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RANDOM CONTRACTOR DIRECTIONS AND SOLUTION FOR A SYSTEM OF SET - VALUED RANDOM OPERATOR EQUATIONS WITH STOCHASTIC DOMAIN

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Abstract

In this paper, the concept of random contractor directions is introduced in order to study the solvability of a system of nonlinear set - valued random operator equations with stohastic domain.

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1. Introduction

Recently, Reddy and Subrahmanyam [13], [14], [15] studied the relationship between Altman's theory of contractors and directional contractors and Matkowski's fixed point theorem and unified their apparently disconected theorems.

As a random generalization of Altman's contractor theory, Lee and Padgett [8], [9], [12] considered the concept of random contractors and its applications to the solution of nonlinear random operator equations. In [16], [17],

[18], we have made a new contribution to the theory of random contractors and random directional contractors and offered some existence theorems of random solution for a system of point-valued and/or set-valued random operator equations.

Engl [5], [6] and Ding [19] obtained some general random fixed point theorems for continuous random operators with stochastic domain and given some applications to random integral and differential equations.

In this paper, as random generalization of Altman's contractor directions, the concept of random contractor directions is introduced in order to study the solvability of a system of nonlinear set-valued random operator equations with stochastic domain. Next, using the new concept and transfinite induction we prove several existence theorems to a system of nonlinear set-valued random operator equations with stochastic domain. As consequence, the solvability theorems to a system of nonlinear point-valued random operator equations with stochastic domain can be obtained. These theorems improve and extend some results of Altman [1], [2], [3], Matkowski [10], [11], and Reddy and Subrahmanyam [15]. Finally some applications of our results will be given to a system of nonlinear random integral and differential equations.

2. Preliminaries

Let $(\Omega, \mathcal{A}, P)$ be a complete probability measure space, and let (X_i, d_1), $i = 1, \ldots, n$ be Polish spaces, i. e. separable complete metric spaces. Let $(Y_i, \|\cdot\|_i)$, $i = 1, \ldots, n$ be separable Banach spaces. $CL(Y_i)$ and $CL(X_i)$ denote the families of all nonempty closed subsets of Y_i and X_i, respectively. The function $H_i(\cdot,\cdot)$ denotes the generalized Hausdorff distance for $CL(Y_i)$ induced by the norm of Y_i and for $y \in Y_i$, $A \subset Y_i$, $D_i(y, A) = \inf\{\|y - z\| : z \in A\}$. $\mathcal{B}_i$ and $\mathcal{C}_i$ are σ-algebras of subsets of X_i and Y_i respectively.

Definition 2.1. *A mapping $E_i : \Omega \to CL(X_i)$ is said to be a random subset of X_i if for each $B \in \mathcal{B}_i$,*

$$E_i^{-1}(B) = \{\omega \in \Omega : E_i(\omega) \cap B \neq \emptyset\} \in \mathcal{A}.$$

A mapping

$$E : \Omega \to CL(X_1) \times \ldots \times CL(X_n)$$

is said to be a random subset of $X = X_1 \times \ldots \times X_n$, *if for each* $\omega \in \Omega$, $E(\omega) = E_1(\omega) \times \ldots \times E_n(\omega)$ *where* $E_i : \Omega \to CL(X_i)$ *is a random subset of* X_i, $i = 1, \ldots, n$. *A function* $x_i : \Omega \to X_i$ *is said to be an* X_i*-valued random variable if for each* $B \in \mathcal{B}_i$,

$$x_i^{-1}(B) = \{\omega \in \Omega : \ x_i(\omega) \in B\} \in \mathcal{A}.$$

We denote the graph of the random set $E = E_1 \times \ldots \times E_n$ by

$$G_r(E) = \{(\omega, x_1, \ldots, x_n) \in \Omega \times X_1 \times \ldots \times X_n : \ x_i \in E_i(\omega), \ i = 1, \ldots, n\}.$$

Definition 2.2. *A mapping* $T : G_r(E) \to CL(Y_i)$ *is said to be set-valued random operator with stochastic domain* E, *if for all* $x_i \in X_i$, $i = 1, \ldots, n$ *and for all* $c \in C_i$,

$$\{\omega \in \Omega : \ x_i \in E_i(\omega), \ i = 1, \ldots, n, \ \textit{and} \ T(\omega, x_1, \ldots, x_n) \cap C \neq \emptyset\} \in \mathcal{A}.$$

T will be called almost surely (a.s.) closed if

$$x_i^m(\omega) \in E_i(\omega) \text{ a.s.}, \ i = 1, \ldots, n, \ m = 1, 2, \ldots;$$

$$x_i^m(\omega) \to x_i^0 \text{ a.s.};$$

$$y^m(\omega) \in T(\omega, x_1^m(\omega), \ldots, x_n^m(\omega)) \text{ a.s.}, \ m = 1, \ldots,$$

$$\text{and } y^m(\omega) \to y^0(\omega) \text{ a.s.}$$

imply

$$(x_1^0(\omega), \ldots, x_n^0(\omega)) \in E(\omega) \text{ a.s., and } y^0(\omega) \in T(\omega, x_1^0(\omega), \ldots, x_n^0(\omega)) \text{ a.s.}$$

T will be called a.s. continuous if

$$x_i^m(\omega) \in E_i(\omega) \text{ a.s.}, \ i = 1, \ldots, n, \ m = 1, 2, \ldots;$$

$$\text{and } x_i^m(\omega) \to x_i^0(\omega) \text{ a.s.}, \ i = 1, \ldots, n$$

imply

$$T(\omega, x_1^m(\omega), \ldots, x_n^m(\omega)) \to T(\omega, x_1^0(\omega), \ldots, x_n^0(\omega)) \text{ a.s.}$$

under the Hausdorff metric H_i.

Remark 2.1. *Obviously,* T *is a.s. closed whenever* T *is a.s. continuouns.*

Definition 2.3. *Let $E_i : \Omega \to CL(X_i)$ be a random subset of X_i. A mapping $x_i : \Omega \to X_i$ will be said to be a measurable selection of E_i if $x_i(\omega)$ is an X_i-valued random variable and $x_i(\omega) \in E_i(\omega)$, $\forall \omega \in \Omega$.*

Lemma 2.1. *([20]) Let $T : G_r(E) \to CL(Y_i)$ be an a.s. continuous set-valued random operator. Then for all X_i-valued random variable $x_i(\omega) \in E_i(\omega)$ a.s. $i = 1, \ldots, n$, $T(\omega, x_1(\omega), \ldots, x_n(\omega))$ is a random subset of Y_i.*

Lemma 2.2. *([20]) Let $S, T : \Omega \to CL(Y_i)$ be random subsets of Y_i and let $u : \Omega \to Y_i$ be a measurable selection of S. Then for each real-valued random variable $k : \Omega \to (1, \infty)$, there exists a measurable selection $v : \Omega \to Y_i$ of T such that*

$$\|u(\omega) - v(\omega)\| \leq k(\omega) H_i(S(\omega), T(\omega)).$$

Let $(a_{i,j}(\omega))$ be an $n \times n$ matrix where $a_{i,j} : \Omega \to [0, \infty)$, $i, j = 1, \ldots, n$ are real-valued random variables.

Define

$$a^1_{i,j}(\omega) = \begin{cases} a_{i,j}(\omega), & i \neq j \\ 1 - a_{i,j}(\omega), & i = j,\ i, j = 1, \ldots, n, \end{cases} \tag{2.1}$$

$$a^{l+1}_{i,j}(\omega) = \begin{cases} a^l_{1,1}(\omega) a^l_{i+1,j+1}(\omega) + a^l_{i+1,1}(\omega) a^l_{1,j+1}(\omega), & i \neq j \\ a^l_{1,1}(\omega) a^l_{i+1,j+1}(\omega) - a^l_{i+1,1}(\omega) a^l_{1,j+1}(\omega), & i = j, \end{cases} \tag{2.2}$$

$$l = 1, \ldots, n-1,\ i, j = 1, \ldots, n-l.$$

Lemma 2.3. *([16], [20]) Let $a_{i,j}(\omega)$, $i, j = 1, \ldots, n$, be nonnegative real-valued random variables. Then there exist positive real-valued random variables $r_i(\omega)$, $i = 1, \ldots, n$, such that*

$$\sum_{i=1}^{n} a_{i,j}(\omega) r_j(\omega) < r_i(\omega) \text{ a.s., } i = 1, \ldots, n, \tag{2.3}$$

if and only if

$$a^l_{i,j}(\omega) > 0 \text{ a.s., } i = 1, \ldots, n+1-l,\ l = 1, \ldots, n. \tag{2.4}$$

Suppose $(r_1(\omega), \ldots, r_n(\omega))$ is a random positive solution of the system of random inequalities (2.3).

Defining

$$q(\omega) = \max_{1 \leq i \leq n} \{r_i^{-1}(\omega) \sum_{j=1}^{n} a_{i,j}(\omega) r_j(\omega)\},$$

we have

$$\sum_{j=1}^{n} a_{i,j}(\omega) r_j(\omega) \leq q(\omega) r_i(\omega) \text{ a.s, } i = 1, \ldots, n, \tag{2.5}$$

and

$$0 < q(\omega) < 1 \text{ a.s.} \tag{2.6}$$

Lemma 2.4. *([14]) Let α be an ordinal number of first or second class and let $\{t_\nu\}_{0 \leq \nu \leq \alpha}$ be naturally well-ordered sequence of real numbers provided that for numbers β of second kind we have*

$$t_\beta = \lim_{\nu \to \beta} t_\nu.$$

Then the following equality holds

$$t_\alpha = t_0 + \sum_{0 \leq \nu < \alpha} (t_{\nu+1} - t_\nu).$$

Lemma 2.5. *([14]) Let α be an ordinal number of first or second class and let $\{x_\nu\}_{0 \leq \nu \leq \alpha}$ be naturally well-ordered sequence of a metric space X provided that for numbers β of second kind we have*

$$x_\beta = \lim_{\nu \to \beta} x_\nu.$$

Then denote by $d(\cdot, \cdot)$ the distance

$$d(x_\alpha, x_0) \leq \sum_{0 \leq \nu < \alpha} d(t_{\nu+1}, t_\nu).$$

Definition 2.4. *Let $T_i : G_r(E) \to CL(Y_i)$, $i = 1, \ldots, n$ be random operators with stochastic domain E, and let $y_i(\omega)$, $i = 1, \ldots, n$, be Y_i-valued random variables. Any X_i-valued random variables $x_i(\omega)$, $i = 1, \ldots, n$ which satisfy the following conditions*

$$x_i(\omega) \in E(\omega) \text{ a.s., } i = 1, \ldots, n,$$

and

$$y_i(\omega) \in T(\omega, x_1(\omega), \ldots, x_n(\omega)) \text{ a.s., } i = 1, \ldots, n$$

will be said to be a random solution for a system of nonlinear setvalued random operators

$$(2.7) \qquad y_i(\omega) \in T(\omega, x_1(\omega), \ldots, x_n(\omega)) \text{ a.s., } i = 1, \ldots, n.$$

3. Random contractor directions

In this section we will introduce the concept of random contractor directions. Using the new concept, we consider the solvability of a system of nonlinear set-valued random operators with stochastic domain.

Definition 3.1. *Let $T_i : G_r(E) \to CL(Y_i)$, $i = 1, \ldots, n$, be a.s. continuous set-valued random operators, and $x_i(\omega) \in E_i(\omega)$, $\forall \omega \in \Omega$, be X_i-valued random variable, $i = 1, \ldots, n$ (by Theorem III.30 of [4], such $x_i(\omega)$, $i = 1, \ldots, n$, exist). Then we define the sets $\Gamma_{x(\omega)}(T)$ of random contractor directions for $T = (T_1, \ldots, T_n)$ at $x(\omega) = (x_1(\omega), \ldots, x_n(\omega))$ as follows: for each $y(\omega) = (y_1(\omega), \ldots, y_n(\omega)) \in \Gamma_{x(\omega)}(T)$ where $y_i(\omega)$ is Y_i-valued random variable, $i = 1, \ldots, n$, there exist a positive real-valued random variable $\varepsilon(\omega) = \varepsilon(\omega, x_1(\omega), \ldots, x_n(\omega); y_1(\omega), \ldots, y_n(\omega)) \le 1$ a.s. and X_i-valued random variable $\bar{x}_i(\omega) \in E_i(\omega)$, $\forall \omega \in \Omega$, $i = 1, \ldots, n$, such that*

$$(3.1) \quad H_i(T_i(\omega, \bar{x}_1(\omega), \ldots, \bar{x}_n(\omega)), T_i(\omega, x_1(\omega), \ldots, x_n(\omega)) + \varepsilon(\omega) y_i(\omega))$$

$$\le \varepsilon(\omega)\Big(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j(\omega)\|_j + \sum_{j=1}^{n} c_{i,j}(\omega) D_j(\theta_j, T_i(\omega, x_1(\omega), \ldots, x_n(\omega)))\Big)$$

$$\text{a.s., } i = 1, \ldots, n$$

and

$$(3.1)' \qquad d_i(x_i(\omega), \bar{x}_i(\omega)) \le \varepsilon(\omega) B_i(\omega, \|y_i(\omega)\|_i) \text{ a.s., } i = 1, \ldots, n,$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$, $i, j = 1, \ldots, n$, are nonnegative real-valued random variables and $a_{i,j}(\omega) = b_{i,j}(\omega) + c_{i,j}(\omega)$, $i.j = 1, \ldots, n$, are such that the system (2.3) has random positive solution; $B_i : \Omega \times [0, \infty) \to [0, \infty)$ is such that for each $t \in [0, \infty)$, $B_1(\cdot, t)$ is a real-valued random variable and for each

$\omega \in \Omega$, $B_i(\omega,\cdot)$ is a continuous increasing function satisfying $B_i(\omega,0)=0$, $B_i(\omega,t)>0$, $\forall t>0$, and

$$\int_0^a s^{-1}B_i(\omega,s)ds<\infty \text{ a.s., } j=1,\dots,n, \tag{3.2}$$

for $a>0$, and θ_i is zero element of Y_i, $i=1,\dots,n$.

Theorem 3.1. *Let $T_i : G_r(E)\to CL(Y_i)$, $i=1,\dots,n$ be a.s. continuous set-valued random operators with stochastic domain E. If for X_i-valued random variables $x_i(\omega)\in E_i(\omega)$, $\forall\omega\in\Omega$, $i=1,\dots,n$,*

$$-(T_1(\omega,x_1(\omega),\dots,x_n(\omega)),\dots,T_n(\omega,x_1(\omega),\dots,x_n(\omega)))\subset\Gamma_{x(\omega)}(T),$$

then the system of nonlinear set-valued random operator equations

$$\theta_i\in T_i(\omega,x_1,\dots,x_n),\ i=1,\dots,n \tag{3.3}$$

has a random solution $(x_1(\omega),\dots,x_n(\omega))\in E(\omega)$ a.s.

Proof. Since E is a random subset of $X=X_1\times\ldots\times X_n$, by Theorem III.30 of [4], there exists a measurable selection $x_i^0(\omega)$ of E_i, $i=1,\dots,n$. Now we construct well-ordered sequences of real-valued random variables $t_\alpha(\omega)$, X_i-valued random variables $x_i^\alpha\in E_i(\omega)$ and Y_i-valued random variables y_i^α, $i=1,\dots,n$, $\alpha\in\Lambda$, where Λ is the set of all ordinal numbers less than the first uncountable ordinal. Let $t_0(\omega)=0$ a.s. and $x_i^0(\omega)$ is a measurable selection of E_i, $i=1,\dots,n$. It follows from Lemma 2.1 that $T_i(\omega,x_1(\omega),\dots,x_n(\omega))$ is a random subset of Y_i, $i=1,\dots,n$. By Theorem III.30 of [4], there exists Y_i-valued random variable $y_i^0(\omega)\in T_i(\omega,x_1(\omega),\dots,x_n(\omega))$, $\forall\omega\in\Omega$, $i=1,\dots,n$. since the set of random positive solutions to the system (2.3) is closed with respect to multiplication by positive scalars, without less generality, we can assume that

$$\|y_i^0(\omega)\|_i\le r_i(\omega) \text{ a.s., } i=1,\dots,n, \tag{3.4}$$

where $(r_1(\omega),\dots,r_n(\omega))$ is a random positive solution of the system (2.3).

Suppose that $t_\gamma(\omega)$, $x_i^\gamma(\omega)$ and $y_i^\gamma(\omega)\in T_i(\omega,x_1^\gamma(\omega),\dots,x_n^\gamma(\omega))$, $\forall\omega\in\Omega$, $i=1,\dots,n$ have been constructed for all ordinal numbers $\gamma<\alpha$ satisfying the following conditions:

For arbitrary ordinal number $\gamma < \alpha$ inequalities (3.5_γ) are satisfied

$$(3.5_\gamma) \qquad \|y_i^\gamma(\omega)\|_i \leq e^{-(1-q(\omega))t_\gamma(\omega)} r_i(\omega) \text{ a.s.}, \; i = 1, \dots, n$$

For the first kind ordinal numbers $\gamma + 1 < \alpha$ the following inequalities hold:

$$(3.6_{\gamma+1}) \qquad \|y_i^{\gamma+1}(\omega) - y_i^\gamma(\omega)\|_i \leq 2e^{-(1-q(\omega))t_\gamma(\omega)} \cdot (t_{\gamma+1}(\omega) - t_\gamma(\omega)) r_i(\omega) \text{ a.s.}, \; i = 1, \dots, n$$

$$(3.7_{\gamma+1}) \qquad d_i(x_i^{\gamma+1}(\omega), x_i^\gamma(\omega)) \leq B_i(\omega, r_i(\omega), e^{-(1-q(\omega))t_\gamma(\omega)}) \cdot (t_{\gamma+1}(\omega) - t_\gamma(\omega)) \text{ a.s.}, \; i = 1, \dots, n$$

$$(3.8_{\gamma+1}) \qquad 0 < (t_{\gamma+1}(\omega) - t_\gamma(\omega)) \leq 1$$

For the second kind ordinal numbers $\gamma < \alpha$ the following relations hold:

$$(3.9_\gamma) \qquad t_\gamma(\omega) = \lim_{\beta \to \gamma} t_\beta(\omega); \; x_i^\gamma(\omega) = \lim_{\beta \to \gamma} x_i^\beta(\omega)$$

$$\text{and } y_i^\gamma(\omega) = \lim_{\beta \to \gamma} y_i^\beta(\omega) \text{ a.s.}, \; i = 1, \dots, n.$$

Then it follows from (3.7), (3.9), Lemma 2.4 and 2.5 that for arbitrary $\lambda < \gamma < \alpha$, we have

$$d_i(x_i^\gamma(\omega), x_i^\lambda(\omega)) \leq \sum_{\lambda \leq \beta < \gamma} d_i(x_i^{\beta+1}(\omega), x_i^\beta(\omega))$$

$$\leq \sum_{\lambda \leq \beta < \gamma} B_i(\omega, r_i(\omega) e^{-(1-q(\omega))t_\beta(\omega)})(t_{\beta+1}(\omega) - t_\beta(\omega))$$

$$= \sum_{\lambda \leq \beta < \gamma} B_i(\omega, r_i(\omega) e^{(1-q(\omega))(t_{\beta+1}(\omega) - t_\beta(\omega))} \cdot e^{-(1-q(\omega))t_{\beta+1}(\omega)})(t_{\beta+1}(\omega) - t_\beta(\omega))$$

$$< \sum_{\lambda \leq \beta < \gamma} B_i(\omega, r_i(\omega) e^{(1-q(\omega))} \cdot e^{-(1-q(\omega))t_{\beta+1}(\omega)})(t_{\beta+1}(\omega) - t_\beta(\omega))$$

$$< \sum_{\lambda \le \beta < \gamma} \int_{t_\beta(\omega)}^{t_{\beta+1}(\omega)} B_i(\omega, r_i(\omega) e^{(1-q(\omega))} e^{-(1-q(\omega))t}) dt$$

$$= \int_{t_\lambda(\omega)}^{t_\gamma(\omega)} B_i(\omega, r_i(\omega) e^{(1-q(\omega))} e^{-(1-q(\omega))t}) dt \text{ a.s., } i = 1, \ldots, n.$$

Hence, we obtain the following estimate

$$d_i(x_i^\gamma(\omega), x_i^\lambda(\omega)) \le \int_{t_\lambda(\omega)}^{t_\gamma(\omega)} B_i(\omega, r_i(\omega) \cdot e^{(1-q(\omega)} e^{-(1-q(\omega))t}) dt \text{ a.s., } i = 1, \ldots, n. \tag{3.10}$$

In the same way we obtain from (3.6), (3.8), (3.9), Lemma 2.4 and 2.5 that

$$\|y_i^\gamma(\omega) - y_i^\lambda(\omega)\|_i \le 2e^{1-q(\omega)} r_i(\omega) \cdot \int_{t_\lambda(\omega)}^{t_\gamma(\omega)} e^{-(1-q(\omega))t} dt \text{ a.s., } i = 1, \ldots, n. \tag{3.11}$$

Suppose that α is a firs kind number. If $\theta_i \in T_i(\omega, x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega))$ a.s., $i = 1, \ldots, n$, then the proof of the theorem is complete. If $\theta_i \in T_i(\omega, x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega))$ a.s. doesn't hold,then it follows from Theorem III.30 of [4] that there exist Y_i-valued random variables

$$y_i^{\alpha-1}(\omega) \in T_i(\omega, x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega)),$$

$\forall \omega \in \Omega$, $i = 1, \ldots, n$. By the hypothesis of the theorem, there exist a positive real-valued random variable

$$\varepsilon(\omega) = \varepsilon(\omega, x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega); -y_1^{\alpha-1}(\omega), \ldots, -y_n^{\alpha-1}(\omega)) \le 1$$

and X_i-valued random variables $\bar{x}_i(\omega) \in E_i(\omega)$, $\forall \omega \in \Omega$, $i = 1, \ldots, n$ such that (3.1) holds.

Define

$$t_\alpha(\omega) = t_{\alpha-1}(\omega) + \tau_\alpha(\omega), \quad x_i^\alpha(\omega) = \bar{x}_i(\omega), \; i = 1, \ldots, n, \tag{3.11'}$$

where $\tau_\alpha(\omega) = \varepsilon(\omega)$. Hence (3.8_α) holds.

Replacing $x_i(\omega)$ by $x_i^{\alpha-1}(\omega)$, $y_i(\omega)$ by $-y_i^{\alpha-1}(\omega)$ in (3.1), we get

$$(3.12)\qquad H_i(T_i(\omega,x_1^\alpha(\omega),\ldots,x_n^\alpha(\omega)),T_i(\omega,x_1^{\alpha-1}(\omega),\ldots,x_n^{\alpha-1}(\omega))$$

$$-\tau_\alpha(\omega)y_i^{\alpha-1}(\omega))\le\tau_\alpha(\omega)(\sum_{j=1}^n b_{i,j}(\omega)\|y_j^{\alpha-1}(\omega)\|_j$$

$$+\sum_{j=1}^n c_{i,j}(\omega)D_j(\theta_j,T_j(\omega,x_1^{\alpha-1}(\omega),\ldots,x_n^{\alpha-1}(\omega))))\text{ a.s, } i=1,\ldots,n.$$

Since $(1-\tau_\alpha(\omega))y_i^{\alpha-1}(\omega)$ is a measurable selection of

$$T_i(\omega,x_1^{\alpha-1}(\omega),\ldots,x_n^{\alpha-1}(\omega))-\tau_\alpha(\omega)y_i^{\alpha-1}(\omega),$$

it follows from Lemma 2.2 that for each real-valued random variable $k:\Omega\to(1,\infty)$ there exists a measurable selection $y_i^\alpha(\omega)$ of $T_i(\omega,x_1^\alpha(\omega),\ldots,x_n^\alpha(\omega))$, $i=1,\ldots,n$, such that

$$(3.13)\qquad \|y_i^\alpha(\omega)-(1-\tau_\alpha(\omega))y_i^{\alpha-1}(\omega)\|_i$$

$$\le k(\omega)H_i(T_i(\omega,x_1^\alpha(\omega),\ldots,x_n^\alpha(\omega)),$$

$$T_i(\omega,x_1^{\alpha-1}(\omega),\ldots,x_n^{\alpha-1}(\omega))-\tau_\alpha(\omega)y_i^{\alpha-1}(\omega)),\ i=1,\ldots,n.$$

From (3.12), (3.13), $(3.5_{\alpha-1})$ and (2.5) we have

$$(3.14)\qquad \|y_i^\alpha(\omega)-(1-\tau_\alpha(\omega)y_i^{\alpha-1}(\omega)\|_i$$

$$\le k(\omega)\tau_\alpha(\omega)(\sum_{j=1}^n b_{i,j}(\omega)\|y_j^{\alpha-1}(\omega)\|_j+\sum_{j=1}^n c_{i,j}(\omega)\|y_j^{\alpha-1}(\omega)\|_j)$$

$$\le k(\omega)\tau_\alpha(\omega)\sum_{j=1}^n a_{i,j}(\omega)e^{-(1-q(\omega))t_{\alpha-1}(\omega)}r_j(\omega)$$

$$\le k(\omega)\tau_\alpha(\omega)e^{-(1-q(\omega))t_{\alpha-1}(\omega)}q(\omega)r_i(\omega)\text{ a.s., } i=1,\ldots,n.$$

Therefore

$$(3.15)\qquad \|y_i^\alpha(\omega)\|_i\le(1-\tau_\alpha(\omega))e^{-(1-q(\omega))t_{\alpha-1}(\omega)}r_i(\omega)$$

$$+k(\omega)\tau_\alpha(\omega)e^{-(1-q(\omega))t_{\alpha-1}(\omega)}q(\omega)r_i(\omega)$$

$$\le(1-(1-k(\omega)q(\omega))\tau_\alpha(\omega))e^{-(1-q(\omega))t_{\alpha-1}(\omega)}r_i(\omega)$$

$$\leq (1-(1-k(\omega)q(\omega))\tau_\alpha(\omega))e^{(1-q(\omega))\tau_{\alpha-1}(\omega)}$$
$$\cdot e^{-(1-q(\omega))t_{\alpha-1}(\omega)}r_i(\omega) \text{ a.s., } i=1,\dots,n.$$

From (3.14) we also have

$$\|y_i^\alpha(\omega)-y_i^{\alpha-1}(\omega)\|_i \leq \tau_\alpha(\omega)e^{-(1-q(\omega))t_{\alpha-1}(\omega)}r_i(\omega) \tag{3.16}$$
$$+k(\omega)\tau_\alpha(\omega)e^{-(1-q(\omega))t_{\alpha-1}(\omega)}q(\omega)r_i(\omega)$$
$$\leq (1+k(\omega)q(\omega))\tau_\alpha(\omega)e^{-(1-q(\omega))t_{\alpha-1}(\omega)}r_i(\omega) \text{ a.s., } i=1,\dots,n.$$

Now we choose the real-valued random variable $k:\Omega\to(1,\infty)$ as follows

$$k(\omega)=\min\{\frac{1}{q(\omega)},1+\frac{(1-q(\omega))^2\tau_\alpha(\omega)^2}{q(\omega)\tau_\alpha(\omega)}(\frac{1}{2}-\frac{(1-q(\omega))\tau_\alpha(\omega)}{6}).$$

With the choice of $k(\omega)$, we have from (3.15) and (3.16),

$$\|y_i^\alpha(\omega)\|_i \leq e^{-(1-q(\omega))t_\alpha(\omega)}r_i(\omega) \text{ a.s, } i=1,\dots,n \tag{3.5_α}$$

$$\|y_i^\alpha(\omega)-y_i^{\alpha-1}(\omega)\|_i \leq 2e^{-(1-q(\omega))t_{\alpha-1}(\omega)} \tag{3.6_α}$$
$$\cdot(t_\alpha(\omega)-t_{\alpha-1}(\omega))r_i(\omega) \text{ a.s, } i=1,\dots,n$$

It follows from (3.1)', (3.11)' and $(3.5_{\alpha-1})$ that

$$d_i(x_i^\alpha(\omega),x_i^{\alpha-1}(\omega)) \leq \tau_\alpha(\omega)B_i(\omega,\|y_i^{\alpha-1}(\omega)\|_i) \tag{3.7_α}$$
$$\leq B(\omega,e^{-(1-q(\omega))t_{\alpha-1}(\omega)}r_j(\omega))$$
$$\cdot(t_\alpha(\omega)-t_{\alpha-1}(\omega)) \text{ a.s, } i=1,\dots,n.$$

Thus, relations (3.5_α) - (3.8_α) are satisfied for the first kind ordinal number α. Now suppose that α is an ordinal number of second kind. Put $t_\alpha(\omega)=\lim_{\gamma\to\alpha}t_\gamma(\omega)$. Let γ_n be an increasing sequence converging to α. It follows from (3.10) and (3.11) that $\{x_i^{\gamma_n}(\omega)\}$ and $\{y_i^{\gamma_n}(\omega)\}$ are a.s. Cauchy sequences and so are $\{x_i^\gamma(\omega)\}$ and $\{y_i^\gamma(\omega)\}$. Denote by $x_i^\alpha(\omega)$ and $y_i^\alpha(\omega)$ their limits. As the limits of sequences of random variables, $x_i^\alpha(\omega)$ and $y_i^\alpha(\omega)$ are X_i-valued and Y_i-valued random variables, respectively. Since $x_i^\gamma(\omega)\in E_i(\omega)$, $i=1,\dots,n$ and T_i, $i=1,\dots,n$ are a.s. continuous it follows that $x_i^\alpha(\omega)\in E_i(\omega)$ a.s. $i=1,\dots,n$, and $y_i^\alpha(\omega)\in T_i(\omega,x_1^\alpha(\omega),\dots,x_n^\alpha(\omega))$ a.s. $i=1,\dots,n$. If $t_\alpha(\omega)<\infty$, then passage in (3.5_γ) yields (3.5_α). The relations (3.9_α) are satisfied by the definitions of $t_\alpha(\omega)$, $x_i^\alpha(\omega)$ and $y_i^\alpha(\omega)$, $i=1,\dots,n$. This process will terminate if $t_\alpha(\omega) = \infty$ a.s., where α is an ordinal number of second kind. By (3.5_α), we have $\theta_i \in T_i(\omega, x_1^\alpha(\omega), \ldots, x_n^\alpha(\omega))$ a.s. $i = 1, \ldots, n$. The limits $x_i^\alpha(\omega) \in E_i(\omega)$ a.s., $i = 1, \ldots, n$, exist, by (3.10), since

$$\int_0^\infty B_i(\omega, r_i(\omega)e^{(1-q(\omega))}e^{-(1-q(\omega))t})dt$$

$$= (1-q(\omega))^{-1}\int_0^{a_i(\omega)} s \cdot B_i(\omega, s)ds < \infty \text{ a.s., } i = 1, \ldots, n,$$

where $a_i(\omega) = r_i(\omega)e^{(1-q(\omega))}$. This completes the proof of the theorem. □

Definition 3.2. *Let $T_i : G_r(E) \to Y_i$, $i = 1, \ldots, n$ be a.s. continuous point-valued random operators, and let $x_i(\omega)$ be a measurable selection of E_i, $i = 1, \ldots, n$. Then we define sets $\Gamma_{x(\omega)}(T)$ of random contractor directions for $T = (T_1, \ldots, T_n)$ at $x(\omega) = (x_1(\omega), \ldots, x_n(\omega))$ as follows: for each $y(\omega) = (y_1(\omega), \ldots, y_n(\omega)) \in \Gamma_{x(\omega)}(T)$ where $y_i(\omega)$ is an Y_i-valued random variable, $i = 1, \ldots, n$, there exist a positive real-valued random variable*

$$\varepsilon(\omega) = \varepsilon(\omega, x_1(\omega), \ldots, x_n(\omega); y_1(\omega), \ldots, y_n(\omega)) \le 1 \text{ a.s.}$$

and measurable selections $\bar{x}_i(\omega)$ of E_i, $i = 1, \ldots, n$ such that

$$\|T_i(\omega, \bar{x}_1(\omega), \ldots, \bar{x}_n(\omega)) - T_i(\omega, x_1(\omega), \ldots, x_n(\omega)) - \varepsilon(\omega)y_i(\omega)\|_i \le \varepsilon(\omega)\Big(\sum_{j=1}^n b_{i,j}(\omega)\|y_j(\omega)\|_j + \sum_{j=1}^n c_{i,j}(\omega)\|T_j(\omega, x_1(\omega), \ldots, x_n(\omega))\|_j\Big) \text{ a.s, } i = 1, \ldots, n \tag{3.17}$$

and

$$d_i(\bar{x}_i(\omega), x_i(\omega)) \le \varepsilon(\omega)B_i(\omega, \|y_i(\omega))\|_i) \text{ a.s.,} i = 1, \ldots, n \tag{3.18}$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$ and B_i, $i, j = 1, \ldots, n$ satisfy the hypothesis in Definition 3.1.

As consequence of Theorem 3.1, we have

Theorem 3.2. *Let $T_i : G_r(E) \to Y_i$, $i = 1, \ldots, n$, be a.s. continuous point-valued random operators with stochastic domain E. If for all measurable selections $x_i(\omega)$ of E_i, $i = 1, \ldots, n$,*

$$-(T_1(\omega, x_1(\omega), \ldots, x_n(\omega)), \ldots, T_n(\omega, x_1, \ldots, x_n(\omega))) \in \Gamma_{x(\omega)}(T),$$

then the system of nonlinear point-valued random operator equations

$$\theta_i = T_i(\omega, x_1, \ldots, x_n),\ i = 1, \ldots, n$$

has a random solution. This means that there exist measurable selections $x_i^*(\omega)$ of E_i, $i = 1, \ldots, n$, such that

$$\theta_i = T_i(\omega, x_1^*(\omega), \ldots, x_n^*(\omega)),\ a.s.,\ i = 1, \ldots, n.$$

Theorem 3.3. *Let $T_i : G_r(E) \to CL(Y_i)$, $i = 1, \ldots, n$, be a.s. continuous set-valued random operators with stochastic domain E, and let $u_i(\omega)$ is an Y_i-valued random variable, $i = 1, \ldots, n$. If for all measurable selections $x_i(\omega)$ of E_i, $i = 1, \ldots, n$,*

$$(u_1(\omega) - T_1(\omega, x_1(\omega), \ldots, x_n(\omega)), \ldots,$$

$$u_n(\omega) - T_n(\omega, x_1(\omega), \ldots, x_n(\omega))) \in \Gamma_{x(\omega)}(T).$$

In such a case, the inequalities (3.1) are replaced by the following inequalities for the definition 3.1 of $\Gamma_{x(\omega)}(T)$:

$$H_i(T_i(\omega, \bar{x}_1(\omega), \ldots, \bar{x}_n(\omega)), T_i(\omega, x_1, \ldots, x_n(\omega)) + \varepsilon(\omega) y_i(\omega))$$

$$\leq \varepsilon(\omega)(\sum_{j=1}^{n} b_{i,j}(\omega) \|y_j(\omega)\|_j$$

$$+ \sum_{j=1}^{n} c_{i,j}(\omega) D_j(u_j(\omega), T_j(\omega, x_1(\omega), \ldots, x_n(\omega))))\ a.s.,\ i = 1, \ldots, n.$$

Then the system of nonlinear set-valued random operator equations

$$u_i(\omega) \in T_i(\omega, x_1, \ldots, x_n),\ i = 1, \ldots, n$$

has a random solution. This means that there exist measurable selection $x_i^*(\omega)$ of E_i, $i = 1, \ldots, n$ such that

$$u_i(\omega) \in T_i(\omega, x_1^*(\omega), \ldots, x_n^*(\omega))\ a.s,\ i = 1, \ldots, n.$$

Proof. Letting $\hat{T}_i(\omega, x_1(\omega), \ldots, x_n(\omega)) = u_i - T_i(\omega, x_1(\omega), \ldots, x_n(\omega))$, $i = 1, \ldots, n$, it is easy to check that $\hat{T}_i$, $i = 1, \ldots, n$, satisfy the hypothesis of the Theorem 3.1. It follows from Theorem 3.1 that there exist measurable selections $x_i^*(\omega)$ of E_i, $i = 1, \ldots, n$, such that

$$\theta_i \in \hat{T}_i(\omega, x_1^*(\omega), \ldots, x_n^*(\omega)) \text{ a.s, } i = 1, \ldots, n.$$

Hence, we have

$$u_i(\omega) \in T_i(\omega, x_1^*(\omega), \ldots, x_n^*(\omega)) \text{ a.s, } i = 1, \ldots, n. \square$$

Theorem 3.4. *Let $T_i : G_\tau(E) \to Y_i$, $i = 1, \ldots, n$, be a.s. continuous point-valued random operators with stochastic domain E, let $u_i(\omega)$ is an Y_i-valued random variable, $i = 1, \ldots, n$ and for all measurable selections $x_i(\omega)$ of E_i, $i = 1, \ldots, n$,*

$$(u_1 - T_1(\omega, x_1(\omega), \ldots, x_n(\omega)), \ldots, u_n - T_n(\omega, x_1(\omega), \ldots, x_n(\omega)) \in \Gamma_{x(\omega)}(T).$$

In such a case, the inequalities (3.17) are replaced by the following inequalities for the Definition 3.2 of $\Gamma_{x(\omega)}(T)$:

$$\|T_i(\omega, \bar{x}_1(\omega), \ldots, \bar{x}_n(\omega)) - T_i(\omega, x_1(\omega), \ldots, x_n(\omega)) - \varepsilon(\omega) y_i(\omega)\|_i$$
$$\leq \varepsilon(\omega)(\sum_{j=1}^n b_{i,j}(\omega)\|y_j\|_j + \sum_{j=1}^n c_{i,j}(\omega)$$
$$\cdot \|u_j(\omega) - T_j(\omega, x_1(\omega), \ldots, x_n(\omega))\|_j) \text{ a.s., } i = 1, \ldots, n.$$

Then the system of nonlinear point-valued random operator equations

$$u_i = T_i(\omega, x_1(\omega), \ldots, x_n(\omega)),\ i = 1, \ldots, n$$

has a random solution. This means that there exists measurable selection $x_i^*(\omega)$ of E_i, $i = 1, \ldots, n$, such that

$$u_i = T_i(\omega, x_1^*(\omega), \ldots, x_n^*(\omega)), \text{ a.s., } i = 1, \ldots, n.$$

Proof. Using Theorem 3.2 and same argument as in Theorem 3.3, it is easy to prove that the conclusion of this theorem holds.□

Remark 3.1. *Obviously, Theorem 3.1-3.4 may be regarded as the random generalizations of [1], Theorem 2.1 and 2.2 and [2], Theorem 2.1 and 2.2, pp. 93-97.*

In the following, suppose that $(X_i, \|\cdot\|_i)$, $i = 1, \dots, n$ are also separable Banach spaces.

Definition 3.3. *Let E be a random subset of $X = X_1 \times \dots \times X_n$, and let $T_i : G_r(T) \to CL(Y_i)$, $i = 1, \dots, n$, be a.s. continuous set-valued random operators with stochastic domain E. For each measurable selection $x_i(\omega)$ of E_i, $i = 1, \dots, n$, we define sets $\Gamma^*_{x(\omega)}(T)$ of random contractor directions for $T = (T_1, \dots, T_n)$ at $x(\omega) = (x_1(\omega), \dots, x_n(\omega))$ as follows: for each $y(\omega) = (y_1(\omega), \dots, y_n(\omega)) \in \Gamma^*_{x(\omega)}(T))$ where $y_i(\omega)$ is Y_i-valued random variable, $i = 1, \dots, n$, there exist a positive real-valued random variable $\varepsilon(\omega) = \varepsilon(\omega, x_1(\omega), \dots, x_n(\omega); y_1(\omega), \dots, y_n(\omega)) \le 1$ and X_i-valued random variables $h_i(\omega) = h_i(\omega, x_1(\omega), \dots, x_n(\omega); y_1(\omega), \dots, y_n(\omega))$, $i = 1, \dots, n$, (called random strategic direction) such that*

$$x_i(\omega) + \varepsilon(\omega) \cdot h_i(\omega) \in E_i(\omega),\ \forall \omega \in \Omega,\ i = 1, \dots, n,$$

$$\begin{aligned} H_i(T_i(\omega, x_1(\omega) + \varepsilon(\omega)h_1(\omega), \dots, x_n(\omega) + \varepsilon(\omega)h_n(\omega)),\\ T_i(\omega, x_1(\omega), \dots, x_n(\omega)) + \varepsilon(\omega)y_i(\omega))\\ \le \varepsilon(\omega)(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j(\omega)\|_j + \sum_{j=1}^{n} c_{i,j}(\omega)\\ \cdot D_j(\theta_j, T_j(\omega, x_1(\omega), \dots, x_n(\omega)))) \text{ a.s., } i = 1, \dots, n \end{aligned}$$

and

$$\|h_i(\omega)\|_i \le B_i(\omega, \|y_i(\omega)\|_i) \text{ a.s., } i = 1, \dots, n$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$, $i, j = 1, \dots, n$, satisfy the hypothesis in Definition 3.1.

Theorem 3.5. *Let $T_i : G_r(E) \to CL(Y_i)$, $i = 1, \dots, n$, be a.s. continuous set-valued random operators with stochastic domain E. If for all measurable selections $x_i(\omega)$ of E_i, $i = 1, \dots, n$,*

$$-(T_1(\omega, x_1(\omega), \dots, x_n(\omega)), \dots, T_1(\omega, x_1(\omega), \dots, x_n(\omega))) \subset \Gamma^*_{x(\omega)}(T)$$

then the system of nonlinear set-valued random operator equations

$$\theta_i \in T_i(\omega, x_1, \dots, x_n),\ i = 1, \dots, n$$

has a random solution. That means that there exists measurable selection $x_i^*(\omega)$ of E_i, $i = 1, \dots, n$, such that

$$\theta_i \in T_i(\omega, x_1^*(\omega), \dots, x_n^*(\omega)),\ \text{a.s., } i = 1, \dots, n.$$

Proof. Let $\bar{x}_i(\omega) = x_i(\omega) + \varepsilon(\omega)h_i(\omega)$, $i = 1, \ldots, n$. By the hypothesis of this theorem, we have that $\bar{x}_i(\omega)$ is a measurable selection of E_i, $i = 1, \ldots, n$, and

$$\|\bar{x}_i(\omega) - x_i(\omega)\|_i \leq \varepsilon(\omega)\|h_i(\omega)\|_i$$
$$\leq \varepsilon(\omega)B_i(\omega, \|y_i(\omega)\|_i) \text{ a.s., } i = 1, \ldots, n.$$

Thus, it follows from Theorem 3.1 that the conclusion of this theorem holds. □

Remark 3.2. *Theorem 3.5 can be regarded as the random set-valued generalization of Theorem of [3], p. 12.*

4. Random operators with deterministic domain

In this section, we suppose that $D = D_1 \times \ldots \times D_n$, where $D_i \subset X_i$ is a closed vector space, $i = 1, \ldots, n$.

Lemma 4.1. *Let* $\mathcal{E} : \Omega \times D_1 \times \ldots \times D_n \times Y_1 \times \ldots \times Y_n \to (0,1]$ *be such that for each* $(x_1, \ldots, x_n, y_1, \ldots, y_n) \in D_1 \times \ldots \times D_n \times Y_1 \times \ldots \times Y_n$, $\mathcal{E}(\cdot, x_1, \ldots, x_n, y_1, \ldots, y_n)$ *is a real-valued random variable and for each* $\omega \in \Omega$, $\mathcal{E}(\omega, \ldots; \ldots)$ *is continuous in each coordinate variable. Then for all* X_i*-valued and* Y_i*-valued random variables* $x_i(\omega) \in D_i$ *and* $y_i(\omega)$, $i = 1, \ldots, n$,

$$\mathcal{E}(\omega, x_1(\omega), \ldots, x_n(\omega); y_1(\omega), \ldots, y_n(\omega)) = \mathcal{E}(\omega, x(\omega); y(\omega))$$

is also a real-valued random variable.

Lemma 4.2. *Let* $h_i : \Omega \times D_1 \times \ldots \times D_n \times Y_1 \times \ldots \times Y_n \to X_i$ *be such that for each* $(x_1, \ldots, x_n; y_1, \ldots, y_n) \in D_1 \times \ldots \times D_n \times Y_1 \times \ldots \times Y_n$, $h_i(\cdot, x_1, \ldots, x_n; y_1, \ldots, y_n)$ *is an* X_i*-valued random variable and for each* $\omega \in \Omega$, $h_i(\omega, \ldots; \ldots)$ *is continuous in each coordinate variable. Then for all* X_i*-valued and* Y_i*-valued random variables* $x_i(\omega) \in D_i$ *and* $y_i(\omega)$, $i = 1, \ldots, n$,

$$h_i(\omega, x_1(\omega), \ldots, x_n(\omega); y_1(\omega), \ldots, y_n(\omega)) = h_i(\omega, x(\omega); y(\omega))$$

is an X_i*-valued random variable.*

Using similar argument as in Lemma 2.1 of [20], the conclusion of lemma 4.1 and 4.2 easily hold. Thus we omit the proofs.

Definition 4.1. *Let $T_i : \Omega \times D \to CL(Y_i)$, $i = 1, \dots, n$, be a.s. continuous set-valued random operators. For each $x = (x_1, \dots, x_n) \in D$, we define sets $\Gamma_x(T)$ of random contractor directions for $T = (T_1, \dots, T_n)$ at x as follows: for each $y = (y_1, \dots, y_n) \in \Gamma_x(T)$, there exist mappings $\varepsilon(\omega, x; y)$ and $h_i(\omega, x; y)$, $i = 1, \dots, n$, which satisfy the hypothesis in Lemma 4.1 and 4.2 respectively such that*

$$x_i + \varepsilon(\omega, x; y) h_i(\omega, x; y) \in D, \ i = 1, \dots, n, \tag{4.1}$$

$$\begin{aligned} H_i(T_i(\omega, x_1 + \varepsilon(\omega, x; y) h_1(\omega, x; y), \dots, x_n + \varepsilon(\omega, x; y) h_n(\omega, x; y)), \\ T_i(\omega, x_1, \dots, x_n) + \varepsilon(\omega, x; y) y_i) \leq \varepsilon(\omega, x; y) (\sum_{j=1}^{n} b_{i,j}(\omega) \|y_j\|_j \\ + \sum_{j=1}^{n} c_{i,j}(\omega) D_j(\theta_j, T_j(\omega, x_1, \dots, x_n))) \ a.s., \ i = 1, \dots, n \end{aligned} \tag{4.2}$$

and

$$\|h_i(\omega, x; y)\|_i \leq B_i(\omega, \|y_i\|_i) \ a.s, \ i = 1, \dots, n, \tag{4.3}$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$ and B_i, $i, j = 1, \dots, n$, satisfy the hypothesis in Definition 3.1.

Theorem 4.1. *Let $T_i : \Omega \times D \to CL(Y_i)$, $i = 1, \dots, n$, be a.s. continuous set-valued random operators. If for all $x = (x_1, \dots, x_n) \in D$,*

$$-(T_1(\omega, x_1, \dots, x_n), \dots, T_n(\omega, x_1, \dots, x_n)) \subset \Gamma_x(T) \tag{4.4}$$

then the system of nonlinear set-valued random operator equations

$$\theta_i \in T_i(\omega, x_1, \dots, x_n), \ i = 1, \dots, n$$

has a random solution $(x_1^*(\omega), \dots, x_n^*(\omega)) \in D$.

Proof. Let $E_i(\omega) = D_i$, $\forall \omega \in \Omega$, $i = 1, \dots, n$. Clearly, $E = E_1 \times \dots \times E_n$ is a random subset of $X = X_1 \times \dots \times X_n$. Thus $T_i : G_r(E) \to CL(Y_i)$, $i = 1, \dots, n$, are a.s. continuous random operators with stochastic domain E. For each measurable selection $x_i(\omega)$ of E_i, $i = 1, \dots, n$, we have $x_i(\omega) \in D$, $\forall \omega \in \Omega$, $i = 1, \dots, n$. Now let $y_i(\omega)$ be a Y_i - valued random variable, $i = 1, \dots, n$, and suppose that $y(\omega_0) = (y_1(\omega_0), \dots, y_n(\omega_0)) \in \Gamma_{x(\omega_0)}(T)$ for any fixed $\omega_0 \in \Omega$. It follows from Definition 4.1 that there exist $\varepsilon(\omega, x(\omega_0); y(\omega_0))$ and $h_i(\omega, x(\omega_0); y(\omega_0))$, $i = 1, \ldots, n$, such that

$$(4.5) \quad x_i(\omega_0) + \varepsilon(\omega, x(\omega_0); y(\omega_0))h_i(\omega, x(\omega_0); y(\omega_0)) \in D_i = E_i(\omega),\ i = 1, \ldots, n,$$

$$(4.6) \quad H_i(T_i(\omega, x_1(\omega_0) + \varepsilon(\omega, x(\omega_0); y(\omega_0))h_1(\omega, x(\omega_0); y(\omega_0)), \ldots, x_n(\omega_0) + \varepsilon(\omega, x(\omega_0); y(\omega_0))h_n(\omega, x(\omega_0); y(\omega_0))), T_i(\omega, x_1(\omega_0), \ldots, x_n(\omega_0)) + \varepsilon(\omega, x(\omega_0); y(\omega_0))y_i(\omega_0))) \leq \varepsilon(\omega, x(\omega_0); y(\omega_0))(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j(\omega_0)\|_j + \sum_{j=1}^{n} c_{i,j}(\omega)D_j(\theta_j, T_j(\omega, x_1(\omega_0), \ldots, x_n(\omega_0)))) \text{ a.s., } i = 1, \ldots, n$$

and

$$(4.7) \quad \|h_i(\omega, x(\omega_0); y(\omega_0))\|_i \leq B_i(\omega, \|y_i(\omega_0)\|_i) \text{ a.s, } i = 1, \ldots, n.$$

By the arbitrarity of $\omega_0 \in \Omega$, replacing ω_0 by ω, the relations (4.5)-(4.7) still hold. Hence, it follows from lemma 4.1 and 4.2 and Definition 3.3 that

$$y(\omega) = (y_1(\omega), \ldots, y_n(\omega)) \in \Gamma^*_{x(\omega)}(T).$$

Hence from (4.4) it follows that

$$-(T_1(\omega, x_1(\omega), \ldots, x_n(\omega)), \ldots, T_n(\omega, x_1(\omega), \ldots, x_n(\omega))) \subset \Gamma^*_{x(\omega)}(T).$$

Then all hypotheses of Theorem 3.5 are satisfied. It follows that the conclusion of this theorem hold. □

As a concequence of Theorem 4.1, we also have

Theorem 4.2. *Let $T_i : \Omega \times D \to Y_i$, $i = 1, \ldots, n$, be a.s. continuous point-valued random operators. If for all $x = (x_1, \ldots, x_n) \in D$,*

$$-(T_1(\omega, x_1, \ldots, x_n), \ldots, T_n(\omega, x_1, \ldots, x_n)) \subset \Gamma_x(T)$$

then the system of nonlinear set-valued random operator equations

$$\theta_i \in T_i(\omega, x_1, \ldots, x_n),\ i = 1, \ldots, n$$

has a random solution $(x_1^*(\omega), \ldots, x_n^*(\omega)) \in D$.

Remark 4.1. *Letting $c_{i,j}(\omega) = 0$, $i, j = 1, \ldots, n$, in Theorem 4.2, we obtain also the random generalization of Theorem in [3], p. 12.*

5. Random directional contractor and directional contractions

Let X_i, Y_i, $i = 1, \dots, n$, be separable Banach spaces and $T_i : \Omega \times D \subset \Omega \times X \to CL(Y_i)$, $i = 1, \dots, n$, be a.s. continuous set-valued random operators, where $D = D_1 \times \dots D_n$, $X = X_1 \times \dots X_n$ and $D_i \subset X_i$ is a closed vector space, $i = 1, \dots, n$. Let $\Gamma_i(\cdot, x_i) : \Omega \times Y_i \to X_i$, corresponding each $x_i \in X_i$, be a bounded linear random operator, $i = 1, \dots, n$. (cf. [11]).

Theorem 5.1. *Let $T_i : \Omega \times D \to CL(Y_i)$, $i = 1, \dots, n$, be a.s. continuous set-valued random operators satisfying the following hypotheses: for $x = (x_1, \dots, x_n) \in D$ and $y = (y_1, \dots, y_n) \in Y_1 \times \dots \times Y_n$, there exists mapping $\varepsilon(\omega, x; y)$ satisfying the hypotheses in Lemma 4.1 such that*

$$x_i + \varepsilon(\omega, x; y)\Gamma_i(\omega, x_i)y_i \in D_i, \ i = 1, \dots, n, \tag{5.1}$$

$$\begin{aligned} H_i(T_i(\omega, x_1 + \varepsilon(\omega, x; y)\Gamma_1(\omega, x_1)y_1, \dots, x_n + \varepsilon(\omega, x; y)\Gamma_n(\omega, x_n)y_n),\\ T_i(\omega, x_1, \dots, x_n) + \varepsilon(\omega, x; y)y_i) \le \varepsilon(\omega, x; y)(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j\|_j\\ + \sum_{j=1}^{n} c_{i,j}(\omega) D_j(\theta_j, T_j(\omega, x_1, \dots, x_n))) \ a.s., \ i = 1, \dots, n, \end{aligned} \tag{5.2}$$

and

$$\|\Gamma_i(\omega, x_i)\|_i \le B_i(\omega) \ a.s., \ i = 1, \dots, n, \tag{5.3}$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$, $i, j = 1, \dots, n$, satisfy the hypotheses in Definition 3.1 and $B_i(\omega)$, $i = 1, \dots, n$, are positive real-valued random variables. Then the system of nonlinear set-valued random operator equations

$$\theta_i \in T_i(\omega, x_1, \dots, x_n), \ i = 1, \dots, n$$

has a random solutuion $(x_1^*(\omega), \dots, x_n^*(\omega)) \in D$.

Proof. For $x = (x_1, \dots, x_n) \in D$ and $y = (y_1, \dots, y_n) \in Y_1 \times \dots \times Y_n$, define $h_i : \Omega \times D \times Y_1 \times \dots \times x_n \to X_i$, $i = 1, \dots, n$, by

$$h_i(\omega, x; y) = \Gamma_i(\omega, x_i)y_i, \ i = 1, \dots, n.$$

Then h_i, $i = 1, \ldots, n$, satisfy the hypotheses of Lemma 4.2. By (5.3), we have

$$\|h_i(\omega, x; y)\|_i = \|\Gamma_i(\omega, x_i)\| \cdot \|y_i\|_i \leq B_i(\omega)\|y_i\|_i, \text{ a.s., } i = 1, \ldots, n.$$

Let $B_i(\omega, t) = B_i(\omega)t$ for all $(\omega, t) \in \Omega \times [0.\infty)$, $i = 1, \ldots, n$. Then $B_i : \Omega \times [0, \infty) \to [0, \infty)$ satisfies the conditions in Definition 3.1. Hence, it follows that for each $x \in D$, $\Gamma_x(T) = Y_1, \ldots, Y_n$. Theorem 4.1 can be applied to obtain the conclusions of this theorem. □

Remark 5.1. *Theorem 5.1 is the random generalization of Theorem 3.1 in [15].*

Theorem 5.2. *Let $X = X_1 \times \ldots \times X_n$ and $F_i : \Omega \times X \to CL(Y_i)$, $i = 1, \ldots, n$, be a.s. continuous set-valued random operators. Suppose that for arbitrary $x, y \in X$ there exists a mapping $\varepsilon(\omega, x; y)$ satisfying the hypotheses in Lemma 4.1 such that*

$$(5.4) \qquad H_i(F_i(\omega, x_1 + \varepsilon(\omega, x; y)y_1, \ldots, x_n + \varepsilon(\omega, x; y)y_n), F_i(\omega, x_1, \ldots, x_n)) \leq \varepsilon(\omega, x; y)(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j\|_j + \sum_{j=1}^{n} c_{i,j}(\omega)D_j(x_j, F_j(\omega, x_1, \ldots, x_n))) \text{ a.s., } i = 1, \ldots, n,$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$, $i, j = 1, \ldots, n$, satisfy the hypotheses in Definition 3.1. Then $T = (T_1, \ldots, T_n)$ has a random fixed point, i.e. there exist X_i-valued random variable $x_i^*(\omega)$, $i = 1, \ldots, n$, such that

$$x_i^*(\omega) \in F_i(\omega, x_1^*(\omega), \ldots, x_n^*(\omega)) \text{ a.s., } i = 1, \ldots, n.$$

Proof. Let $T_i = I - F_i$, $i = 1, \ldots, n$, where I denote the identity operator. Then, by (5.4), we have

$$H_i(T_i(\omega, x_1 + \varepsilon(\omega, x; y)y_1, \ldots, x_n + \varepsilon(\omega, x; y)y_n), T_i(\omega, x_1, \ldots, x_n) + \varepsilon(\omega, x; y)y_i)$$
$$= H_i(F_i(\omega, x_1 + \varepsilon(\omega, x; y)y_1, \ldots, x_n + \varepsilon(\omega, x; y)y_n),$$

$$F_i(\omega,x_1,\ldots,x_n)) \le \varepsilon(\omega,x;y)(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j\|_j$$

$$+\sum_{j=1}^{n} c_{i,j}(\omega)D_j(\theta_j,T_j(\omega,x_1,\ldots,x_n))) \text{ a.s., } i=1,\ldots,n.$$

Using the Theorem 5.1 with $D = X$ and $\Gamma_i = I$, $i = 1,\ldots,n$, we obtain that there exists X_i-valued random variable $x_i^*(\omega)$, $i = 1,\ldots,n$, such that

$$\theta_i \in T_i(\omega, x_1^*(\omega),\ldots,x_n^*(\omega)) \text{ a.s, } i=1,\ldots,n.$$

Thus we have

$$x_i^*(\omega) \in F_i(\omega, x_1^*(\omega),\ldots,x_n^*(\omega)) \text{ a.s, } i=1,\ldots,n.\square$$

As a consequence of Theorem 5.1 and 5.2, we have

Theorem 5.3. *Let $T_i : \Omega \times D \to Y_i$, $i = 1,\ldots,n$, be a.s. continuous point-valued random operators satisfying the following hypotheses: for $x \in X$ and $y \in Y_1 \times \ldots \times Y_n$, there exists a mapping $\varepsilon(\omega,x;y)$ satisfying the conditions of Lemma 4.1 such that*

$$x_i + \varepsilon(\omega,x;y)\Gamma_i(\omega,x_i)y_i(\omega) \in D_i,\ i=1,\ldots,n,$$

$$\|T_i(\omega, x_1 + \varepsilon(\omega,x;y)\Gamma_1(\omega,x_1)y_1,\ldots,x_n + \varepsilon(\omega,x;y)\Gamma_n(\omega,x_n)y_n)$$

$$-T_i(\omega,x_1,\ldots,x_n) - \varepsilon(\omega,x;y)y_i\|_i \le \varepsilon(\omega,x;y)(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j\|_j$$

$$+\sum_{j=1}^{n} c_{i,j}(\omega)\|T_j(\omega,x_1,\ldots,x_n)\|_i) \text{ a.s., } i=1,\ldots,n,$$

and

$$\|\Gamma_i(\omega,x_i)\|_i \le B_i(\omega) \text{ a.s., } i=1,\ldots,n,$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$, $i,j = 1,\ldots,n$, satisfy the hypotheses in Theorem 5.1. Then, there exists X_i-valued random variable $x_i^*(\omega)$, $i = 1,\ldots,n$, such that

$$\theta_i = T_i(\omega, x_1^*(\omega),\ldots,x_n^*(\omega)) \text{ a.s., } i=1,\ldots,n.$$

Theorem 5.4. *Let $F_i : \Omega \times X \to X_i$, $i = 1, \ldots, n$, be a.s. continuous point-valued random operators. Suppose that for arbitrary $x, y \in X$ there exists a mapping $\varepsilon(\omega, x; y)$ satisfying the conditions of Lemma 4.1 such that*

$$\|F_i(\omega, x_1 + \varepsilon(\omega, x; y)y_1, \ldots, x_n + \varepsilon(\omega, x; y)y_n) - F_i(\omega, x_1, \ldots, x_n)\|_i$$

$$\leq \varepsilon(\omega, x; y)(\sum_{j=1}^{n} b_{i,j}(\omega)\|y_j\|_j$$

$$+ \sum_{j=1}^{n} c_{i,j}(\omega)\|x_j - F_j(\omega, x_1, \ldots, x_n)\|) \text{ a.s., } i = 1, \ldots, n,$$

where $b_{i,j}(\omega)$, $c_{i,j}(\omega)$, $i, j = 1, \ldots, n$, satisfy the hypotheses in Definition 3.1. Then there exist X_i-valued random variable $x_i^*(\omega)$, $i = 1, \ldots, n$, such that

$$x_i^* = F_i(\omega, x_1^*(\omega), \ldots, x_n^*(\omega)) \text{ a.s., } i = 1, \ldots, n.$$

Remark 5.2. *Theorems 5.2-5.4 improve and extend Theorems 3.2, 3.3 of [15], Theorem 1.4 of [10], [11]; and the corresponding results in [1], [2].*

6. Some applications

In this section, we shall give some applications of our results to a system of nonlinear random integral and differential equations in Banach spaces.

Let $(X_i, \|\cdot\|)$, $i = 1, \ldots, n$ be a separable Banach spaces, and let

$$C_J(X_i) = \{x : J \to X_i | \ x \text{ continuous, } \|x(t)\|_i^J = \max_{t \in J} \|x(t)\|_i\}$$

where $J = [0, a] \subset \mathbf{R}$. Obviouslu, $(C_J(X_i), \|\cdot\|_i^J)$, $i = 1, \ldots, n$, are separable Banach spaces.

Lemma 6.1. *([7]) A mapping $x_i : \Omega \times J \to X_i$ satisfies that for each $\omega \in \Omega$, $x_i(\omega, \cdot)$ is continuous and for each $t \in J$, $x_i(\cdot, t)$ is an X_i-valued random variable if and only if $x_i(\omega, t)$ is an $C_J(X_i)$-valued random variable.*

Lemma 6.2. *Let $K_i : \Omega \times J \times J \times X_1 \times \ldots X_n \to X_i$ be such that $K_i(\omega, \cdot, \cdot, \ldots, \cdot)$ is continuous for each $\omega \in \Omega$ and $K_i(\cdot, t, s, x_1, \ldots, x_n)$ is X_i-valued random* *variable for each $(t,s,x_1,\dots,x_n)\in J\times J\times X_1\times\dots X_n$. Then for any fixed $C_J(X_i)$-valued random variable $x_i(\omega,s)$, $i=1,\dots,n$,*

$$\int_0^t K_i(\omega,t,s,x_1(\omega,s),\dots,x_n(\omega,s))ds$$

is an $C_J(X_i)$-valued random variable.

Lemma 6.3. *Let $K_i:\Omega\times J\times J\times X_1\times\dots\times X_n\to X_i$ satisfies the hypotheses in the Lemma 6.2 and there exists a positive real-valued function $M_i(\omega)$ such that for all $(t,s,x_1,\dots,x_n)\in J\times J\times X_1\times\dots X_n\to X_i$,*

$$\|K_i(\omega,t,s,x_1,\dots,x_n)\|_i\le M_i(\omega)\ \text{a.s.}$$

Then $F_i:\Omega\times C_J(X_1)\times\dots C_J(X_n)\to C_J(X_i)$ defined by

$$F_i(\omega,x_1(t),\dots,x_n(t))=x_i^0(\omega,t)\int_0^t K_i(\omega,t,s,x_1(s),\dots,x_n(s))ds \tag{6.1}$$

is an a.s. continuous point-valued random operator, where $x_i^0(\omega,t)$ is a given $C_J(X_i)$-valued random variable.

Proof. It follows from Lemma 6.2 that for each $(x_1(s),\dots,x_n(s))\in C_J(X_1)\times\dots C_J(X_n)$, $F_i(\cdot,x_1(t),\dots,x_n(t))$ is an $C_J(X_i)$-valued random variable. By similar argument as in the Lemma 4 of [21], we have also that F_i is a.s. continuous. Hence F_i is an a.s. continuous point-valued random operator. □

Theorem 6.1. *Let $x_i^0(\omega,t)$ be given $C_J(X_i)$-valued random variable, $i=1,\dots,n$, and let $K_i:\Omega\times J\times J\times X_1\times\dots X_n$, $i=1,\dots,n$, satisfy the hypotheses in the Lemma 6.3. If for $x_i(t),y_i(t)\in C_J(X_i)$, $i=1,\dots,n$, there exists a mapping $\varepsilon(\omega,x(t);y(t))=\varepsilon(\omega,x_1(t),\dots,x_n(t);y_1(t),\dots,y_n(t))$ satisfying the hypotheses in Lemma 4.1 such that for all $s\in J$*

$$\begin{aligned}\tag{6.2} &\|K_i(\omega,t,s,x_1(s)+\varepsilon(\omega,x(t);y(t))y_1(s),\dots,x_n(s)\\ &+\varepsilon(\omega,x(t);y(t))y_n(s)-K_i(\omega,t,s,x_1(s),\dots,x_n(s))\|_i\\ &\le\varepsilon(\omega,x(t);y(t))(\sum_{j=1}^n b_{i,j}(\omega,s)\|y_j\|_j^J\\ &+\sum_{j=1}^n c_{i,j}(\omega,s)\|x_j-F_j(\omega,x_1(t),\dots,x_n(t))\|_j^J)\ \text{a.s.},\ i=1,\dots,n,\end{aligned}$$

where $b_{i,j},c_{i,j}:\Omega\times J\to[0.\infty)$, $i,j=1,\dots,n$, are such that *(i) for each $t \in J$, $b_{i,j}(\cdot,t)$ and $c_{i,j}(\cdot,t)$ are real-valued random variable and for each $\omega \in \Omega$, $b_{i,j}(\omega,\cdot)$ and $c_{i,j}(\omega,\cdot)$ are integrable on J,*

(ii) letting $b_{i,j}(\omega) = \int_0^a b_{i,j}(\omega,s)ds$ and $c_{i,j}(\omega) = \int_0^a c_{i,j}(\omega,s)ds$, $c_{i,j}$ and $b_{i,j}$ satisfy the hypotheses in Definition 3.1.

Then the system of nonlinear random Volterra integral equations

$$(6.3)\quad x_i(t) = x_i^0(\omega,t) + \int_0^t K_i(\omega,t,s,x_1(s),\dots,x_n(s))ds,\ i = 1,\dots,n$$

has a random solution, what means that there exists $C_J(X_i)$-valued random variable $x_i^*(\omega,t)$, $i = 1,\dots,n$, such that

$$x_i(\omega,t) = x_i^0(\omega,t) + \int_0^t K_i(\omega,t,s,x_1^*(\omega,s),\dots,x_n^*(\omega,s))ds,\ \text{a.s.},\ i = 1,\dots,n.$$

Proof. From Lemma 6.3 it follows that the operators F_i, $i = 1,\dots,n$, defined by (6.1) are a.s. continuous point-valued random operators.

By (6.2), we have

$$\|F_i(\omega, x_1(t) + \varepsilon(\omega,x(t);y(t))y_1(t),\dots,x_n(t) + \varepsilon(\omega,x(t);y(t))y_n(t)$$
$$-F_i(\omega,x_1(t),\dots,x_n(t))\|_i$$
$$\le \int_0^t \|K_i(\omega,t,s,x_1(s) + \varepsilon(\omega,x(t);y(t))y_1(s),$$
$$\dots,x_n(s) + \varepsilon(\omega,x(t);y(t))y_n(s) - K_i(\omega,t,s,x_1(s),\dots,x_n(s))\|_i ds$$
$$\le \varepsilon(\omega,x(t);y(t))(\sum_{j=1}^n b_{i,j}(\omega)\|y_j(t)\|_j^J$$
$$+\sum_{j=1}^n c_{i,j}(\omega,s)\|x_j(t) - F_j(\omega,x_1(t),\dots,x_n(t))\|_j^J)\ \text{a.s.},\ i = 1,\dots,n.$$

Hence

$$\|F_i(\omega, x_1(t) + \varepsilon(\omega,x(t);y(t))y_1(t),\dots,x_n(t) + \varepsilon(\omega,x(t);y(t))y_n(t)$$
$$-F_i(\omega,x_1(t),\dots,x_n(t))\|_i^J \le \varepsilon(\omega,x(t);y(t))(\sum_{j=1}^n b_{i,j}(\omega)\|y_j(t)\|_j^J$$

$$+\sum_{j=1}^{n} c_{i,j}(\omega)\|x_j(t) - F_j(\omega, x_1(t), \ldots, x_n(t))\|_j^J) \text{ a.s., } i = 1, \ldots, n.$$

Then, it follows from the Theorem 5.4 that there exists $C_J(X_j)$-valued random variable $x_i^*(\omega, t)$, $i = 1, \ldots, n$, such that

$$x_i^*(\omega, t) = F_i(\omega, x_1^*(\omega, t), \ldots, x_n^*(\omega, t)) \text{ a.s., } i = 1, \ldots, n.$$

Obviously $(x_1^*(\omega, t), \ldots, x_n^*(\omega, t))$ is a random solution of the system (6.3). □

Theorem 6.2. *Let $f_i : \Omega \times J \times X_1 \times \ldots X_n \to X_i$, $i = 1, \ldots, n$, be such that for each $(s, x_1, \ldots, x_n) \in J \times X_1 \times \ldots X_n$, $f_i(\cdot, s, x_1, \ldots, x_n)$ is an X_i-valued random variable and for each $\omega \in \Omega$, $f_i(\omega, \cdot, \ldots, \cdot)$ is continuous. Suppose that there exists positive real-valued function $M_i(\omega)$, $i = 1, \ldots, n$, such that for all $(s, x_1, \ldots, x_n) \in J \times X_1 \times \ldots X_n$,*

$$\|f_i(\omega, s, x_1, \ldots, x_n)\|_i \leq M_i(\omega) \text{ a.s., } i = 1, \ldots, n.$$

If for $x_i(t), y_i(t) \in C_J(X_i)$, $i = 1, \ldots, n$, there exists a mapping

$$\varepsilon = \varepsilon(\omega, x_1(t), \ldots, x_n(t); y_1(t), \ldots, y_n(t))$$

satisfying the hypotheses in Lemma 4.1 such that for all $s \in J$,

$$\|f_i(\omega, s, x_1(s) + \varepsilon y_1(s), \ldots, x_n(s) + \varepsilon y_n(s)$$

$$-f_i(\omega, s, x_1(s), \ldots, x_n(s))\|_i \leq \varepsilon(\sum_{j=1}^{n} b_{i,j}(\omega, s)\|y_j(t)\|_j^J$$

$$+\sum_{j=1}^{n} c_{i,j}(\omega, s)\|x_j(t) - F_j(\omega, x_1(t), \ldots, x_n(t))\|_j^J) \text{ a.s., } i = 1, \ldots, n,$$

where $F_i(\omega, x_1(t), \ldots, x_n(t)) = x_i^0(\omega) + \int_0^t f_i(\omega, s, x_1(s), \ldots, x_n(s))ds$ and $x_i^0(\omega)$ is a given X_i-valued random variable, $i, j = 1, \ldots, n$; $b_{i,j}(\omega, s)$ and $c_{i,j}(\omega, s)$, $i, j = 1, \ldots, n$, satisfy the condition in Theorem 6.2. Then the random initial value problem

$$(6.4) \qquad \begin{cases} \frac{dx_i(t)}{dt} = f_i(\omega, t, (x_1(\omega), \ldots, x_n(\omega)), \\ x_i(\omega, 0) = x_i^0, i = 1, \ldots, n, \end{cases}$$

has a random solution. That means that there exists $C_J(X_i)$-valued random variable $x_i^*(\omega,t)$, $i=1,\dots,n$, such that

$$\frac{dx_i^*(\omega,t)}{dt} = f_i(\omega,t,x_1^*(\omega,t),\dots,x_n^*(\omega,t)) \text{ a.s.,}$$

$$x_i(\omega,0) = x_i^0,\ i=1,\dots,n.$$

Proof. In order to find the random solution of the random initial problem (6.4), we can consider the equivalent system of nonlinear random Volterra integral equations

$$(6.5) \qquad x_i(t) = x_i^0(\omega) + \int_0^t f_i(\omega,s,x_1(s),\dots,x_n(s))ds,\ i=1,\dots,n.$$

From Theorem 6.1 with $x_i^0(\omega,t) = x_i^0(\omega)$,

$$K_i(\omega,t,s,x_1,\dots,x_n) = f_i(\omega,s,x_1,\dots,x_n),$$

$i=1,\dots,n$, for all $t\in J$, it follows that there exists a random solution $(x_1^*(\omega,t),\dots,x_n^*(\omega,t))$ of the system (6.5). Since (6.5) is equivalent to (6.4), therefore $(x_1^*(\omega,t),\dots,x_n^*(\omega,t))$ is also a random solution of the system (6.4). □

Remark 6.1. *Theorem 6.2 is the improvement and random generalization of Theorem 5.1 in [2], p. 105.*

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REZIME

SLUČAJNI KONTRAKTORSKI PRAVCI I REŠENJE SISTEMA SLUČAJNIH SKUPOVNIH OPERATORSKIH JEDNAČINA SA SLUČAJNIM DOMENOM

U ovom radu se uvodi koncept slučajnih kontraktorskih pravaca da bi se ispitala rešivost sistema nelinearnih skupovnih slučajnih operatorskih jednačina sa slučajnim domenom.

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