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ON THE EXISTENCE OF A SELF-RECURRENT SW-ON

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Abstract

A recent paper treats the relations between the basic objects P, Q, π, $\tilde{\pi}$, $\hat{\pi}$ of a self-recurrent SW-On. There is a number of such relations in a Riemannian space if it is to serve as a basis for a self-recurrent SW-On.

Introduction

A space of the general regular connection, defined and investigated by T. Otsuki ([2], [3], [4], [5]) is a differentiable manifold μ, supplied by the basic covariant differentiation, given by

$$T^i_{jk|h} = \frac{\partial T^i_{jk}}{\partial x^h} + {}'\Gamma^i_{sh} T^s_{jk} - {}''\Gamma^s_{jh} T^i_{sk} - {}''\Gamma^s_{kh} T^i_{js} \tag{0.1}$$

and by the general covariant differentiation, given by

$$T^i_{jk,h} = P^i_a \, T^a_{bc|h} \, P^b_j \, P^c_k \tag{0.2}$$

where (P^i_j) denotes a C^∞ field of a nonsingular tensor of type (1,1). (Q^i_j) denotes the inverse of the tensor (P^i_j). The coefficients of two classical affine connections $({}'\Gamma^i_{jk})$ and $({}''\Gamma^i_{jk})$ (usually called the contravariant and covariant part of the regular general connection) are connected mutually by the fact

$$Q^i_{j|k} = 0 \, . \tag{0.3}$$

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If the space of the regular general connection is a metric space, provided with a metric tensor (g_{ij}), we define an SW-On in the following way:

a) $g_{ij|k} = \gamma_k m_{ij}$ (γ_k is a vector field and m_{ij} is a symmetric tensor field)

b) $'\Gamma^i_{jk}$ is symmetric

c) $P_{jk} = g_{ja} P^a_k$ is symmetric

According to the formulae expressing $'\Gamma^i_{jk}$ and $''\Gamma^i_{jk}$ ([6], [7]) we have always paid special attention to the connections $'\overset{m}{\Gamma}{}^i_{jk}$ and $''\overset{m}{\Gamma}{}^i_{jk}$, which correspond to $\gamma_k=0$. $\{''\overset{m}{\Gamma}{}^i_{jk}\}$ is an object of a nonsymmetric metric affine connection and $'\overset{m}{\Gamma}{}^i_{jk}$ is its contravariant mate, which is not a metric connection.

Since we have given a metric tensor (g_{ij}) and a differentiable manifold M, then we have given a Riemannian geometry. We have named it the adjoint Riemannian space to SW-On.

The SW-On is self-recurrent if its fundamental (in some sense) tensor (P^i_j) is recurrent to the unit tensor in the adjoint Riemannian space

$$\nabla_k P^i_j = \pi_k \delta^i_j \tag{0.4}$$

and if the vector $\tilde{\pi}_i = Q^a_i \pi_a$ satisfies the concircularity condition

$$\nabla_k \tilde{\pi}_h = \tilde{\pi}_k \tilde{\pi}_h + \rho g_{kh} \qquad (\rho \text{ is a scalar function}) \tag{0.5}$$

Then the corresponding $(''\overset{m}{\Gamma}{}^i_{jk})$ is a concircularly semi-symmetric metric connection

$$''\overset{m}{\Gamma}{}^i_{jk} = \left\{{}^{\,i}_{jk}\right\} + \tilde{\pi}_j \delta^i_k - \tilde{\pi}^i g_{jk} \tag{0.6}$$

and, besides

$$\nabla_k \pi_h = \tilde{\pi}_k \pi_h + \pi_k \tilde{\pi}_h + \rho P_{kh} \tag{0.7}$$

In [8], we proved that concircular and projective curvature tensor of a self-recurrent SW-On (that is, of the connection $''\overset{m}{\Gamma}$) are equal to the same tensors in the adjoint Riemannian space. In [9], we gave some properties of its Ricci and scalar curvature. Now, we want to give some conditions which the basic objects of a SW-On must satisfy.

1. Rieman-Cristoffel tensors, Ricci tensors and scalar curvatures

Taking into account (0.5) and (0.6), we can easily get

$$''\overset{\blacksquare}{R}_{ijkh} = R_{ijkh} + (2\rho + \omega)(g_{ik}g_{jh} - g_{ih}g_{jk}), \tag{1.1}$$

where $''\overset{\blacksquare}{R}_{ijkh}$ denote the Rieman-Cristoffel tensor of $''\overset{\blacksquare}{\Gamma}$, R_{ijkh} the same tensor of the adjoint Riemannian space, ρ is the scalar function from (0.5) and ω denotes the length of the vector $(\tilde{\pi}_i)$, which is also a scalar function.

Contracting (2.1) by g^{ih}, we get

$$''\overset{\blacksquare}{R}_{jk} = R_{jk} - (n-1)(2\rho+\omega)g_{jk} \tag{2.2}$$

which gives the relation between the Ricci tensor of $''\overset{\blacksquare}{\Gamma}$ and the same tensor in the adjoint Riemannian space.

If we contract (2.2) now by g^{jk}, we are getting

$$''\overset{\blacksquare}{R} = R - n(n-1)(2\rho+\omega) \tag{1.3}$$

which gives us the relation between the scalar curvature of $''\overset{\blacksquare}{\Gamma}$ and the scalar curvature of the adjoint Riemannian space.

2. Integrability conditions for the existence of a self-recurrent SW-On

Let us now consider the scalar function ω. Its partial derivative can be expressed in this way

$$\omega_k = \frac{\partial\omega}{\partial x^k} = \nabla_k\omega = 2(\rho+\omega)\tilde{\pi}_k \tag{2.1}$$

taking into account (0.5) and (0.6). The system of partial differential equations (2.1) is integrable if the following integrability condition

$$\frac{\partial^2\omega}{\partial x^k \partial x^h} = \frac{\partial^2\omega}{\partial x^h \partial x^k}$$

is satisfied. Differentiating (2.1) one more time, we get

$$(2.2)\qquad \omega_{kh} = \frac{\partial^2 a}{\partial x^k \partial x^h} = 2(\rho_h + 2(\rho+\omega)\tilde{\pi}_h)\tilde{\pi}_k + 2(\rho+\omega)(\tilde{\pi}_k\tilde{\pi}_h + \rho g_{kh} + \{^s_{kh}\}\tilde{\pi}_s)\ .$$

Alternating the indices k and h, it follows from (2.2) and the integrability condition

$$(2.3)\qquad \rho_h\tilde{\pi}_k = \rho_k\tilde{\pi}_h,$$

which is itself an integrability condition. ρ_k stands for $\frac{\partial_s}{\partial x^k}$. Besides, we can see that

$$(2.4)\qquad \frac{\partial\tilde{\pi}_k}{\partial x^h} = \tilde{\pi}_k\tilde{\pi}_h + \rho g_{hk} + \{^s_{hk}\}\tilde{\pi}_s = \frac{\partial\tilde{\pi}_h}{\partial x^k}$$

Now, we can formulate

Lemma 1. *The necessary condition for a self-recurrent SW-On to exist is* (2.3). *Expect for this, the vector* $(\tilde{\pi}_k)$ *is locally a gradient.*

Now, we are going to examine the integrability condition for the scalar function μ. μ denotes the scalar product of vectors (π_i) and $(\tilde{\pi}_i)$.

$$(2.5)\qquad \mu_k = \frac{\partial\mu}{\partial x^k} = \nabla_k\mu = \nabla_k(\tilde{\pi}^s\pi_s) = 2\mu\tilde{\pi}_k + (2\rho+\omega)\pi_k\ .$$

The partial derivative of the second order is expressed in this way

$$(2.6)\qquad \frac{\partial^2\mu}{\partial x^k\partial x^h} = 2\mu_h\tilde{\pi}_k + 2\mu(\tilde{\pi}_k\tilde{\pi}_h + \rho g_{kh} + \{^s_{kh}\}\tilde{\pi}_s) +$$

$$+ 2(\rho_h+\omega_h)\pi_k + (2\rho+\omega)(\tilde{\pi}_k\pi_h + \pi_k\tilde{\pi}_h + \rho P_{kh} + \{^s_{kh}\}\tilde{\pi}_s)\ .$$

After alternating the indices k and h, we get the integrability condition:

$$(2.7)\qquad (\rho\tilde{\pi}_k + \rho_k)\pi_h = (\rho\tilde{\pi}_h + \rho_h)\pi_k$$

and

Lemma 2. *The necessary condition for a self-recurrent SW-On to exist is* (2.7).

Let us denote the scalar product $\pi_s\pi^s$ by $\mathcal{H}$. It is a scalar function. Differentiating $\mathcal{H}$ in the same manner as we have already done with functions ω and μ, we get

(2.8) $$\mathcal{H}_k = 2(\mathcal{H}\tilde{\pi}_k + \mu\pi_k + \rho\hat{\pi}_k),$$

where $\hat{\pi}_k$ denotes $\pi_a P^a_k$. For such a vector, there hold

(2.9) $$\nabla_h\hat{\pi}_k = 2\pi_k\pi_h + \tilde{\pi}_h\hat{\pi}_k + \rho\overset{2}{P}_{kh}$$

$(\overset{2}{P}_{kh} = P_{ka}P^a_h)$

Differentiating (2.8) one more time, we get

(2.10) $$\frac{\partial^2\mathcal{H}}{\partial x^h\partial x^k} = 2\Big[\mathcal{H}_h\tilde{\pi}_k + \mathcal{H}(\tilde{\pi}_h\tilde{\pi}_k + \rho g_{kh} + \{^{s}_{kh}\}\tilde{\pi}_s + + \mu_h\pi_k + \mu(\tilde{\pi}_h\pi_k + \tilde{\pi}_k\pi_h + \rho P_{kh} + \{^{s}_{kh}\}\pi_s) + + \rho_h\tilde{\pi}_k + \rho(2\pi_k\pi_h + \tilde{\pi}_h\hat{\pi}_k + \rho\overset{2}{P}_{kh})\Big]$$

Alternating the indices k and h in (2.10) and using the general integrability condition, we get

(2.11) $$2\rho\hat{\pi}_h\tilde{\pi}_k + \rho_h\hat{\pi}_k = 2\rho\hat{\pi}_k\tilde{\pi}_h + \rho_k\hat{\pi}_h$$

Then, there follows

Lemma 3. *The necessary condition for a self-recurrent SW-On to exist is* (2.11).

3. Other conditions

The other group of existence conditions can be obtained by the Ricci identities. For a covector field (v_m) and the connection $''\overset{m}{\Gamma}$, the Ricci identity is

(3.1) $$''\overset{m}{\nabla}_l\, ''\overset{m}{\nabla}_k v_h - ''\overset{m}{\nabla}_k\, ''\overset{m}{\nabla}_l v_h = ''\overset{m}{R}{}^s_{mlk} v_s + ''\overset{m}{T}{}^s_{lh}\, ''\overset{m}{\nabla}_s v_h.$$

Transforming (3.1) by formula (1.1), the Ricci identity for the Riemannian connection and by the fact that the connection $''\overset{m}{\Gamma}$ is semi-symmetric, we get

$$(3.2)\quad {}''\overset{m}{\nabla}_l {}''\overset{m}{\nabla}_k v_h - {}''\overset{m}{\nabla}_k {}''\overset{m}{\nabla}_l v_h = \nabla_l \nabla_k v_h - \nabla_k \nabla_l v_h + (2\rho+\omega)(v_l g_{kh} - v_k g_{lh})$$

$$+ \tilde{\pi}_l {}''\overset{m}{\nabla}_k v_h - \tilde{\pi}_k {}''\overset{m}{\nabla}_l v_h .$$

We shall apply formula (3.2) to vectors $(\tilde{\pi}_h)$ and (π_h) first. For the vector $(\tilde{\pi}_h)$, we have from (0.5)

$$(3.3)\quad \nabla_l \nabla_k \tilde{\pi}_h - \nabla_k \nabla_l \tilde{\pi}_h = \rho\tilde{\pi}_k g_{lh} - \rho\tilde{\pi}_l g_{hk} + \rho_l g_{kh} - \rho_k g_{lh} ;$$

$$(3.4)\quad {}''\overset{m}{\nabla}_k \tilde{\pi}_h = (\rho+\omega) g_{kh}$$

and

$$(3.5)\quad {}''\overset{m}{\nabla}_l {}''\overset{m}{\nabla}_k \tilde{\pi}_h - {}''\overset{m}{\nabla}_k {}''\overset{m}{\nabla}_l \tilde{\pi}_h = (\rho_l+\omega_l) g_{kh} - (\rho_k+\omega_k) g_{lh} .$$

Combining the results of (3.3), (3.4) and (3.5) with (3.2) and (2.1) we get an identity and no new conditions.

For the vector (π_h), it follows from (0.7) that

$$(3.6)\quad \nabla_l \nabla_k \pi_h - \nabla_k \nabla_l \pi_h = P_{lh}(\rho\tilde{\pi}_k - \rho_k) - P_{kh}(\rho\tilde{\pi}_l - \rho_l)$$

$$(3.7)\quad {}''\overset{m}{\nabla}_k \pi_h = \tilde{\pi}_k \pi_h + \rho P_{kh} + \mu g_{kh}$$

and

$$(3.8)\quad {}''\overset{m}{\nabla}_l {}''\overset{m}{\nabla}_k \pi_h - {}''\overset{m}{\nabla}_k {}''\overset{m}{\nabla}_l \pi_h = (\mu\tilde{\pi}_k - \mu_k) g_{lh} - (\mu\tilde{\pi}_l - \mu_l) g_{kh}$$

Combining (3.6), (3.7) and (3.8) with (3.2), we get

$$(3.9)\quad \left[2\mu\tilde{\pi}_k - \mu_k + (2\rho+\omega)\pi_k\right] g_{lh} - \left[2\tilde{\mu}\pi_l - \mu_l + (2\rho+\omega\pi_l)\right] g_{kh} =$$

$$= \rho_l P_{kh} - \rho_k P_{lh} .$$

Applying (2.5) to (3.9), we get an identity again and no new conditions for the existence of a self-recurrent SW-On. The fact, that (3.5) and (3.9) are identities means that the structure of the self-recurrent SW-On is relatively compatible.

In the same fashion, we can investigate the fundamental (in some sense) tensors P and Q. The investigation of the tensor Q would be very simple because there hold

(3.10) $$''\overset{m}{\nabla}_k Q_{ij} = Q_{ij|k} = 0$$

and

(3.11) $$''\overset{m}{\nabla}_l\, ''\overset{m}{\nabla}_k Q_{ij} - ''\overset{m}{\nabla}_k\, ''\overset{m}{\nabla}_l Q_{ij} = ''\overset{m}{R}{}^s_{ilk} Q_{sj} + ''\overset{m}{R}{}^s_{jlk} Q_{is} = 0 \;.$$

Besides, from (0.4), we can easily get

(3.12) $$\nabla_k Q_{ij} = -\pi_k Q_{ia} Q^a_j$$

and, using (0.7)

(3.13) $$\nabla_l \nabla_k Q_{ij} - \nabla_k \nabla_l Q_{ij} = R^s_{ilk} Q_{sj} + R^s_{jlk} Q_{is} = 0 \;.$$

Combining (3.13) with (3.11) and (1.1), we get

(3.14) $$(2\rho+\omega)(g_{ik} Q_{lj} - g_{il} Q_{kj} + g_{jk} Q_{il} - g_{jl} Q_{ik}) = 0$$

Then, we can state

Lemma 4. (3.14) *is an existence condition for a self-recurrent SW-On.*

For the tensor P, there hold, from (0.4),

(3.15) $$\nabla_l \nabla_k P_{ij} = g_{ij} \nabla_l \pi_k$$

and, consequently, by (0.7)

(3.16) $$\nabla_l \nabla_k P_{ij} - \nabla_k \nabla_l P_{ij} = R^s_{ilk} P_{sj} + R^s_{jlk} P_{is} = 0$$

By (0.4) and (0.6), we have

$$''\overset{m}{\nabla}_k P_{ij} = \pi_k g_{ij} + \pi_i g_{jk} + \pi_j g_{ik} - \tilde{\pi}_i P_{kj} - \tilde{\pi}_j P_{ik}$$

and

(3.17) $$''\overset{m}{\nabla}_l\, ''\overset{m}{\nabla}_k P_{ij} - ''\overset{m}{\nabla}_k\, ''\overset{m}{\nabla}_l P_{ij} = g_{jk}\tilde{\pi}_l \pi_i - g_{jl}\tilde{\pi}_k \pi_i + g_{ik}\tilde{\pi}_l \pi_j - g_{il}\tilde{\pi}_k \pi_j +$$
$$+ (2\rho+\omega)(P_{il} g_{jk} - P_{kl} g_{jl} + P_{lj} g_{ik} - P_{kj} g_{il}) \;.$$

But,

$$''\overset{m}{\nabla}_l\, ''\overset{m}{\nabla}_k P_{ij} - ''\overset{m}{\nabla}_k\, ''\overset{m}{\nabla}_l P_{ij} = (2\rho+\omega)(P_{lj} g_{ik} - P_{kj} g_{il} + P_{il} g_{jk} - P_{ik} g_{jl})$$
$$+ (\tilde{\pi}_l \delta^s_k - \tilde{\pi}_k \delta^s_l)\, ''\overset{m}{\nabla}_s P_{ij}$$

Then,

$$g_{ij}\tilde{\pi}_l\pi_l - g_{jl}\tilde{\pi}_k\pi_l + g_{lk}\tilde{\pi}_l\pi_j - g_{ll}\tilde{\pi}_k\pi_j = (\tilde{\pi}_l\delta^s_k - \tilde{\pi}_k\delta^s_l)''\overset{*}{\nabla}_s P_{lj}$$

and, finally

(3.18) $$\tilde{\pi}_l\pi_k g_{lj} - \tilde{\pi}_l\tilde{\pi}_l P_{lk} - \tilde{\pi}_l\tilde{\pi}_j P_{lk} = \tilde{\pi}_k\pi_l g_{lj} - \tilde{\pi}_k\tilde{\pi}_l P_{lj} - \tilde{\pi}_k\tilde{\pi}_j P_{ll}$$

Then, we can state

Lemma 5. (3.18) *is an existence condition for a self-recurrent SW-On.*

4. Some special cases for functions ω, $\mathcal{H}$ and μ

Let us consider a possible case: that (π_h) is an eigen vector of the isomorphism Q, with an eigen value λ (λ is a C^r function of several variables). Then, $\tilde{\pi}_h=\lambda\pi_h$ and

(4.1) $$\nabla_k\tilde{\pi}_h = \lambda^2\pi_k\pi_h + \rho g_{kh}$$

(4.2) $$\nabla_k\tilde{\pi}_h = \nabla_k(\lambda\pi_h) = \lambda(2\lambda\pi_k\pi_h + \rho p_{kh}) + \lambda_k\pi_h,$$

where λ_k denotes the k-th partial derivative of the function λ.

If (π_h) is an eigen vector of the isomorphism Q, it is an eigen vector of the isomorphism P, with an eigen value $\frac{1}{\lambda}$.

Combining (4.2) with (4.1) and transvecting by π^h, we get

(4.3) $$\lambda_k = -\lambda^2\pi_k .$$

Now, we have the following relations for the function $\mathcal{H}$, μ and λ

(4.4) $$\mu = \lambda\mathcal{H}, \quad \omega = \lambda^2\mathcal{H} .$$

From (4.3) we can see that λ is a constant function if and only if $\lambda=0$ globally, which can never happen,

Then, there follows:

(4.5) $$\mathcal{H}_k = 2(\mathcal{H}\tilde{\pi}_k + \mu\pi_k + \rho\tilde{\pi}_k) = (4\mathcal{H}\lambda + \frac{2\rho}{\lambda})\pi_k$$

(4.6) $$\mu_k = 2\mu\tilde{\pi}_k + (2\rho + \omega)\pi_k = (3\lambda^2 + 2\rho)\pi_k$$

(4.7) $$\omega_k = 2(\rho + \omega)\tilde{\pi}_k = 2\lambda(\rho + \lambda^2\mathcal{H})\pi_k .$$

Now, we have the next

Proposition 1. *If the vector (π_k) is an eigen vector of the isomorphism Q, with an eigen value λ, then*

(a) the vector (π_k) is of constant length if and only if $\rho=-2\mathcal{H}\lambda^2$ ($\mathcal{H}$ is the square of the length)

(b) the vector $(\tilde{\pi}_k)$ is of constant length if and only if $\rho=-\lambda^2\mathcal{H}$

(c) the scalar product of vectors (π_k) and $(\tilde{\pi}_k)$ is constant if $\rho= -\dfrac{3\mathcal{H}\lambda^2}{2}$ or $\rho= C\mu$.

But, since the vector (π_k) and $(\tilde{\pi}_k)$ are collinear, their scalar product can be constant if and only if the lengths are constant. Then, the conjuction of (a) and (b) is equivalent to (c). But it is possible if and only if $\lambda^2\mathcal{H}=0$. Since λ is never zero, then $\mathcal{H}=0$ and there follows:

Theorem 1. *If the vector (π_i) is an eigen vector of the isomorphism Q, with a C^r eigen value, λ then*

(i) λ is never a constant

(ii) (π_i) and $(\tilde{\pi}_i)$ are of constant length and their scalar product is constant if and only they are isotropic.

5. Some considerations of an indefinite metric form of underlying Riemannian geometry

Surely, the vector (π) should not be an eigen vector of the isomorphism Q. In a more common case, vectors (π_k) and $(\tilde{\pi}_k)$ are linearly independent and form a 2-plane Π. Now, if we want the vector $(\tilde{\pi}_h)$ to be of constant length, then

$$\omega_k = (2\rho + \omega)\tilde{\pi}_k = 0$$

and, consequently,

$$\rho = -\omega . \tag{5.1}$$

Then, applying (5.1) to (1.1), we get

$$''\overset{*}{R}{}^i_{jkh} = R^i_{jkh} - c(\delta^i_k g_{jk} - \delta^i_h g_{jk}), \tag{5.2}$$

where c denotes ω, for the sake of its invariability.

Then, taking into account formula (1.3), we have

Lemma 6. *If the length of the vector* (π_h) *is constant, then the difference between the scalar curvatures of* $''\overset{m}{\Gamma}$ *in a self-recurrent SW-On and its adjoint Riemannian space is constant.*

An even more interesting result is gained for the case μ =const. As we have supposed, (π_k) and $(\tilde{\pi}_k)$ are linearly independent and consequently, (μ_k) is an element of the 2 plane Π. Then, it is evident that if μ =const, then $\mu=0$ and $2\rho+\omega=0$.

Lemma 7. *If the vectors* (π_k) *and* $(\tilde{\pi}_k)$ *are linearly independent, their scalar product is a constant if and only if they are orthogonal.*

But, if they are orthogonal, $2\rho+\omega=0$ too. Then, we can state

Lemma 8. *If the vectors* (π_k) *and* $(\tilde{\pi}_k)$ *are orthogonal, the curvature tensor of* $''\overset{m}{\Gamma}$ *in a self-recurrent SW-On is equal to the curvature tensor of the adjoint Riemannian space.*

For the tensor $'\overset{m}{R}$ and $''\overset{m}{R}$, there holds the relation

$$''\overset{m}{R}{}^{s}_{pkl}Q^{i}_{s}P^{p}_{j} = {}'\overset{m}{R}{}^{i}_{jkl} \qquad ([1]) \;. \tag{5.3}$$

But, in the case of the orthogonality of the vectors (π_k) and $(\tilde{\pi}_k)$, $''R=R$ and, from the Ricci identity for the tensor Q (3.13), we have

$$R^{s}_{pkl}Q^{i}_{s} = R^{si}_{lk}Q_{sp}$$

and, transvecting by P^{p}_{j} ,

$$R^{s}_{pkl}Q^{i}_{s}P^{p}_{j} = R^{si}_{lk}Q_{sp}P^{p}_{j}$$

i.e.

$$'R^{i}_{jkl} = R^{si}_{lk}g_{sj} = R^{i}_{jlk} \;. \tag{5.4}$$

We can see that the curvature tensor $'\overset{m}{R}$ is not equal to the curvature tensor of the adjoint Riemannian space, but lowering the superscript i, we get the relation

$$'\overset{m}{R}_{ijkl} = R_{jilk} = R_{ijkl} \tag{5.5}$$

Now, we can state

Theorem 2. *If the vectors* (π_k) *and* $(\tilde{\pi}_k)$ *are orthogonal, the Rieman--Cristoffel tensors* $'\overset{m}{R}$ *and* $''\overset{m}{R}$ *are both equal to the Rieman-Cristoffel tensor of the adjoint Riemannian space. The same relation holds for their Ricci tensor and scalar curvatures.*

In that case, we call R_{ijkl} a Rieman-Cristoffel tensor of a self-recurrent SW-On, R_{jk} a Ricci tensor of a self-recurrent SW-On and $R=R_{jk}g^{jk}$ a scalar curvature of a self-recurrent SW-On.

We have a similar situation for $\omega=0$ ($\tilde{\pi}_h$ an isotropic vector). Then, from (2.1)

$$\omega_k = 2(\rho+\omega)\tilde{\pi}_k = 0$$

and $\rho=\omega=0$.

Proposition 2. *If* $\tilde{\pi}_h$ *is an isotropic vector field, Lemma 8. and Theorem 2. hold for such a self-recurrent SW-On. Moreover, such a vector is a harmonic vector field in the adjoint Riemannian space (pseudo-Riemannian space) and it is parallel with respect to connection* $''\overset{m}{\Gamma}$.

For the vector $\hat{\pi}_k = \pi_a P^a_k$, *we have two cases: it may be an element of* Π, *if* Π *is an invariant 2-plane or* π_k, $\tilde{\pi}_k$ *and* $\hat{\pi}_k$ *may be linearly independent.*

If these vectors are linearly independent, then (π_k) *can be of a constant length if and only if*

(a) π_k *is an isotropic vector field*

(b) π_k *and* $\tilde{\pi}_k$ *are orthogonal*

(c) $\rho = -\frac{\omega}{2} = 0$

(*from* (2.1), (2.5) *and* (2.8)).

Proposition 3. *If* Π *is not an invariant 2-plane of the isomorphism Q and if the vector* (π_k) *is of a constant length, then*

(1) π_k *and* $\tilde{\pi}_k$ *are orthogonal*

(2) π_k *is an isotropic vector field and the adjoint Riemannian space is a pseudo-Riemannian space*

(3) $\tilde{\pi}_k$ *is an isotropic vector field*

(4) *the Riemann-Cristoffel tensor of the adjoint pseudo-Riemannian space is equal to both* $''\overset{m}{R}_{ijkh}$ *and* $'\overset{m}{R}_{ijkh}$.

If the 2-plane Π is an invariant plane for the isomorphism Q, then the fact that π_k is of a constant length does not cause its isotropy.

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Rezime

O POSTOJANJU SAMOREKURENTNOG SW-On

U radu su ispitani neki neophodni uslovi koje moraju zadovoljavati osnovni objekti P, Q, $\tilde{\pi}$, $\hat{\pi}$, da bi se nad datim prostorom kao nad baznim mogao konstruisati SW-On. Takođe, ispitani su i neki slučajevi kada priduženi Rimanov prostor ima idefinitnu metriku.

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