Univ. u Novom Sadu
Zb. Rad. Prirod.–Mat. Fak.
Ser. Mat. 21, 1 (1991), 203-215

Review of Research
Faculty of Science
Mathematics Series

CONTINUOUS DEPENDENCE OF THE FIXED POINTS ON PARAMETERS IN RANDOM NORMED SPACES

Olga Hadžić
Institute of Mathematics, University of Novi Sad
Trg Dositeja Obradovića 4, 21000 Novi Sad, Yugoslavia

Abstract

Continuous dependence of the fixed points on parameters is investigated in many papers ([1], [8], [14], [26]). In this paper we shall give a generalization of Vorel's result from [25] to random normed spaces. As a corollary we shall give a generalization of the well known fixed point theorem of Krasnoselskiĭ in random normed spaces.

AMS Mathematics Subject Classification (1980): 47H10
Key words and phrases: Fixed points, random normed spaces.

1. Introduction

The notion of a probabilistic metric space was introduced by K.Menger [17] and the notion of a random normed space by A.N.Sherstnev in [22]. The first result about the existence of the fixed point of a probabilistic contraction on a probabilistic metric space $(S, F, \min)$ is obtained by V.Sehgal and A. Bharucha - Reid in [21]. Some fixed point theorems for singlevalued and multivalued mappings in probabilistic metric spaces are obtained in [2], [3], [4], [5], [6], [7], [9], [10], [18], [21], [24].

The continuous dependence of the fixed points on parameters for densifying mappings is investigated in [1], [8], [14] and [26]. Using the probabilistic functions α and β ([6], [25]) we shall give in this paper a probabilistic generalization of Z.Vorel's results from [26]. As an application we shall obtain a generalization of the Krasnoselskii fixed point theorem in random normed spaces.

2. Preliminaries

Let Δ denote the set of all distribution functions F such that $F(0) = 0$(F is a nondecreasing, left continuous mapping from the set of real numbers $\mathbf{R}$ into $[0,1]$ so that $\sup_{x\in\mathbf{R}} F(x) = 1$) and $\mathbf{R}^+ = [0,\infty)$.

The ordered pair (S, F) is a probabilistic metric space if S is a nonempty set and $F : S \times S \to \Delta$ ($F(p,q)$ for $p, q \in S$ being denoted by $F_{p,q}$) so that the following conditions are satisfied:

1. $F_{u,v}(x) = 1$, for every $x > 0 \Rightarrow u = v((u,v) \in S \times S)$,

2. F is symmetric,

3. $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1 \Rightarrow F_{u,w}(x + y) = 1((u,v,w) \in S \times S \times S$ and $(x,y) \in \mathbf{R} \times \mathbf{R})$.

A Menger space is a triple (S, F, t), where (S, F) is a probabilistic metric space and t is a $T-$norm so that:

$$F_{u,w}(x + y) \geq t(F_{u,v}(x), F_{v,w}(y)), \quad \text{for every } (u,v,w) \in S \times S \times S$$

and every $(x,y) \in \mathbf{R} \times \mathbf{R}$.

Recall that a mapping $t : [0,1] \times [0,1] \to [0,1]$ is called a $T-$norm if the following conditions are satisfied:

1. For every $a \in [0,1]$, $t(a,1) = a$.

2. For every $(a,b) \in [0,1] \times [0,1] : t(a,b) = t(b,a)$.

3. For every $(a,b,c,d) \in [0,1] \times [0,1] \times [0,1] \times [0,1] : a \geq b, \ c \geq d \Rightarrow t(a,c) \geq t(b,d)$.

4. For every $a,b,c \in [0,1] \times [0,1] \times [0,1]$:

$$t(a, t(b,c)) = t(t(a,b), c).$$

Examples of such functions are:
$t_m(a,b) = \max\{a+b-1,0\}$ and $t_{\min}(a,b) = \min\{a,b\}, (a,b) \in [0,1] \times [0,1]$.

The (ϵ, λ) - topology in a probabilistic metric space is introduced by the family of neighbourhoods given by :

$$U = \{U_v(\epsilon,\lambda)\}_{(v,\epsilon,\lambda)\in S\times \mathbf{R}^+ \times (0,1)}$$

where $U_v(\epsilon,\lambda) = \{u \mid u \in S, F_{u,v}(\epsilon) > 1-\lambda\}$.

A random normed space (S,F,t) is an ordered triple where S is a real or complex vector space, t is a $T-$norm which is stronger then the $T-$norm $t_m (t \geq t_m)$ and the mapping $F : S \to \Delta$ satisfies the following conditions, where

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

(a) $F_p = H \iff p = \mathbf{0}$ ($\mathbf{0}$ is the neutral element of S)

(b) For every $p \in S$, every $u > 0$ and every $r \in K\backslash\{0\}$(K is the scalar field) :$F_{rp}(u) = F_p(\frac{u}{|r|})$.

(c) For every $(p,q) \in S \times S$ and every $(u,v) \in \mathbf{R}^+ \times \mathbf{R}^+$:

$$F_{p-q}(u+v) \geq t(F_p(u), F_q(v)).$$

Every random normed space is a Menger space, where $F : S \times S \to \Delta$ is defined by $F(p,q) = F_{p-q}$.

If the $T-$norm t is continuous then S is, in the $(\epsilon,\lambda)-$ topology, a topological vector space.

Let (S,F) be a probabilistic metric space. The following two definitions are given in [6].

Definition 1. *Let A be a nonempty subset of S. The function $D_A(\cdot)$, defined on $\mathbf{R}^+$ by*

$$D_A(u) = \sup_{s<u} \inf_{p,q\in A} F_{p,q}(s), \quad u \in \mathbf{R}^+,$$

is called the probabilistic diameter of the set A and the set A is probabilistic bounded if and only if

$$\sup_{u\in\mathbf{R}^+} D_A(u) = 1.$$

Definition 2. *Let A be a probabilistic bounded subset of S. The Kuratowski function $\alpha_A(\cdot)$ is defined by:*

$\alpha_A(u) = \sup\{s \mid s > 0$, *there is a finite family* $A_j (j \in J)$ *such that* $A = \cup_{j \in J} A_j$ *and* $D_{A_j}(u) \geq s$, *for every* $j \in J\}$ $(u \in \mathbf{R})$.

The Kuratowski function has the following properties :

1. $\alpha_A \in \Delta$,
2. $\alpha_A(u) \geq D_A(u)$, for every $u \in \mathbf{R}$,
3. $\emptyset \neq A \subset B \subset S \Rightarrow \alpha_A(u) \geq \alpha_B(u)$, for every $u \in \mathbf{R}$,
4. $\alpha_{A \cup B}(u) = \min\{\alpha_A(u), \alpha_B(u)\}$, for every $u \in \mathbf{R}$,
5. $\alpha_A(u) = \alpha_{\bar{A}}(u)$, for every $u \in \mathbf{R}$, where $\overline{A}$ is the closure of A,
6. $\alpha_A = H \Rightarrow A$ is precompact.

In [25] the function $\beta_A(\cdot)$ is defined in the following way:
$\beta_A(u) = \sup\{r \mid r > 0$, there exists a finite subset A_f of S such that $\tilde{F}_{A,A_f}(u) \geq r\}$, where for every probabilistic bounded subsets A and B of S

$$\tilde{F}_{A,B}(u) = \sup_{s<u} \inf_{x \in A} \sup_{y \in B} F_{x,y}(s).$$

The function β has properties 1) - 6) for β instead of α.

If the $T-$norm t is $t_{\min}$ then for every $u \in \mathbf{R}^+$

$$\beta_A(u) \geq \alpha_A(u) \geq \beta_A(\frac{u}{2}).$$

Let (S, F) be a probabilistic metric spaces, K a probabilistic bounded subset of S and T a mapping from K into the family of all nonempty subsets of S. Let for every probabilistic bounded subset A of S, $\gamma_A : \mathbf{R} \to [0,1]$. If $T(K)$ is a probabilistic bounded subset of S and for every $B \subset K$:

$$\gamma_{T(B)}(u) \leq \gamma_B(u), \quad \text{for every } u > 0 \Rightarrow B \text{ is precompact}$$

then we say that the mapping T is densifying on the set K with respect to the functio γ.

3. Continuous dependence of the fixed points on parameters

In the next theorem we suppose that $\gamma_A(\cdot) = \alpha_A(\cdot)$, for every probabilistic bounded subset A of S or $\gamma_A(\cdot) = \beta_A(\cdot)$ for every probabilistic bounded subset A of S.

Theorem 1 in this paper is a generalization of Z.Vorel's result from [26] to random normed spaces.

Theorem 1. *Let (S, F, t) be a complete random normed space, t a continuous $T-$norm, K a nonempty closed and probabilistic bounded subset of $S, T_k : K \to S(k \in \mathrm{N} \cup \{0\})$, x_k a fixed point of the mapping $T_k (k \in \mathrm{N})$ and x_0 the unique fixed point of the mapping T_0. If the mapping T_0 is continuous and densifying on K with respect to the function γ then the following holds:*

$$\lim_{k\to\infty} x_k = x_0 \Longleftrightarrow \lim_{k\to\infty} (T_k - T_0)x_k = 0.$$

Proof. If $\lim_{k\to\infty} x_k = x_0$ then $T_k x_k - T_0 x_k = x_k - x_0 + T_0 x_0 - T_0 x_k$ converges to 0 since T_0 is a continuous mapping. Suppose that $\lim_{k\to\infty}(T_k - T_0)x_k = 0$. We shall prove that an arbitrary subsequence $\{x_{n_k}\}_{k\in\mathrm{N}}$ has a convergent subsequence y_{k_r} with the limit x_0. Let for every $k \in \mathrm{N}, y_k = x_{n_k}$. Then

$$y_k = T_{n_k} y_k = (T_{n_k} - T_0)y_k + T_0 y_k$$

hich implies that $\lim_{k\to\infty} y_k - T_0 y_k = 0$. This means that for every $s > 0$, $\lim_{k\to\infty} F_{y_k - T_0 y_k}(s) = 1$. We shall prove that:

$$\gamma_{\{y_k|k\in\mathrm{N}\}}(u) = \gamma_{\{T_0 y_k|k\in\mathrm{N}\}}(u) \tag{1}$$

for every $u \in \mathrm{R}^+$.

First, we shall prove that:

$$\gamma_{\{y_k|k\in\mathrm{N}\}}(u) \le \gamma_{\{T_0 y_k|k\in\mathrm{N}\}}(u), \quad \text{for every } u \in \mathrm{R}^+. \tag{2}$$

Let $\gamma = \beta$. In order to prove (2) for $\gamma = \beta$ we shall prove that for every $u > 0$ and every $s \in (0, u)$:

$$\beta_{\{y_k|k\in\mathrm{N}\}}(u - s) \le \beta_{\{T_0 y_k|k\in\mathrm{N}\}}(u). \tag{3}$$

Suppose that for some $u > 0$ and $s \in (0, u)$:

$$\beta_{\{y_k | k\in \mathrm{N}\}}(u-s) > 0. \tag{4}$$

If (4) is not satisfied then (3) holds. In order to prove (3) we shall prove the following implication:

$$0 < r < \beta_{\{y_k | k\in \mathrm{N}\}}(u-s) \Rightarrow r \leq \beta_{\{T_0 y_k | k\in \mathrm{N}\}}(u). \tag{5}$$

Let $r < \beta_{\{y_k | k\in N\}}(u-s)$. From the definition of the function $\beta_A(\cdot)$ it follows that there exists a finite set A_f from S such that

$$\tilde{F}_{\{y_k | k\in \mathrm{N}\}, A_f}(u-s) > r.$$

Hence, from the definition of the function $\tilde{F}$ it follows that for every $n \in \mathrm{N}$ there exists $z(n) \in A_f$ so that

$$F_{y_n, z(n)}(u-s) > r.$$

Let δ be an arbitrary element from the interval $(0, r)$. We shall prove that

$$\beta_{\{T_0 y_k | k\in \mathrm{N}\}}(u) \geq r - \delta. \tag{6}$$

Since the mapping t is continuous and $t(1, r) = r$ it follows that there exists $\tilde{\delta}$ so that the following implication holds

$$1 \geq h > 1 - \tilde{\delta} \Rightarrow t(h, r) > r - \delta.$$

Let $n_0(s, \tilde{\delta})$ be such a natural number that for every $k > n_0(s, \tilde{\delta})$

$$F_{y_k - T_0 y_k}\left(\frac{s}{2}\right) > 1 - \tilde{\delta}.$$

Then for every $k > n_0(s, \tilde{\delta})$ we have that

$$F_{T_0 y_k - z(k)}\left(u - \frac{s}{2}\right) \geq t\left(F_{T_0 y_k - y_k}\left(\frac{s}{2}\right), F_{y_k - z(k)}(u-s)\right)$$

$$\geq t\left(F_{T_0 y_k - y_k}\left(\frac{s}{2}\right), r\right) > r - \delta.$$

From the definition of the function $\beta_A(\cdot)$ and the relation

$$\beta_{\{T_0 y_k | k\in \mathrm{N}\}}(u) = \beta_{\{T_0 y_k | k > n_0(s,\tilde{\delta})\}}(u)$$

it follows that (6) is satisfied. Since δ is an arbitrary element from the interval $(0,r)$ we obtain that the right side in (5) holds.

Hence (3) holds. Since the function $\beta_A(\cdot)$ is left continuous we have that

$$\lim_{s\to 0+} \beta_{\{y_k|k\in \mathrm{N}\}}(u-s) = \beta_{\{y_k|k\in \mathrm{N}\}}(u) \leq \beta_{\{T_0 y_k|k\in \mathrm{N}\}}(u)$$

for every $u > 0$.

Similarly we can prove that

$$\beta_{\{y_k|k\in \mathrm{N}\}}(u) > \beta_{\{T_0 y_k|k\in \mathrm{N}\}}(u)$$

and so

$$\beta_{\{y_k|k\in \mathrm{N}\}}(u) = \beta_{\{T_0 y_k|k\in \mathrm{N}\}}(u).$$

Suppose now that $\gamma = \alpha$ and prove that

$$\alpha_{\{y_k|k\in \mathrm{N}\}}(u) = \alpha_{\{T_0 y_k|k\in \mathrm{N}\}}(u)$$

for every $u \in \mathbf{R}^+$.

As in the case $\gamma = \beta$ we shall prove that for every $s \in (0,u)$:

$$\alpha_{\{y_k|k\in \mathrm{N}\}}(u-s) \leq \alpha_{\{T_0 y_k|k\in \mathrm{N}\}}(u).$$

Let $0 < r < \alpha_{\{y_k|k\in N\}}(u-s)$. Then there exists a finite family $\{Y_1, Y_2, ..., Y_n\}$ $(Y_i \subset S,\ i \in \{1,2,...,n\})$ so that $\{y_k \mid k \in \mathrm{N}\} = \cup_{i=1}^n Y_i$ and

$$D_{Y_i}(u-s) > r, \quad \text{for every } i \in \{1,2,...,n\}. \tag{7}$$

From (7) and the definition of $D_A(\cdot)$ we obtain that for every $x, y \in Y_i$, $F_{x-y}(u-s) > r$. Suppose that δ is an arbitrary element from the interval $(0,r)$. The mapping t is continuous and since $t(1,t(r,1)) = r$ it follows that there exists $\tilde{\delta}$ so that the following implication holds

$$1 \geq v,\ w > 1-\tilde{\delta} \Rightarrow t(v,t(r,w)) > r-\delta.$$

For every $j \in \{1,2,...,n\}$ let $Z_j \subset S$ be defined by

$Z_j = \{z \mid z \in S,$ there exists some $y \in Y_j$ so that $F_{z-y}(\frac{s}{4}) > 1-\tilde{\delta}\}$.
Further, let $n_1(s,\tilde{\delta}) \in \mathrm{N}$ so that for every $k > n_1(s,\tilde{\delta})$

$$F_{y_k - T_0 y_k}(\frac{s}{4}) > 1-\tilde{\delta}.$$

Then it is easy to see that $\{T_0 y_k \mid k > n_1(s, \tilde{\delta})\} \subset \cup_{j=1}^{n} Z_j$. In order to prove that

$$r - \delta \leq \alpha_{\{T_0 y_k | k > n_1(s,\tilde{\delta})\}}(u)$$

we shall prove that $D_{Z_j}(u) > r - \delta$, for every $j \in \{1, 2, ..., n\}$.

Let $x, y \in Z_j$. Then there exist $\tilde{x}$ and $\tilde{y}$ from Y_j so that

$$F_{x-\tilde{x}}(\frac{s}{4}) > 1 - \tilde{\delta}, \quad F_{y-\tilde{y}}(\frac{s}{4}) > 1 - \tilde{\delta}.$$

Since $F_{\tilde{x}-\tilde{y}}(u - s) > r$ we have that

$$F_{x-y}(u - \frac{s}{2}) \geq t(F_{x-\tilde{x}}(\frac{s}{4}), t(F_{\tilde{x}-\tilde{y}}(u - s), F_{y-\tilde{y}}(\frac{s}{4})))$$

$$\geq t(F_{x-\tilde{x}}(\frac{s}{4}), t(r, F_{y-\tilde{y}}(\frac{s}{4}))) > r - \delta$$

and so

$$D_{Z_j}(u) = \sup_{s<u} \inf_{x,y\in Z_j} F_{x-y}(u) \geq \inf_{x,y\in Z_j} F_{x-y}(u - \frac{s}{2}) > r - \delta.$$

Similarly as in the case $\gamma = \beta$ it can be prove that

$$\alpha_{\{y_k | k\in \mathrm{N}\}}(u) = \alpha_{\{T_0 y_k | k\in \mathrm{N}\}}(u), \quad \text{for every } u \in \mathbf{R}^+.$$

Since T_0 is densifying on K in respect to γ then $\{y_k \mid k \in \mathrm{N}\}$ is precompact. If it is not the case for some $u \in \mathbf{R}^+$ then

$$\gamma_{\{y_k | k\in \mathrm{N}\}}(u) < \gamma_{\{T_0 y_k | k\in \mathrm{N}\}}(u) = \gamma_{\{y_k | k\in \mathrm{N}\}}(u)$$

whish is a contradiction.

Hence, there exists a subsequence $\{y_{k_r}\}_{r\in N}$ such that

$$\lim_{r\to\infty} y_{k_r} = x \in K \text{ and } x = \lim_{r\to\infty} y_{k_r} = \lim_{r\to\infty} T_{n_{k_r}} y_{k_r}$$

$$= \lim_{r\to\infty} (T_{n_{k_r}} y_{k_r} - T_0 y_{k_r}) + \lim_{r\to\infty} T_0 y_{k_r} = T_0 x.$$

This means that x is a fixed point of the mapping T_0 and since x_0 is the unique fixed point of the mapping T_0 we have that $x = x_0$. Hence $x_0 = \lim_{r\to\infty} y_{k_r}$ and so $\lim_{k\to\infty} x_k = x_0$.

From Theorem 1 we obtain the following corollary.

Corollary 1. *Let (S, F, t) be a complete random normed space with a continuous $t-$norm t, K a nonempty closed and probabilistic bounded subset of $S, T_0 : K \to K$ a probabilistic $k-$contraction, $T_k : K \to S(k \in \mathbf{N})$ and $x_k = T_k x_k (k \in \mathbf{N} \cup \{0\})$. If*

$$\lim_{k \to \infty} (T_k - T_0) x_k = 0$$

then $\lim_{k \to \infty} x_k = x_0$.

Proof. Since T_0 is a probabilistic $k-$contraction and K is closed and probabilistic bounded from Theorem 1 [12] it follows that there exists one and only element $x_0 \in K$ such that $x_0 = T_0 x_0$. Namely, the existence of x_0 follows from Theorem 1 [12] and the uniqueness follows easily. In Theorem 1 [12] it is proved that T_0 is densifying in respect to the function β. Hence, all the conditions of Theorem 2 are satisfied and so $\lim_{k \to \infty} x_k = x_0$.

In order to obtain a generalization of the Krasnoselskii fixed point theorem [27] we shall give the definition of the notion of an admissible subset in a topological vector space which is given in the paper of V.Klee [26].

Definition 3. *Let X be a topological vector space and K a nonempty subset of X. The subset K is said to be admissible if for every compact subset A of K and every neighbourhood U of zero in X there exists a continuous mapping $h : A \to K$ so that the following conditions are satisfied:*

1. *dim $Linh(A) < \infty$, (Lin- linear hull).*
2. *For every $x \in A$, $x - h(x) \in U$.*

If $K = X$ then X is an admissible space.

The notion of the admissibility is very important in the fixed point theory, because by means of this notion many results from the fixed point theory in locally convex topological vector spaces can be generalized to topological vector spaces which are not necessarily locally convex. The admissibility of a class of random normed spaces is investigated in [20].

Further information about the admissibility can be found in the book [19].

The admissibility of the space $S(0,1)$ (the space of all classes of Lebesgues measurable real functions defined on the interval $(0,1)$) is proved by T.Riedrich in [35].

In [21] S.Hahn and K.F.Pötter obtained the following generalization of the well known Tihonov fixed point theorem.

Theorem 2. *Let X be a Hausdorff topological vector space, K a nonempty closed convex and admissible subset of X and $f : K \to K$ a continuous mapping such that $\overline{f(K)}$ is compact. Then there exists $x \in K$ such that $x = fx$.*

This theorem will be used in the proof of the next corollary. Since every nonempty, closed and convex subset of a topological vector space, which is locally convex, is admissible the above fixed point theorem is a generalization of Tihonov's fixed point theorem.

Corollary 2. *Let (S, F, t) be a complete random normed space with a continuous $T-$norm $t, \emptyset \neq K \subset S$ a closed, convex, admissible and probabilistic bounded set, T_1 and T_2 continuous mappings from K into S so that the following conditions are satisfied:*

i) $T_1K + T_2K \subset K$.

ii) T_1 is a probabilistic $k-$contraction.

iii) $\overline{T_2(K)}$ is a compact subset of S.
Then there exists $x \in K$ such that $x = T_1x + T_2x$.

Proof. Since T_1 is a probabilistic $k-$contraction it follows that for every $z \in \overline{T_2K}$ there exists one and only one element $x(z)$ from K such that $x(z) = T_1x(z) + z$. We shall prove that the mapping $z \mapsto x(z)(z \in \overline{T_2K})$ is continuous using Theorems 1 and 2. Suppose that $\{z_k\}_{k\in N}$ is a sequence from $\overline{T_2(K)}$ such that $\lim_{k\to\infty} z_k = z$. We have to prove that $\lim_{k\to\infty} x(z_k) = x(z)$. Let, for every $k \in N : T_kx = Tx + z_k(x \in K)$ and $T_0x = Tx + z$. The mapping T_0 is a probabilistic $k-$contraction and from Theorem 1 [12] it follows that T_0 is densifying on K in respect to the function β. Further, $\lim_{k\to\infty}(T_k - T_0)u = \lim_{k\to\infty} z_k - z = 0$ for every $u \in K$ and hence $\lim_{k\to\infty}(T_k - T_0)x(z_k) = 0$. This implies that all the conditions of Theorem 1 are satisfied and so $\lim_{k\to\infty} x(z_k) = x(z)$. Let $fu = x(T_2u)$, for every $u \in K$. The mapping $f : K \to K$ satisfies the conditions of the Theorem of Hahn and Pötter and so there exists $x \in K$ such that $x = fx$. It is easy to see that $x = T_1x + T_2x$.

References

[1] Artstein, Z.: Continuous dependence of solutions of operator equations, I. Trans. Amer. Math. Soc. 231 (1977), 143-166.

[2] Bocsan, Gh.: Some applications of functions of Kuratowski, Sem. Teor. Funct. si. Mat. Apl. , Timisoara, RS Romania, No 5, 1973.

[3] Bocsan, Gh.: On the Kuratowski function in random normed spaces, Sem. Teor. Funct. si. Mat. Apl., Timisoara, RS Romania, No 8, 1974.

[4] Bocsan, Gh.: On some fixed point theorems in random normed spaces, Sem. Teor. Funct. si. Mat. Apl. , Timisoara, RS Romania, No 13, 1974.

[5] Bocsan, Gh.: On some fixed point theorems in probabilistic metric spaces, Sem. Teor. Funct. si. Mat. Apl. , Timisoara, RS Romania, No 24, 1974.

[6] Bocsan, Gh., Constantin, Gh.: The Kuratowski function and some applications to the probabilistic metric spaces, Sem. Teor. Funct. si. Mat. Apl., Timisoara, RS Romania , No 1, 1973.

[7] Cain, G.L., Kasriel, R.H.: Fixed and periodic points of local contraction mappings on probabilistic metric spaces, Math. Systems Theory, Vol. 9, No. 4 (1976), 289-297.

[8] Cheng, M: On certain condensing operators and the behaviour of their fixed points with respect to parameters, J. Math. Anal. Appl. 64 (1978), 505-517.

[9] Hadžić, O.: Fixed point theorems for multivalued mappings in probabilistic metric spaces, Mat. vesnik 3(16)(31), (1979), 125-133.

[10] Hadžić, O.: Some theorems on the fixed point in probabilistic metric and random normed spaces, Boll. Unione Mat. Ital., (6) , 1-B (1982), 381-391.

[11] Hadžić, O., Nikolić - Despotović, D.: A proof of the admissibility of a class of random normed spaces, Mat. vesnik 3 (16) (31) (1979), 267-271.

[12] Hadžić, O.: Fixed point theorems for multivalued mappings in some classes of fuzzy metric spaces, Fuzzy Sets and Systems 29,1(1989), 115-125.

[13] Hahn, S., Pötter, F.: Über Fixpunkte kompakter Abbildungen in topologischen Vektorräunem, Studia Math. L (1974), 1-16.

[14] Hale, J.: Continuous dependence of fixed points of condensing maps, J. Math. Anal. Appl. 46 (1974), 388-393.

[15] Klee, V.: Leray - Schauder theory without local convexity, Math. Ann. 141 (1960), 286-296.

[16] Krasnoselskiĭ, M.: Two observations about the method of succesive approximations, U. M. N. 10, B. 1 (63) (1955), 123-127.

[17] Menger, K.: Statistical metric, Proc. Nat. Acad. Sci. , USA 28 (1942), 535-537.

[18] Radu, V.: On some fixed point theorems in probabilistic metric spaces, Sem. Teor. Prob. Apl. , Timisoara, RS Romania, No 74, 1984.

[19] Riedrich, T.: Der Raum $S(0,1)$ ist zulässig, Wiss. Z. Technische Universität Dresden , 13 (1964), 1-6.

[20] Schweizer,4B., Sklar, A.: Statistical metric spaces, North - Holland Series in Probability and Applied Mathematics 5, 1983.

[21] Sehgal, V., Bhauicha - Reid, A.: Fixed points of contraction mappings on probabilistic metric spaces, Math. System Theory 6 (1972), 97-102.

[22] Sherstnev, A.N.: The notion of random normed spaces, DAN USSR 149 (2) (1963), 280-283.

[23] Sherwood, H.: Complete probabilistic metric spaaces, Z. Wahrscheinlichkeitstheorie Verw. Geb. 63 (1983), 463-474.

[24] Shih - Sen Chang, On some fixed point theorems in probabilistic metric spaces with applications, Z. Wahrscheinlichkeitstheorie Verw. Geb. 63 (1983), 463-474.

[25] Tan, D.H.: On probabilistic condensing mappings, Rev. Roum. Math. Pures Appl. 26, 10 (1981), 1305-1317.

[26] Vorel, Z.: Necessary and sufficient conditions for continuous dependence of fixed points of $\alpha-$ condensing maps, in the book: Nonlinear Phenomena in Mathematical Statistics, Ed. V.Lakshmikantham, Academic Press, 1982, 987-989.

REZIME

NEPREKIDNA ZAVISNOST NEPOKRETNE TAČKE OD PARAMETARA U SLUČAJNIM NORMIRANIM PROSTORIMA

Neprekidna zavisnost nepokretne tačke od parametara je ispitivana u mnogim radovima ([1], [8], [14], [26]). U ovom radu je dato uopštenje Vorelovog rezultata iz [25] na slučajne normirane prostore. Kao posledica dobijeno je uopštenje poznate teoreme o nepokretnoj tački Krasnoseljskog u slučajnim normiranim prostorima.

Received by the editors March 16, 1989.