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ON SEBASTIAO E SILVA'S ORDER OF GROWTH OF DISTRIBUTIONS

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Abstract

Sebastiao e Silva introduced in [7] the "order of growth" of distributions, which fitted very well into his axiomatic approach to the distribution theory. This notion enabled him to define a limit of distributions (both at finite and infinite points), the Landau "oh" symbols for them, and most important, the definite integral which led naturally to the convolution and Fourier transformation. In this paper the "equivalence at infinity" (analysed in [10]) is compared with the "order of growth" of distributions, and using both notions an asymptotic expansion of distributions is applied to the distributional Stieltjes transformation in the sense of [3].

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1. Order of growth and equivalence at infinity of distributions

The letters m and n will always denote natural, while α and $\rho > -1$ will denote real numbers. The set of negative integers will be denoted by Z_-. The distributional detivation operator in x will be denoted by D.

We rewrite Sebastiao e Silva's extension of the Landau symbols first:

Definition 1.1. *([7], 8.3) Let I be an unbounded interval to the right and let $r \in C^{\infty}(I)$. We write $T = O(r)$ (respectively $T = o(r)$) as $x \to +\infty$ iff there exist an $a \in \mathbf{R}$, a continuous function F and an $n \in \mathbf{N}_0$ such that*

$$T = rD^n F \quad on \quad (a, \infty),$$

and

$$\frac{F(x)}{x^n} \text{ is bounded on } (a,\infty), \text{ (respectively } \lim_{x\to+\infty} \frac{F(x)}{x^n} = 0) \tag{1}$$

If $r = 1$ on I we get the definition of " $\lim\limits_{x \to +\infty} T = 0$", or, more generally, if the limit in (1) is equal to $\frac{c}{n!}$, then we write $\lim\limits_{x \to \infty} T \overset{D'}{=} c$. As can be expected, this notion generalizes the usual Landau symbols and limits for continuous functions, while the opposite is not true. For instance, $\sin x = o\,(1)$ (i.e. $\lim\limits_{x \to \infty} \sin x \overset{D'}{=} 0$) in the sense of Definition 1. In the same manner one can define the Landau symbols and the limits at $-\infty$ or at finite points. The latter was defined by Lojasiewicz in 1957 (see [4]).

The case $r(x) = x^{\alpha}, x > 0$ for $\alpha > -1$ is the archetype of the functions r. For such an r we have

Lemma 1.1. *([7], 8.4) If T is $O(x^{\alpha})$ (resp. $o(x^{\alpha})$) as $x \to \infty$, then DT is $O(x^{\alpha-1})$) (resp. $o(x^{\alpha-1})$) as $x \to \infty$ for any $\alpha \in \mathbf{R}$.*

A more precise notion was used in [2], [3] and [10], which is an asymptotic behaviour of distributions:

Definition 1.2. *A distribution T is* **equivalent at infinity** *with a regularly varying function $r(x) = x^{\alpha}L(x)$ if*

i. for $\alpha \notin Z_-$ there exist $a \in \mathbf{R}, n \in \mathbf{N}_0, n + \alpha > 0$ and a continuous function F on $\mathbf{R}$ such that $T = D^n F$ on (a, ∞) and

$$\lim_{x\to\infty} \frac{F(x)}{x^{n+\alpha}L(x)} = \frac{1}{(n+1)\dots(n+\alpha)} =: C_{\alpha,n}$$

ii. for $\alpha \in Z_-$ there exist $a \in \mathbf{R}$ and a continuous function F on $\mathbf{R}$ such that $T = F$ on (a, ∞) and..

$$\lim_{x\to+\infty} \frac{F(x)}{x^n L(x)} = 1.$$

We then write $T \overset{E}{\sim} r(x)$ as $x \to \infty$.

We recall that a function $r(x) = x^\alpha L(x)$ is termed regularly varying at infinity if the function $L : (a, \infty) \to (0, \infty)$ is measurable and satisfies the condition

$$lim_{x\to\infty} \frac{L(\lambda x)}{L(x)} = 1. \text{ for each } \lambda > 0.$$

(L is then called slowly varying at infinity.)

We shall now prove a few "expected" properties of these notions.

Theorem 1.1. *If a distribution T is equivalent at infinity with $r(x) = x^\alpha$, then $T = O(x^\alpha)$ and $T \neq o(x^\alpha)$ as $x \to \infty$.*

Proof. The statement is obvious for $\alpha \in Z_-$. So, let $T \overset{E}{\sim} x^\alpha$ as $x \to \infty, \alpha \notin Z_-$ and let a, n and F be as in Definition 1.2. This means that on (a, ∞)

$$T = D^n(C_{\alpha,n} x^{\alpha+n}(1 + \omega(x))),$$

where ω is a continuous function such that $\omega(x) \to 0$ as $x \to \infty$.

From the Leibniz formula it follows that

$$\begin{aligned} T &= D^{n-1}[x^{\alpha+n-1}(C_{\alpha,n-1}(1 + \omega(x)) + C_{\alpha,n} D(x \cdot \omega(x)) - \omega(x))] = \\ &= D^{n-1}(x^{\alpha+n-1} D(\omega_1(x))) \text{ on } (a, \infty), \end{aligned}$$

where ω_1 is again a continuous function such that $|\omega_1(x)| \leq C_1 x$ for x sufficiently large. Continuiting in this manner, we get

$$T = x^\alpha D^n(\omega_n(x))$$

where the continuous function ω_n satisfies

(2) $$|\omega_n(x)| \leq C_n x^n, \text{ for } x \text{ sufficiently large.}$$

So, $T = O(x^n)$.

If $T = o(x^\alpha)$, then $T = x^\alpha D^m G(x)$, where G is a continuous function such that $G(x)/x^m \to 0$ as $x \to \infty$. So, we obtain

$$T = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \alpha(\alpha - 1) \ldots (\alpha - k + 1) D^{m-k}(x^{\alpha-k} G(x))$$

on (a,∞) and $x^{\alpha-k}G(x) = o(x^{\alpha+m-k})$ as $x \to \infty$ in the ordinary sense. But, then, it is impossible to find an n and a continuous function F such that

$$F(x) \sim Cx^{\alpha+n} \text{ as } x \to \infty.$$

From the proof of Theorem 1.1. we get

Theorem 1.2. *If $T \overset{E}{\sim} x^\alpha$ as $x \to \infty$, then on some interval (a,∞) we have*

$$T = x_+^\alpha + R,$$

where $R \in D'$ satisfies $R = o(x^\alpha)$ as $x \to \infty$.

As usual, x_+^α is the tempered distribution which regularizes the function $H(x)\cdot x^\alpha$, H is the Heaviside function.

If x^α is replaced with a regularly varying function, then we can only prove

Theorem 1.3. *If $T \overset{E}{\sim} x^\alpha L(x)$ as $x \to \infty$, then $T = o(x^{\alpha+\varepsilon})$ and $T = o(x^{\alpha-\varepsilon})$ for any $\varepsilon > 0$, such that $\alpha+\varepsilon$ (resp. $\alpha-\varepsilon$) is not a negative integer if $\alpha \notin Z_-$.*

Proof. It is well known ([6]) that for a given $\varepsilon > 0$ a slowly varying function L at infinity satisfies the inequalities

$$C_1x^{-\varepsilon} \le L(x) \le C_2x^\varepsilon \text{ for } x \ge a_1 \tag{3}$$

for some positive constants a_1, C_1 and C_2 which depend on ε.

We shall give only the proof for $\alpha \notin Z_-$ of the statement "$T = O(x^{\alpha+\varepsilon})$", the others are similar. We can write on (a,∞)

$$T = D^n(x^{\alpha+n+\varepsilon'}(C_{\alpha,n}\frac{L(x)}{x^{\varepsilon'}} + \frac{L(x)}{x^{\varepsilon'}}\omega(x)))$$

for $a \ge a_1$ and $n \ge -\alpha$ as in Definition 1.2. and $\omega(x) \to 0$ as $x \to \infty$ (we choose $0 < \varepsilon' < \min(1,\varepsilon)$ such that $\alpha+n+\varepsilon' \notin \mathbf{N}$). In the same way as in the proof of Theoren 1.1, using (3), we get

$$T = x^{\alpha+\varepsilon'}D^n(\omega_n(x))$$

where $\omega_n(x)$ is a continuous function satisfying (2).

A regularly varying function does not need to be continuous. However, by [6], p.17, for a given slowly varying function L there exists another, infinitely differentiable slowly varying function L_1 such that $L_1(x) \sim L(x)$ as $x \to \infty$ and $L_1(n) = L(n)$ for all sufficiently large integers n. If L_1 has this property, then we have

Theorem 1.4. *If $r_1(x) = x^\alpha L_1(x)$ is an infinitely differentiable regularly varying function at infinity, and $T \overset{E}{\sim} r_1$ as $x \to \infty$, then $T = O(r_1)$ as $x \to \infty$.*

Proof. As in the proof of Theorem 1.1, we can write $T = D^n F$ on (a, ∞) for some $a, n > -\alpha$ and some continuous function F such that $F(x) = C_{\alpha,n} x^{\alpha+n} L_1(x)(1 + \omega(x))$, where $\omega(x) \to 0$ as $x \to \infty$. So we obtain (compare to the proof of Theorem 1.1)

$$T = D^n F = x^\alpha L_1(x) D^n \omega_n(x)$$

where ω_n is a continuous function such that $|\omega_n(x)| \leq C_n x^\alpha$ for x sufficiently large; we used the fact that $\lim_{x\to\infty} \frac{xL_1'(x)}{L_1(x)} = 0$ (see [6], pp.6-7).

2. Asymptotic expansion of distributions at infinity

Let a distribution T and an increasing sequence of real numbers $(n_k)_{k=0}^{\infty}$ be given. We say that T has the **asymptotic expansion** $\sum_{k=0}^{\infty} A_k^{-n_k}$ at ∞ related to $(x^{-n_k})_{k=0}^{\infty}$, if

i. either $T \overset{E}{\sim} A_0 x^{-n_0}$ as $x \to \infty$, or $T = o(x^{-n_0})$ (by definition we put $A_0 = 0$);

ii. if the conplex numbers $A_0, \ldots, A_m (m \in \mathrm{N}_0)$ are already chosen, then either

$$T_{m:} = T - \sum_{k=0}^{m} A_k x^{-n_k} \overset{E}{\sim} A_{m+1} x^{-n_{m+1}} \tag{1}$$

as $x \to \infty$ or

$$T = o(x^{-n_{m+1}}); \text{ we put then } A_{m+1} = 0.$$

Then we write

$$(2) \qquad T \overset{AE}{\sim} \sum_{k=0}^{\infty} A_k x^{-n_k} \text{ as } x \to \infty.$$

It is clear that the asymptotic expansion of a distribution T is unique, provided it exists; however two different distributions can have the same expansion. In view of Definition 1.2 it is clear that if

$$T \overset{AE}{\sim} \sum_{m=0}^{\infty} A_m x^{-n_m} \text{ as } x \to \infty,$$

and at least one of the numbers n_m is a natural number with $A_m \neq 0$, then T_m is a locally integrable function on some interval (a, ∞) and the asymptotic expansion of T_m can be taken in the usual sense.

Similarly as the equivalence at infinity, this asymptotic expansion is a local property of a distribution which generalizes the classical asymptotic expansion and preserves some "expected" properties of an asymptotic expansion. For instance, the distribution T_m from (2) is $o(x^{-n_m})$ as $x \to \infty$. Furthermore, the asymptotic expansion can be added or multiplied with nonzero constants. As in the classical case, it is possible to "integrate" the asymptotic expansion (i.e. to convolve it with the Heaviside function), provided that $T \in D'_+$, which means that the support of T is in $[0, \infty]$. However, in general, one can not differentiate it.

On using Theorem 1.2, we obtain

Theorem 2.1. *If $T \in D'_+$ satisffies $T \overset{AE}{\sim} \sum_{k=0}^{\infty} A_k x^{-n_k}$ as $x \to \infty$ where $n_k \notin Z_-$ for each $k = 0, 1, \ldots$, and $n_0 < n_1 < \ldots$, then on some interval (a_m, ∞) $(0 \leq a_0 \leq a_1 \leq \ldots)$*

$$(3) \quad T = \sum_{k=0}^{\infty} A_k x_+^{-n_k} + R_m(x), \text{ where } R_m(x) = o(x^{-n_m}) \text{ as } x \to \infty.$$

From Theorem 1.2 and Theorem 3 form [10] we get an important property of the asymptotic expansion:

Theorem 2.2. *If T satisties the conditions of Theorem 2.1., then $R_m = o(x^{-n_m})$ satisfies*

$$(4) \qquad \lim_{\lambda \to \infty} \langle R_m(x), \frac{\varphi(x/\lambda)}{\lambda^{n_m+1}} \rangle = 0$$

for each $\varphi \in D$ with a support in (a_m, ∞).

A special case of the asymptotic expansion gives

Theorem 2.3. *A distribution $T \in E'$ has the asymptotic expansion at infinity related to any sequence $(x^{-n_k})_{k=0}^{\infty}, n_0 < n_1 < \dots,$ just zero.*

This statement remains true if T is rapidly decreasing at infinity, i.e. for each $\alpha < 0$ there exist continuous functions F_l and natural numbers $n_l, l = 1, \dots, m$ such that

$$T = \sum_{l=1}^{m} D^{n_l} F_l \text{ on some interval } (a, \infty)$$

and

$$F_l = o(x^\alpha) \text{ for } l = 1, \dots, m.$$

3. Integrals of distributions

The limit of distributions from Section 1 allows a definition of their integral.

Definition 3.1. *Let $[a, b] \subset \mathbf{R}$ be interval and T a distribution. The integral of T over $[a, b]$ is defined by*

$$S - \int_a^b T(x)dx := T^{-1}(b+) - T^{-1}(a-),$$

where T^{-1} is a primitive of T (i.e. $D(T^{-1}) = T$)), provided that the limits

$$T^{-1}(b+) = \lim_{x \to b+0} T^{-1}(x), \ \lim T^{-1}(a-) = \lim_{x \to a-0} T^{-1}(x).$$

exist in the distributional sense.

(The letter "S" stands for Sebastiao E Silva, since he gave this definition in [7]).
Especially the integral over $[a, \infty]$ is defined as

$$(1) \qquad S - \int_a^\infty T(x)dx := T^{-1}(+\infty) - T^{-1}(a-),$$

where

$$T^{-1}(+\infty) := \lim_{x\to\infty} T^{-1}(x).$$

It was shown in [7] that this integral generalizes properly the usual definite one. We need the following two statements.

Lemma 3.1. *([7], Section 9) The integral (1) exists if* $T = O((x-a)^\beta)$ *as* $x \to a+0$ *for some* $\beta > -1$ *and* $T = O(x^\alpha)$ *as* $x \to \infty$ *for some* $\alpha < -1$.

Lemma 3.2. *(Compare to [8], Section 2.) Let* $T \in D'_+$ *and* $\varphi \in C^\infty(\mathbf{R})$. *If* $T = O(x^\alpha)$ *and* $\varphi^{(k)} = O(x^{\beta-k})$ *as* $x \to \infty$, *for* $k = 0, 1, \ldots,$ *and some* $\alpha, \beta \in \mathbf{R}$, *then* $T \cdot \varphi = O(x^{\alpha+\beta})$ *and for each* $n \le 0$ *with the property* $\alpha + \beta < n - 1$ *we have*

(2)
$$S - \int_{-0}^{\infty} \varphi D^n T = (-1)^n (S - \int_{-0}^{\infty} \varphi^{(n)} T)$$

Observe that $\psi(x) = \frac{1}{(x+s)^{\rho+1}}$ satisfies the conditions of this lemma.

4. Distributional Stieltjes transformation

The classical Stieltjes transformsof a function f is defined by the integral

$$\sigma_0(f)(s) = \int_0^\infty \frac{f(x)}{x+s} dx, \; s \in \Delta := C/(-\infty, 0],$$

or more generaly

(1)
$$\sigma_\rho(f)(s) = \int_0^\infty \frac{f(x)}{(x+s)^{\rho+1}} dx, \; s \in \Delta,$$

where $\rho > -1$ is called the order of the transform. A sufficient condition for the convergence of (1) is that f is locally integrable on $[0, \infty)$ and satisfies the condition

(2)
$$f = O(x^{\rho-\xi}) \text{ as } x \to \infty \text{ for some } \xi > 0.$$

There are several definitions of the Stieltjes transform of a distribution T from D'_+. In [2] the following one was given:

Definition 4.1. *If $T \in D'_+$ can be written as $T = D^k F$ for some $k \in \mathbf{N}$, continuous function F on $\mathbf{R}$ which is zero on $(-\infty, 0)$ and satisfies the condition.*

$$\sup\{|F(x)x^{-k+\alpha}|, x \geq b\} \leq C < \infty, \tag{3}$$

then the Stieltjes transforms of T is given by

$$\sigma_\rho(T)(s) = (\rho+1)_k \cdot \int_0^\infty \frac{F(x)dx}{(x+s)^{\rho+k+1}},\ s \in \Delta. \tag{4}$$

Here $(\rho+1)_0 := 1, (\rho+1)_k = (\rho+1)\dots(\rho+k)$ and $\rho > \alpha$. As usual, we use the same letter in (1) and (4) for the classical and the distributional ("generalized") transform σ_ρ; this makes sense, since if T is defined by a locally integrable function f on $[0,\infty)$ satisfying condition (2), then the two notions coincide.

(In a later paper ([3]), a slightly more general condition on F was given, namely that the integral $\int_b^\infty \frac{|F(x)|dx}{x^{\rho+k+1}}$ converges. However, in what follows condition (3) is sufficient.)

If $F = O(x^{+\rho+k-\xi})$ as $x \to \infty$ for some ξ with the property $\rho + 1 > \xi > 0$, then $T = D^k F = O(x^{\rho-\xi})$. Since, $\lim_{x \to 0} D^l F = 0$, $\lim_{x \to +\infty} \frac{F^{(k-l)}(x)}{x^{\rho+l+1}} = 0$ for $l = 0, 1, \dots, k$, we have by partial integration (see Lemma 3.2)

$$\begin{aligned} S - \int_{0-}^\infty \frac{T(x)}{(x+s)^{\rho+1}}dx &= S - \int_{0-}^\infty \frac{D^k F(x)}{(x+s)^{\rho+1}}dx = \\ &= S - \int_{0-}^\infty \frac{F(x)dx}{(x+s)^{\rho+k+1}} = \int_0^\infty \frac{F(x)dx}{(x+s)^{\rho+1}} = \sigma_\rho(T)(s). \end{aligned}$$

This suggests defining the Stieltjes transform of $T \in D'_+$ as

$$\sigma_\rho(T)(s) := S - \int_{0-}^\infty \frac{T(x)}{(x+s)^{\rho+1}}dx,\ s \in \Delta. \tag{5}$$

Formula (5) can be used for a greater set of distributions than (4); namely $T = O(x^{\rho-\varepsilon})$ as $x \to \infty$ is a sufficient, but not a necessary condition for the existence of (5). We remark that if $T = B \in E'_+$(distributions with a compact support in $[0,\infty)$), then

$$\sigma_\rho(B)(s) = S - \int_{0-}^\infty \frac{B(x)dx}{(x+s)^{\rho+1}} = \langle B(x), \frac{\eta(x)}{(x+s)^{\rho+1}} \rangle,$$

where $\eta \in C^\infty(\mathbf{R})$ has the properties $\eta(x) = 0$ for $x < -2\varepsilon$ and $\eta(x) = 1$ for $x > -\varepsilon, \varepsilon > 0$. Furthermore if $T = x_+^\alpha$, then (see [2])

$$(6) \qquad \sigma_\rho(x_+^\alpha(s) = B(\rho - \alpha, \alpha + 1)s^{\alpha-\rho}, \rho > \alpha,$$

and for $\alpha \in Z_-, s \in \Delta$. As usual Γ and B are the gamma and beta functions, more precisely their analytic continations. From now on we suppose that s is real and positive.

The following theorem was proved in [5] (compare with (6)):

Theorem 4.1. *If $T \in D'_+$ is equivalent at infinity with the regularly varying function $r(x) = x^\alpha L(x)$ as $x \to \infty$, then*

$$\sigma_\rho(T)(s) \sim B(\rho - \alpha, \rho + 1) \cdot s^{\alpha-\rho} L(s) \ \textit{as}\ s \to \infty,$$

provided that $-1 < \alpha < \rho$.

Remark 4.1. The condition $\alpha > -1$ is essential; if $\alpha \leq -1$, then the "quasiasymptotic behaviour" from [1] of distributions is more appropriate for this analysis than equivalence at infinity. Especially if $T = B \in E'_+$, then using the structural theorem from [1] one can show that it has a quasi-asymptotic behaviour of some order $-m_0 (m_0 \in \mathbf{N})$ and

$$(7) \qquad \sigma_\rho(B)(s) \sim C \cdot s^{-(\rho+m_0)} \text{ as } s \to \infty$$

(see the Abelian theorem from [9]).

We can now prove

Theorem 4.2. *If $-1 < \alpha < \rho$ and $T \in D'_+$ satisfies $T = o(x^\alpha)$, then $\sigma_\rho(T) = o_\rho(s^{\alpha-\rho})$ as $s \to \infty$.*

Proof. From $T = o(x^\alpha)$ as $x \to \infty$ it follows $T = x^\alpha D^n F(x)$ on some interval (a, ∞), for some continuous function F on $\mathbf{R}$ and some $n \in \mathbf{N}$ such that $\lim_{x\to\infty} \frac{F(x)}{x^n} = 0$. We put $G(x) = F(x)$ for $x > a, G(x) = 0$ for $x \leq a$ and $T_1 = T - x^\alpha D^n G(x)$. The support of T_1 is a compact subset of $[0, a]$, so bu Lemma 3.1 from [3] follows

$$(8) \qquad |\sigma_\rho(T_1)(s)| \leq C \cdot s^{-(\rho+1)} \text{ as } s \to \infty \text{ for some } C > 0$$

(see also (7)). From (3.2) it follows that

$$S-\int_{a-}^{\infty}\frac{x^{\alpha}D^{n}G(x)}{(x+s)^{\rho+1}}dx=$$

$$=S-\int_{a-}^{\infty}\frac{C_0x^{\alpha}+C_1\cdot s\cdot x^{\alpha-1}+\cdots+C_ns^nx^{\alpha-n}}{(x+s)^{\rho+n+1}}G(x)dx.$$

for some constants $C_i, i=0,1,\ldots,n$. The last S-integral is also an "ordinary" integral, hence, from Theorem 4.1. it follows that

$$|\sigma_\rho(x^\alpha D^n G(x))(s)|\leq C\cdot s^{\rho+1}, \tag{9}$$

s sufficiently large, since $G(x)=x^n\omega(x)$ and $\omega(x)\to 0$. Hence (7) and (8) imply

$$\lim_{s\to\infty} s^{\rho-\alpha}\sigma_\rho(T)(s)=0, \text{ or } \sigma_\rho(T)(s)=o(s^{\alpha-\rho})$$

as $s\to\infty$.

In a similar way one can prove

Theorem 4.3. *If $-1<\alpha<\rho$ and $\dot{T}\in D'_+$ satisfies $T=O(x^\alpha)$, then $\sigma_\rho(T)=(s^{\alpha-\rho})$ as $s\to\infty, s\in\mathbf{R}$.*

A consequence of these statements is

Theorem 4.4. *Let $T\overset{AE}{\sim}\sum_{k=0}^{\infty}A_kx^{-n_k}$ as $x\to\infty$ (in the sense of Definition 2.1) for $-\rho<n_0<n_1<\ldots<1$. Then*

$$\sigma_\rho(T)(s)\sim\sum_{k=0}^{\infty}A_ks^{-n_k-\rho} \tag{10}$$

as $s\to\infty$ in the ordinary sense.

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REZIME

O SEBASTIAO E SILVA "REDU RASTA" DISTRIBUCIJA

U ovom radu se uporedjuje "red rasta" koji je uveo Sebastiao E Silva sa "ekvivalencijom u beskonačnosti" za distribucije. Pomoću ova dva pojma se uvodi asimptotski razvoj distribucija, koji se primenjuje na uopštenu Stieltjesovu transformaciju.

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