\{\frac{\sigma_1 r_1^{\rho_1}-(\sigma_1+\varepsilon)r_o'^{\rho_1}}{\sigma_1 R_1^{\rho_1}}, \frac{\sigma_2 r_2^{\rho_2}-(\sigma_2+\varepsilon)r_o''^{\rho_2}}{\sigma_2 R_2^{\rho_2}}\}$$

is positive. Now, we can choose a positive number t such that $t < u$. By (2.2) there exists $N'\in N$, such that

$$t(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2})>\ln(m+n)$$

for $m+n\geq N'$. Hence, by (2.6)

$$|a_{m,n}||f_1(\lambda_m s_1)||f_2(\mu_n s_2)|<k_1k_2 \exp\{-u(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2})\}<$$

$$<k_1k_2 \exp\{-\frac{u}{t}\ln(m+n)\}=\frac{k_1k_2}{(m+n)^{u/t}}$$

for $m+n>\max\{N_o,N'\}$ and $|s_1|<r_o'$, $|s_2|<r_o''$.

For special entire functions f_i, we have the following

<u>Theorem 2.2.</u> Let

$f_i(z)=E_{\rho_i}(\sigma_i^{1/\rho_i}z)$, $0<\rho<\infty, \sigma\neq 0,\infty, i=1,2$, where E_ρ is the Mittag-Leffler function, i.e.

$$E_\rho(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\frac{n}{\rho}+1)} .$$

The series in (2.1) converges absolutely and uniformly on $D(R_1,R_2)$, iff (2.3) holds.

Proof. By Theorem 2.1. we have to prove only that if the series in (2.1) converges, then (2.3) holds. We suppose the contrary, i.e. that for some series

$$\sum_{m,n=0}^{\infty} a_{m,n}E_{\rho_1}(\sigma_1^{1/\rho_1}s_1)E_{\rho_2}(\sigma_2^{1/\rho_2}s_2)$$

the relation (2.3) does not hold, although the series is an absolute convergent for arbitrary $(s_1,s_2)\in D(R_1,R_2)$.
Let $r_i (i=1,2)$ be such numbers that $r_i<R_i$ holds. Since (2.3) does not hold for r_i' , $0<r_i'<r_i$, there exist two subsequences $\{m_k\}$ and $\{n_k\}$, such that

$$\frac{\ln|a_{m_k n_k}|}{\sigma_1 r_1^{\rho_1}|\lambda_{m_k}|^{\rho_1}+\sigma_2 r_2^{\rho_2}|\lambda_{n_k}|^{\rho_2}} > -1 \quad (k\in\mathbb{N}).$$

Hence,

$$(2.7)\quad |a_{m_k,n_k}|<\exp\{-\sigma_1 r_1^{\rho_1}-\sigma_2 r_2^{\rho_2}|\mu_{n_k}|^{\rho_2}\} .$$

We shall need the following asymptotic behaviour of the function E_ρ (see p. 134 in [4])

$$(2.8)\quad E_{\rho_i}(\sigma_i^{1/\rho_i}s_i)=\rho_i e^{\sigma_i s_i^{\rho_i}}+O(1/s_i),\ (i=1,2)$$

in a small angle $|\arg s_i|<\psi_i$. There exist subsequences $\{s_k\}$ of $\{m_k\}$ and $\{u_k\}$ of $\{n_k\}$, such that $\arg \lambda_{s_k}\to\phi_1$ and $\arg \mu_{u_k}\to\phi_2$ as $k\to\infty$. By (2.8), for $s_i^0=r_i e^{-i\phi_i}$ $(i=1,2)$ for some small $\varepsilon>0$ and $k>k_0$

$$(2.9)\quad \begin{aligned} &|E_{\rho_1}(\sigma_1^{1/\rho_1}\lambda_{s_k}s_2^0)|>\exp\{(\sigma_1-\varepsilon)r_1^{\rho_1}|\lambda_{s_k}|^{\rho_1}\}\\ &|E_{\rho_2}(\sigma_2^{1/\rho_2}\mu_{u_k}s_2^0)|>\exp\{(\sigma_2-\varepsilon)r_2^{\rho_2}|\mu_{u_k}|^{\rho_2}\}.\end{aligned}$$

So for $k>k_0$ (2.7) and (2.9) imply

$$|a_{s_k,u_k}||E_{\rho_1}(\sigma_1^{1/\rho_1}\lambda_{s_k}s_1^0)||E_{\rho_2}(\sigma_2^{1/\rho_2}\mu_{u_k}s_2^0)|$$
$$>\exp\{|\lambda_{s_k}|^{\rho_1}[\sigma_1 r_1^{\rho_1}-(\sigma_1+\varepsilon)r_1'^{\rho_1}] -$$

$$-|\mu_n|^{\rho_2}[\sigma_2 r_2^{\rho_2}-(\sigma_2+\varepsilon)r_2''^{\rho_2}]\} .$$

We can choose a positive real number ε small enough so that the numbers

$$\sigma_1 r_1^{\rho_1}-(\sigma_1+\varepsilon)r_1'^{\rho_1} \quad \text{and} \quad \sigma_2 r_2^{\rho_2}-(\sigma_2+\varepsilon)r_2''^{\rho_2} \quad \text{are}$$

positive.

Hence,

$$|a_{s_k,u_k}||E_{\rho_1}(\sigma_1^{1/\rho_1}\lambda_{s_k}s_1^0)||E_{\rho_2}(\sigma_2^{1/\rho_2}\mu_{u_k}s_2^0)|>1$$

and the series in (2.1) does not converge for the point $(s_1^o,s_2^o)\in D(R_1,R_2)$. This contradiction implies that condition (2.3) must hold.

Using the idea of the preceding proof, we can prove the following theorem.

<u>Theorem 2.3.</u> Let

$$f_i(z)=E_{\rho_i}(\sigma_i^{1/\rho_i}z),\ 0<\rho<\infty,\ 0<\rho<\infty,\quad \sigma\neq 0,\infty, i=1,2.$$

If condition

$$(2.10)\quad \lim_{m+n\to\infty}\sup \frac{\ln|a_{m,n}|}{\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2}} = -1$$

holds for some positive real numbers R_1 and R_2, then for any pair (K_1,K_2) of real numbers such that $K_i>R_i\ (i=1,2)$, there exists a point $s^o=(s_1^o,s_2^o)\in C^2$ such that $R_i<|s_i^o|<K_i\ (i=1,2)$ and the series

$$(2.11)\qquad \sum_{m,n=0}^{\infty} a_{m,n} E_{\rho_1}(\sigma_1^{1/\rho_1} s_1^o) E_{\rho_2}(\sigma_2^{1/\rho_2} s_2^o)$$

does not converge.

Proof. Let K_i $(i=1,2)$ be positive real numbers such that $R_i<K_i (i=1,2)$ holds. We shall find a point $s^o=(s_1^o,s_2^o)$ such that the series (2.11) does not converge. Let $r_i (i=1,2)$ be real numbers such that $R_i<r_i<K_i$ $(i=1,2)$. By (2.10) there exist two subsequences $\{m_k\}$ and $\{n_k\}$ such that

$$\frac{\ln|a_{m_k n_k}|}{\sigma_1 r_1^{\rho_1}|\lambda\ |^{\rho_1}+\sigma_2 r_2^{\rho_2}|\mu_{n_k}|^{\rho_2}} >-1 \quad (k\in\mathbb{N}) .$$

Hence,

$$(2.12)\qquad |a_{m_k,n_k}|<\exp\{-\sigma_1 r_1^{\rho_1}|\lambda_{m_k}|^{\rho_1}-\sigma_2 r_2^{\rho_2}|\mu_{n_k}|^{\rho_2}\} .$$

There exist subsequences $\{s_k\}$ of $\{m_k\}$ and $\{u_k\}$ of $\{n_k\}$ such that $\arg \lambda_{s_k}\to\phi_1$ and $\arg \mu_{u_k}\to\phi_2$ as $k\to\infty$. By the asymptotic behaviour (2.8) of the Mittag-Leffler function $E_{\rho_i}(\sigma_i^{1/\rho_1}s_i)$, we have for $s_i^o=K_i e^{-i\phi_i}$ $(i=1,2)$ for some small $\varepsilon>0$ and $k>k_o$

$$|E_{\rho_1}(\sigma_1^{1/\rho_1}\lambda_{s_k} s_1^0)|>\exp\{(\sigma_1-\varepsilon)K_1^{\rho_1}|\lambda_k|^{\rho_1}$$

$$|E_{\rho_2}(\sigma_2^{1/\rho_2}\mu_{u_k} s_2^0)|>\exp\{(\sigma_2-\varepsilon)K_2^{\rho_2}|\mu_{u_k}|^{\rho_2}\}.$$

Hence, by (2.12),

$$|a_{s_k,u_k}||E_{\rho_1}(\sigma_1^{1/\rho_1}\lambda_{s_k}s_1^0)||E_{\rho_2}(\sigma_2^{1/\rho_2}\mu_{u_k}s_2^0)|$$

$$>\exp\{|\lambda_{s_k}|^{\rho_1}[\sigma_1K_1^{\rho_1}-(\sigma_1+\varepsilon)r_1^{\rho_1}]$$

$$-|\mu_{u_k}|^{\rho_2}[\sigma_2K_2^{\rho_2}-(\sigma_2+\varepsilon)r_2^{\rho_2}]\}.$$

We can choose a positive real number ε small enough so that the numbers

$\sigma_1K_1^{\rho_1} - (\sigma_1+\varepsilon)r_1^{\rho_1}$ and , $\sigma_2K_2^{\rho_2}-(\sigma_2+\varepsilon)r_2^{\rho_2}$ are positive.

positive.

Hence,

$$|a_{s_k,u_k}||E_{\rho_1}(\sigma_1^{1/\rho_1}\lambda_{s_k}s_1^0)||E_{\rho_2}(\sigma_2^{1/\rho_2}\mu_{u_k}s_2^0)|>1$$

and the series (2. 11), does not converge.

3. Linear continuous functionals

Let $X_D(f_1,f_2), f_i \in E_{\rho_i,\sigma_i}$ $(i=1,2)$, denote the set of all the functions defined by the absolute and uniform convergent series from (2.1) on the polydisc $D(R_1,R_2)$, where the sequences $\{\lambda_m\}$ and $\{\mu_n\}$ satisfy condition (2.2) and $\{a_{m,n}\}$ satisfies the condition (2.3).

We can introduce a paranorm (see [9]) on $X_D(f_1,f_2)$ in the following way

$$\|F\| = \sup\{|a_{0,0}|, |a_{m,n}|^{(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2})^{-1}} \quad m+n\neq 0,\ m,n\geq 0\}$$

$(F\in X_D(f_1,f_2))$.

We can prove, in an analogous way as in [2], the following

<u>Theorem 3.1.</u> The set $X_D(f_1,f_2)$ endowed with the metric

$$d(F,G)=\|F-G\| \quad (F,G\in X_D(f_1,f_2)).$$

is a linear metric space.

We shall need the following version of Lemma 2 from [8]

<u>Lemma 3.2.</u> The following conditions are equivalent:

a)

$$\lim_{m+n\to\infty} \frac{\ln(m+n)}{\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2}} = 0;$$

b)

$$\lim_{m\to\infty} \frac{\ln m}{|\lambda_m|^{\rho_1}} = 0 ,$$

$$\lim_{n\to\infty} \frac{\ln n}{|\mu_n|^{\rho_2}} = ;$$

c) for any $\alpha, 0<\alpha<\infty$, the series

$$\sum_{m,n=0}^{\infty} \exp\{ -\alpha(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2})$$

converges.

Proof. The proof is analogous to the proof of Lemma 1 from [1] with some obvious modifications.

Theorem 3.3. Every continuous linear functional Φ on $X_D(f_1,f_2)$ is of the form

$$(3.1) \quad \Phi(F) = \sum_{m,n=0} a_{m,n}c_{m,n}$$

$$F(s_1,s_2) = \sum_{m,n=0}^{\infty} a_{m,n}f_1(\lambda_m s_1)f_2(\mu_n s_2) \;,$$

where

$$\ln|c_{o,o}|<M<1 \qquad \text{and}$$

$$(3.2) \qquad \ln|c_{m,n}|^{(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2})^{-1}} < M<1$$

$(m+n\geqslant 0,\ m+n \neq 0)$.

Conversely, for each infinite matrix $[c_{m,n}]$ with property (3.2), (3.1) represents a continuous linear funcitional on $X_D(f_1,f_2)$.

Proof. Let Φ be a continuous linear functional on $X_D(f_1,f_2)$. We can take

$$\Phi(f_1(\lambda_m s_1)f_2(\mu_n s_2))=c_{m,n} \quad (m,n\in \mathbb{N}\cup\{0\}).$$

Using the fact that the series representing the function F is uncoditionally convergent, we have

$$\Phi(F) = \lim_{m+n\to\infty} \Phi\left(\sum_{i,j=0}^{m,n} a_{ij} f_1(\lambda_m s_1) f_2(\mu_n s_2)\right) =$$

$$= \lim_{m+n\to\infty} \sum_{i,j=0}^{m,n} a_{i,j} c_{i,j} = \sum_{m,n=0}^{\infty} a_{m,n} c_{m,n} .$$

By Lemma 3.2 and (2.3) we have that for the matrix $[c_{m,n}]$ (3.2) holds.

Assume now that for the matrix $[c_{m,n}]$, (3.2) First we have to prove that the series

$$\sum_{m,n=0}^{\infty} a_{m,n} c_{m,n} \quad \text{converges.}$$

By (3.2) we have

$$(3.3)\quad |c_{o,o}|<\exp M,\ |c_{m,n}|<\exp\ (\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2}).$$

By (2.3) there exists a natural number N such that

$$|a_{m,n}|<\exp\{ -\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}-\sigma_2 R_2^{\rho}|\mu_n|^{\rho_2}\}$$

for $m+n>N$. Hence, by (3.3),

$$|a_{m,n}c_{m,n}|<\exp\{ -(1-M)(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2})\}.$$

Since, by Lemma 3.2, the series

$$\sum_{m,n=0}^{\infty} \exp\{-(1-M)(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{\rho_2}|\mu_n|^{\rho_2})$$

converges, we obtain that the series

$$\sum_{m,n=0}^{\infty} a_{m,n}c_{m,n}$$

converges absolutely.

It is obvious that the functional Φ is linear. We have to prove that it is continuous. Let $\{F_k\}$ be a seguence from $X_D(f_1,f_2)$ which tends to zero with respect to the convergence in $X_D(f_1,f_2)$. We have to prove that $\lim_{k\to\infty}\Phi(F_k)=0$. Let

$$F_k(s_1,s_2) = \sum_{m,n=0}^{\infty} a_{m,n}^{(k)} \; f_1(\lambda_m s_1)f_2(\mu_n s_2) \quad (k\in\mathbb{N}) .$$

The condition $F_k \xrightarrow{X_D(f_1,f_2)} 0$, as $k\to\infty$ implies that there exists a natural number k_o such that

$$|a_{o,o}^{(k)}|<\exp\{-k\},\; |a_{m,n}^{(k)}|< \exp\{-k \; (\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{2}|\mu_n|^{\rho_2})$$

for each k such that $k\geq k_o$. Hence, by (3.2),

$$|\Phi(F_k)|< \sum_{m,n=0}^{\infty} |a_{m,n}^{(k)}|\,|c_{m,n}|<\exp\{M-k\}+$$

$$+ \sum_{m,n>0}^{\infty} \exp\{(M-k)(\sigma_1 R_1^{\rho_1}|\lambda_m|^{\rho_1}+\sigma_2 R_2^{2}|\mu_n|^{\rho_2})\}$$

for each k such that $k>k_o$. Letting $k\to\infty$, we obtain by Lemma 3.2 (since $M<1$) that $\Phi(F_k)\to 0$. This completes the proof of the theorem.

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REZIME

PROSTOR UOPŠTENIH DIRICHLETOVIH REDOVA OD VIŠE KOMPLEKSNIH PROMENLJIVIH NA POLI-DISKU

U radu se daje karakterizacija koeficijenata uopštenog Dirichletovog reda od više kompleksnih promenljivih, a koji konvergira apsolutno i uniformno nad poli-diskom $D(R_1,R_2)$. Daje se reprezentacija neprekidne linearne funkcionele nad prostorom $X_D(f_1,f_2)$, funkcija razvijenih u uopšteni Dirichlet red nad poli-diskom $D=D(R_1,R_2)$.

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