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EXPECTATION OF GENERALIZED RANDOM PROCESSES ON THE ZEMANIAN SPACE $\mathcal{A}$

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Abstract

Representation theorems for mathematical expectation and conditional expectation of the generalized random processes on the Zemanian space $\mathcal{A}$ are given.

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1. Introduction

The generalized random processes (g.r.p.) on the Zemanian space $\mathcal{A}$ were studied in [6,7]. In [7] several representation theorems for the g.r.p. on $\mathcal{A}$ were obtained. In [8] M. Ullrich investigated the properties of the expectation of the g.r.p. on space $\mathcal{D}$. The expectation of a g.r.p. on the space $\mathcal{K}\{\mathcal{M}_p\}$ was defined and investigated by I.M. Gel'fand in [2]. In [4] L.J. Kitchens obtained representation theorems for the expectation and conditional expectation of the g.r.p. on the space $\mathcal{K}\{\mathcal{M}_p\}$. In this paper we shall use the representations of a g.r.p. on space $\mathcal{A}$ obtained in [7] to represent the expectation of a g.r.p. on $\mathcal{A}$. We shall also investigate the conditional expectation of the g.r.p. on $\mathcal{A}$ and get a result similiar to Theorem 5. in [4].

2. Spaces $\mathcal{A}$ and $\mathcal{A}'$

For a space of test functions we take space $\mathcal{A}$, whose elements have orthonormal expansion. Space $\mathcal{A}$ and its dual space $\mathcal{A}'$ were introduced in [9]. Our construction of the spaces $\mathcal{A}$ and $\mathcal{A}'$ is different from [9] and details are given in [7]. Let I be an open interval in the set of real numbers $\mathbf{R}$, $L^2(I)$ the space of equivalence classes of square integrable functions with values in the set of complex numbers $\mathbf{C}$. The norm in $L^2(I)$ is defined by

$$\|f\|_0 = (f, f) = [\int_I |f|^2 dt]^{1/2}.$$

Let $C^\infty(I)$ be the set of infinitely differentiable (smooth) functions, $\mathbf{N}$ the set of natural numbers, and $\mathbf{N}_0 = \mathbf{N} \bigcup \{0\}$.

Let $\mathcal{R}$ be a linear differential self-adjoint operator of the form

$$\mathcal{R} = \Theta_0 D^{n_1} \Theta_1 \dots D^{n_\nu} \Theta_\nu,$$

where $D = d/dx, n_k, k = 1, 2, \dots, \nu,$ are non-negative numbers, $\Theta_k, k = 0, 1, \dots, \nu,$ are smooth complex functions with no zeros on I.

Suppose that there exist a sequence of real numbers $\{\lambda_n, n \in \mathbf{N}_0\}$, and a sequence of smooth functions $\{\psi_n, n \in \mathbf{N}_0\}$ in $L^2(I)$ such that $|\lambda_n| \to \infty, n \to \infty$, and $\mathcal{R}\psi_n = \lambda_n \psi_n$, $n \in \mathbf{N}_0$. Furthermore, suppose that $\{\psi_n, n \in \mathbf{N}_0\}$ forms a complete orthonormal sistem (o.n.s.) in $L^2(I)$. We can enumerate the sequence $\{\lambda_n,\ n \in \mathbf{N}_0\}$ and $\{\psi_n,\ n \in \mathbf{N}_0\}$, so that $|\lambda_0| \le |\lambda_1| \le |\lambda_2| \le \cdots$. Put

$$\tilde{\lambda}_n = \begin{cases} |\lambda|_n & \text{if } \lambda_n \neq 0 \\ 1 & \text{if } \lambda_n = 0 \end{cases}, n \in \mathbf{N}_0.$$

The sequence $\{\tilde{\lambda}_n,\ n \in \mathbf{N}_0\}$ is non-decreasing and $\tilde{\lambda}_n \to \infty, n \to \infty$. Let $\mathcal{R}^{k+1} = \mathcal{R}(\mathcal{R}^k)$, $k \in \mathbf{N}_0$, where $\mathcal{R}^0 = \mathcal{I}$, and $\mathcal{I}$ is the identity operator.

We shall define the scale of spaces $\mathcal{A}_k, k \in \mathbf{N}_0$.

$$\mathcal{A}_k = \{\phi \in L^2(I) : \phi = \sum_{n=0}^{\infty} a_n \psi_n,\ \|\phi\|_k = \sum_{n=0}^{\infty} |a_n|^2 \tilde{\lambda}_n^{2k} < \infty\},\ k \in \mathbf{N}_0.$$

$\mathcal{A}_k$ is the Hilbert space with respect to the scalar product

$$(\phi, \psi)_k = \sum_{n=0}^{\infty} a_n \bar{b}_n \tilde{\lambda}_n^{2k},\ \phi, \psi \in \mathcal{A}_k,$$

where $\phi = \sum_{n=0}^{\infty} a_n\psi_n$, $\psi = \sum_{n=0}^{\infty} b_n\psi_n$.
Put

$$S = \{\phi_m = \sum_{n=0}^{m} a_n\psi : m \in \mathbf{N}_0, a_n \in \mathbf{C}\}.$$

The set S is dense in $\mathcal{A}_k$, $k \in \mathbf{N}_0$.
Put

$$\mathcal{A} = \bigcap_{k=0}^{\infty} \mathcal{A}_k = \{\phi \in L^2(I) : \phi = \sum_{n=0}^{\infty} a_n\psi_n, \ \forall k, \ \sum_{n=0}^{\infty} |a_n|^2 \tilde{\lambda}_n^{2k} < \infty\}$$

Note that space $\mathcal{A}$ is dense in $\mathcal{A}_k$, $k \in \mathbf{N}_0$, because it contains the set S which is dense in each $\mathcal{A}_k, k \in \mathbf{N}_0$. Thus, $\mathcal{A}_k$, $k \in \mathbf{N}_0$, is the completion of $\mathcal{A}$ with respect to the norm $\|\cdot\|_k$.

Let $\mathcal{A}'$ be the dual space of space $\mathcal{A}$. The, we have:

$$\mathcal{A} = \bigcup_{k=0}^{\infty} \mathcal{A}'_k.$$

It should be noted that from [9, Lemma 9.3.3.] it follows that the spaces $\mathcal{A}$ and $\mathcal{A}'$ are identical to the spaces defined in [9, ch. 9.3 and 9.6] and denoted by the same letters.

As usual, we denote, for $f \in \mathcal{A}', f(\phi) = \langle f, \phi \rangle = (f, \bar{\phi})$. Also, following [9], we define a generalized differential operator $\mathcal{R}'$ on $\mathcal{A}'$ by

$$(\mathcal{R}'f, \phi) = (f, \mathcal{R}\phi), \ \ \phi \in \mathcal{A}, \ f \in \mathcal{A}'.$$

Since $\mathcal{R}$ is the linear and continuous mapping from $\mathcal{A}$ to $\mathcal{A}, \mathcal{R}'$ is the linear and continuous mapping from $\mathcal{A}'$ to $\mathcal{A}'$. Furthermore, since $\mathcal{R}$ is the self-adjoint operator, we shall write $\mathcal{R}$ instead of $\mathcal{R}'$.

3. Generalized random processes on $\Omega \times \mathcal{A}$

In [7] several representation theorems for the generalized random processes on $\Omega \times \mathcal{A}$ were proved. We shall list two of them, which we shall need in the sequel.

Let $(\Omega, \mathcal{F}, P)$ be a probability space. Throughout the paper we shall assume that $(\Omega, \mathcal{F}, P)$ is fixed.

Definition 3.1. *A generalized random process on $\Omega \times \mathcal{A}$ is a mapping ξ : $\Omega \times \mathcal{A} \to C$ such that*

$$(i)\ \ \forall \phi \in \mathcal{A},\ \ \xi(\cdot,\phi)\ \ \text{is a random variable,}$$
$$(ii)\ \ \forall \omega \in \Omega,\ \ \xi(\omega,\cdot)\ \ \text{is an element from } \mathcal{A}'.$$

Theorem 3.1. *Let ξ be a g.r.p. on $\Omega \times \mathcal{A}$. Suppose there exist random variable r, a set $Z \in \mathcal{F}$ with $P(Z) = 0$, and a non-negative integer k, such that $|\xi(\omega,\phi)| \le r(\omega)\|\phi\|_k$, for $\omega \in \Omega \backslash Z$, $\phi \in \mathcal{A}$. Then, there exists a sequence $\{c_n, n \in \mathrm{N}_0\}$, of random variables such that*

$$\xi(\omega,\phi) = \sum_{n=0}^{\infty} c_n(\omega)(\psi_n,\phi)\ \ \omega \in \Omega\backslash Z,\ \phi \in \mathcal{A},$$

and

$$[\sum_{n=0}^{\infty} |c_n(\omega)|^2 \tilde{\lambda}_n^{-2k}]^{1/2} < r(\omega),\ \ \omega \in \Omega\backslash Z.$$

The proof is given in [7].

We define the differential operator $\tilde{\mathcal{R}}^k$, $k \in \mathrm{N}_0$, on the set of g.r.p. -s by

$$\tilde{\mathcal{R}}^k \xi(\omega,\phi) = \xi(\omega,\mathcal{R}^k\phi)$$
$$\tilde{\mathcal{R}}^{k+1} = \tilde{\mathcal{R}}(\tilde{\mathcal{R}}^k),\ k \in \mathrm{N},\quad \tilde{\mathcal{R}}^0 = \mathcal{J}.$$

We shall denote $\tilde{\mathcal{R}}$ by $\mathcal{R}$.

In Theorem 3.5. [7] a representation of a g.r.p. on $\Omega \times \mathcal{A}$ by a continuous ordinary random process on $\Omega \times I$ was obtained. By a continuous stochastic process on $\Omega \times I$ we shall mean the process that for almost every $\omega \times \Omega$ is a continuous function on I.

Put $\Lambda = \{n \in \mathrm{N}_0 : \lambda_n = 0\}, \Lambda^c = \mathrm{N}_0 \backslash \Lambda$.

To obtain the representation with a continuous random process, in [7] the following assumptions on sequences $\{\psi_n,\ n \in \mathrm{N}_0\}$ and $\{\lambda_n, n \in \mathrm{N}_0\}$ were posed.
(*) there exists $s_0 \in \mathrm{N}_0$ and constant K, such that for $s \ge s_0$

$$\sup\{|\psi_n(t)/\tilde{\lambda}_n^s| : n \in \mathrm{N}_0, t \in I\} < K,$$

(**) there exists $p_0 \in \mathbf{N}_0$ such that for $p \geq p_0$,

$$\sum_{n \in \Lambda^c} \lambda_n^{-2p} < \infty.$$

Conditions (*) and (**) are not too restrictive. For example, Hermite, Fourier and Laguerre complete orthonormal systems satisfy these conditions. For other o.n.s. which satisfy these conditions we refer to [9, ch. 9.8] and [1, ch. 10.18].

Theorem 3.2. *Let ξ be a g.r.p on $\Omega \times \mathcal{A}$. Suppose that there exits a random variable r such that $E(r) < \infty$, a set $Z \in \mathcal{F}$, such that $P(Z) = 0$, a positive integer k_0, such that $|\xi(\omega, \phi)| \leq r(\omega)\|\phi\|_{k_0}$, for $\omega \in \Omega \backslash Z$, $\phi \in \mathcal{A}$. Then, for $k \geq k_0$, there exist a continuous random process $X_k(\omega, t)$ on $\Omega \times I$, and random variables c_n, $n \in \Lambda$, such that*

$$\xi(\omega, \phi) = \int_I X_k(\omega, t)\mathcal{R}^{k+p+s} \phi(t)dt + \sum_{n \in \Lambda} c_n(\omega)(\psi_n, \phi), \ \forall \omega \in \Omega \backslash Z, \ \phi \in \mathcal{A},$$

where $s \geq s_0, s_0$ is from (*), and $p \geq p_0, p_0$ is from (**).

The proof is given in [7]. We only note that X_k is of the form:

$$X_k(\omega, t) = \sum_{n \in \Lambda^c} c_n(\omega)\lambda_n^{-(k+p+s)}\psi_n(t), \ \omega \in \Omega, t \in I.$$

4. Expectation of g.r.p. on $\Omega \times \mathcal{A}$

We shall give representation theorems for the expectation of a g.r.p. on $\Omega \times \mathcal{A}$.

Definition 4.1. *Expectation of a g. r. p. denoted by $E[\xi]$ is a functional on $\mathcal{A}$ defined by*

$$E[\xi](\phi) = \int_\Omega \xi(\omega, \phi)dP(\omega).$$

provided that the integral on the right hand side is finite for every $\phi \in \mathcal{A}$.

Theorem 4.1. *Let ξ be a g.r.p. on $\Omega \times \mathcal{A}$. Suppose there exist random variable r, a set $Z \in \mathcal{F}$ with $P(Z) = 0$, and a non-negative integer k,* *such that* $|\xi(\omega,\phi)| \leq r(\omega)\|\phi\|_k$, *for* $\omega \in \Omega\backslash Z$, $\phi \in \mathcal{A}$. *Then, there exists a sequence* $\{c_n, n \in \mathbf{N}_0\}$, *of random variables such that*

$$\xi(\omega,\phi) = \sum_{n=0}^{\infty} c_n(\omega)(\psi_n,\phi) \;\; \omega \in \Omega\backslash Z, \phi \in calA,$$

and

$$[\sum_{n=0}^{\infty} |c_n(\omega)|^2 \tilde{\lambda}_n^{-2k}]^{1/2} < r(\omega), \omega \in \Omega\backslash Z.$$

Furthermore, if $E(r)$ *exists, then* $E(c_n) < \infty, n \in \mathbf{N}_0$ *and* $E[\xi]$ *exists and is given by*

$$E[\xi(\phi)] = \sum_{n=0}^{\infty} E(c_n)(\psi_n,\phi), \; \phi \in \mathcal{A}.$$

Moreover, there exist $p \in \mathbf{N}_0$ *such that*

$$\sum_{n=0}^{\infty} |E(c_n)|^2 \tilde{\lambda}_n^{-2p} < \infty.$$

Proof. The first part of the proof follows from Theorem 3.1.. Since

$$[\sum_{n=0}^{\infty} |c_n(\omega)|^2 \tilde{\lambda}_n^{-2k}]^{1/2} \leq r(\omega), \; \omega \in \Omega\backslash Z,$$

we have that for every $n \in \mathbf{N}_0$, $|c_n(\omega)| \leq \tilde{\lambda}_n^k r(\omega)$, $\omega \in \Omega\backslash Z$. It follows that

$$|E(c_n)| = |\int_\Omega c_n(\omega)dP(\omega)| \leq \tilde{\lambda}_n^k \int_\Omega r(\omega)dP(\omega) < \infty.$$

Further on, for $\phi = \sum_{n=0}^{\infty} a_n\psi_n \in \mathcal{A}$, $\omega \in \Omega$,

$$|E[\xi(\omega,\phi)]| = |\int_\Omega [\sum_{n=0}^{\infty} c_n(\omega)(\psi_n,\phi)]dP(\omega)| \leq$$

$$\leq \int_{\Omega\backslash Z} (\sum_{n=0}^{\infty} |c_n(\omega)\bar{a}_n|)dP(\omega) \leq$$

$$\leq \int_{\Omega\backslash Z} [\sum_{n=0}^{\infty} |c_n(\omega)|^2 \tilde{\lambda}_n^{-2k}]^{1/2} [\sum_{n=0}^{\infty} |a_n|^2 \tilde{\lambda}_n^{2k}]^{1/2} dP(\omega) \leq$$

$$\leq C \int_{\Omega\backslash Z} |r(\omega)| dP(\omega) < \infty.$$

So, by the Fubini theorem we have, for $\phi \in \mathcal{A}$, $\omega \in \Omega$,

$$E[\xi(\omega,\phi)] = \int_\Omega \xi(\omega,\phi)dP(\omega) =$$

$$= \int_{\Omega\setminus Z}[\sum_{n=0}^{\infty} c_n(\omega)(\psi_n,\phi)]dP(\omega) =$$

$$= \sum_{n=0}^{\infty}[\int_{\Omega\setminus Z} c_n(\omega)dP(\omega)](\psi_n,\phi) =$$

$$= \sum_{n=0}^{\infty} E(c_n)(\psi_n,\phi).$$

From the fact that $\sum_{n=0}^{\infty} E(c_n)(\psi_n,\phi)$ is finite for every $\phi \in \mathcal{A}$ and from [3, ch. 30.] it follows that $\exists p \in \mathbf{N}_0$ such that

$$\sum_{n=0}^{\infty} |E(c_n)|^2 \tilde{\lambda}_n^{-2p} < \infty. \; \square$$

Remark 4.1. *Note that we only claim that $E[\xi(\cdot)]$ is functional on $\mathcal{A}$. Clearly it is linear. It is easy to prove continuity. Namely, let $\phi_m \to \phi$ in $\mathcal{A}$ and $\phi \sum_{n=0}^{\infty} a_n\psi_n, \phi_m = \sum_{n=0}^{\infty} a_n^m \psi_n$. Then,*

$$|E[\xi(\phi_m)] - E[\xi(\phi)]| \le \sum_{n=0}^{\infty} |E(c_n)(\psi_n, \phi_m - \phi)| \le$$

$$\le (\sum_{n=0}^{\infty} |E(c_n)|^2 \; \tilde{\lambda}_n^{-2p})^{1/2} \; (\sum_{n=0}^{\infty} |a_n^m - a_n|^2 \; \tilde{\lambda}_n^{2p})^{1/2} \to 0.$$

Thus, we have that $E[\xi]$ is an element from $\mathcal{A}'$.

Theorem 4.2. *Let ξ be a g.r.p. on $\Omega\times\mathcal{A}$. Suppose that there exist a random variable r such that $E(r) < \infty$, a set $Z \in \mathcal{F}$, such that $P(Z) = 0$, a positive integer k_0, such that $|\xi(\omega,\phi)| \le r(\omega)\|\phi\|_{k_0}$, for $\omega \in \Omega\setminus Z$, $\phi \in \mathcal{A}$. Then, for $k \ge k_0$, there exist a continuous random process $X_k(\omega,t)$ on $\Omega \times I$, and random variables c_n, $n \in \Lambda$, such that*

$$\xi(\omega,\phi) = \int_I X_k(\omega,t)\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} c_n(\omega)(\psi_n,\phi), \forall\omega \in \Omega\setminus Z, \; \phi \in \mathcal{A},$$

where $s \geq s_0$, s_0 is from (*), and $p \geq p_0$, p_0 is from (**).

Further, if $E[X_k(\cdot,t)]$ exists for every $k \geq k_0$; $E[\xi]$ exist and is given by

$$E[\xi(\phi)] = \int_I E[X_k(\cdot,t)]\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} E(c_n)(\psi_n,\phi).$$

Proof. The first part of the proof follows from Theorem 3.2. For $\phi = \sum_{n=0}^{\infty} a_n\psi_n \in \mathcal{A}$ we have that

$$[\sum_{n=0}^{\infty} |a_n|^2 \bar{\lambda}_n^{2(k+p+s)}]^{1/2} = C < \infty.$$

Also, we have:

$$[\sum_{n\in\Lambda^c} |c_n(\omega)|^2 \lambda_n^{-2(k+p+s)}]^{1/2} \leq r(\omega), \ \ \omega \in \Omega\backslash Z.$$

Hence,

$$\int_{\Omega\backslash Z}\int_I |X_k(\omega,t)\mathcal{R}^{k+p+s}\phi(t)|dtdP(\omega) \leq$$

$$\int_{\Omega\backslash Z}[(\int_I |X_k(\omega,t)|^2 dt)^{1/2}(\int_I |\mathcal{R}^{k+p+s}\phi(t)|^2 dt)^{1/2}]dP(\omega) \leq$$

$$\leq \int_{\Omega\backslash Z}[(\sum_{n\in\Lambda^c} |c_n(\omega)|^2\ \lambda_n^{-2(k+p+s)})^{1/2}(\sum_{n\in\Lambda^c} |a_n|^2\ \lambda_n^{2(k+p+s)})^{1/2}]dP(\omega) \leq$$

$$\leq C\int_{\Omega\backslash Z} |r(\omega)|dP(\omega) < \infty.$$

From Theorem 3.1. it follows that $E(c_n)$ exist for every $n \in \mathbf{N}_0$ and applyng Fubini's theorem we have for every $\phi \in \mathcal{A}$,

$$E[\xi(\phi)] = \int_\Omega \xi(\omega,\phi)dP(\omega) = \int_{\Omega\backslash Z} \xi(\omega,\phi)dP(\omega) =$$

$$= \int_{\Omega\backslash Z}[\int_I X_k(\omega,t)\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} c_n(\omega)(\psi_n,\phi)]dP(\omega) =$$

$$= \int_I[\int_{\Omega\backslash Z} X_k(\omega,t)dP(\omega)]\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda}[\int_{\Omega\backslash Z} c_n(\omega)dP(\omega)](\psi_n,\phi) =$$

$$= \int_I E[X_k(\cdot,t)]\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} E(c_n)(\psi_n,\phi).\square$$

5. Conditional expectation of g. r. p. on $\Omega \times \mathcal{A}$

We shall give the representation theorem for the conditional expectation of g.r.p., given a sub σ-field $\mathcal{U}$ of $\mathcal{F}$, denoted, as usual, $E^{\mathcal{U}}$. Our theorem is similar to Theorem 5. of [4].

Lemma 5.1. *Let $X(\omega,t) : \Omega \times I \to \mathbf{C}$ be a continuous random process. Suppose there exists a random variable r such that $E(r) < \infty$, and for every $t \in I$ and almost every $\omega \in \Omega$, $|X(\omega,t)| \le c(t)r(\omega)$, where $c(t)$ is bounded on I. Then for arbitrary sub σ-field $\mathcal{U}$ of $\mathcal{F}$ the function $g(\omega,t) = E^{\mathcal{U}}(X(\omega,t))$, $\omega \in \Omega$, $t \in I$, is $\mathcal{U} \times \mathcal{B}(I)$ measurable, where $\mathcal{B}(I)$ is σ-field generated by Borel sets in I.*

Proof. For every $t \in I$ we have that $g(\cdot,t)$ is $\mathcal{U}$-measurable. We shall show that for almost every $\omega \in \Omega$ $g(\omega,\cdot)$ is continuous on I. Let $\{t_n, n \in \mathrm{N}_0\}$ be a sequence in I such that $t_n \to t_0$ and for each $\omega \in \Omega$ put $Y_n(\omega) = X(\omega,t_n)$. Since $X(\omega,\cdot)$ is continuous for almost every $\omega \in \Omega$, we have

$$Y_n(\cdot) \to Y_0(\cdot) = X(\cdot,t_0), \ \ a.e. \quad n \to \infty.$$

Furthermore, since for $t \in I$, $|X(\cdot,t)| \le c(t)r(\cdot) a.e.$ on Ω according to the Fatou-Lebesgue convergence theorem [5, ch. 25.1], and since r is integrable we have that

$$E^{\mathcal{U}}(Y_n) \to E^{\mathcal{U}}(Y_0) \ a.e.$$

Hence, for almost every $\omega \in \Omega$, $g(\omega,t_n) = E^{\mathcal{U}}(X(\omega,t_n)) = E^{\mathcal{U}}(Y_n)$ converges to $g(\omega,t_0) = E^{\mathcal{U}}(Y_0)$, so $g(\omega,\cdot)$ is continuous. Therefore $g(\cdot,\cdot)$ is $\mathcal{U} \times \mathcal{B}(I)$ measurable. Furthermore, since $X(\omega,\cdot)$ is continuous we have that X is $\mathcal{F} \times \mathcal{B}(I)$ measurable.□

Let $P_{\mathcal{U}}$ the restriction of P to $\mathcal{U}$ defined by

$$P_{\mathcal{U}}(B) = P(B), \ \ B \in \mathcal{U}.$$

Definition 5.1. *Let ξ be g.r.p. on $\Omega \in \mathcal{A}$ and $\mathcal{U}$ be a sub σ-field of $\mathcal{F}$. Conditional expectation of ξ with respect to $\mathcal{U}$, denoted by $E^{\mathcal{U}}[\xi] = E^{\mathcal{U}}[\xi(\phi)]$, is for every $\phi \in \mathcal{A}$, $\mathcal{U}$-measurable function defined up to $P_{\mathcal{U}}$ equivalence by*

$$\int_B E^{\mathcal{U}}[\xi(\phi)] dP_{\mathcal{U}}(\omega) = \int_B \xi(\omega,\phi) dP(\omega), \ \ B \in \mathcal{U}, \ \phi \in \mathcal{A}.$$

Theorem 5.1. *Let* ξ *be a g.r.p. on* $\Omega \times \mathcal{A}$. *Suppose that there exist a random variable* r *such that* $E(r) < \infty$, *a measurable set* $Z \subset \Omega$, *such that* $P(Z) = 0$, *and positive integer* k_0 *such that* $|\xi(\omega, \phi)| \leq r(\omega)||\phi||_{k_0}$ *for* $\omega \in \Omega \backslash Z$, $\phi \in \mathcal{A}$. *Then for* $k \geq k_0$, *there exists a continuous ordinary stochastic process* $X_k(\omega, t)$ *on* $\Omega \times I$, *and random variables on* Ω, c_n, $n \in \Lambda$, *such that, for every* $\omega \in \Omega \backslash Z$ *and* $\phi \in \mathcal{A}$,

$$(1) \qquad \xi(\omega, \phi) = \int_I X_k(\omega, t)\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} c_n(\omega)(\psi_n, \phi),$$

where s *is from (*) and* $p \geq p_0$, p_0 *is from (**). Furthermore,*

$$E^{\mathcal{U}}[\xi(\phi)] = \int_I E^{\mathcal{U}}[X_k(\omega, t)]\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} E^{\mathcal{U}}c_n(\omega)(\psi_n, \phi), \ \omega \in \Omega \backslash Z.$$

Proof. Let $k \geq k_0$ be fixed. Then (1) follows from Theorem 3.2.. According to Lemma 4.1., $E^{\mathcal{U}}(X_k(\cdot,\cdot))$, are measurable functions on $\mathcal{U} \times \mathcal{B}(I)$ and $X_k(\omega, t)$, are $\mathcal{F} \times \mathcal{B}(I)$ measurable. From Theorem 4.1. $E(c_n) < \infty$, $n \in \mathbf{N}_0$.

In the same way as in Theorem 4.2., we can show that

$$|\int_B [\int_I X_k(\omega, t)\mathcal{R}^{k+p+s}\phi(t)dt]dP(\omega)| \leq K\int_B |r(\omega)|dP(\omega) < \infty,$$

and according to the Fubini theorem, for every $\phi \in \mathcal{A}$, we have that

$$\int_B E^{\mathcal{U}}[\xi(\phi)]dP_{\mathcal{U}}(\omega) = \int_B \xi(\omega, \phi)dP(\omega) =$$

$$= \int_B [\int_I X_k(\omega, t)\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} c_n(\omega)(\psi_n, \phi)]dP(\omega) =$$

$$= \int_I [\int_B X_k(\omega, \phi)dP(\omega)]\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} [\int_B c_n(\omega)dP(\omega)](\psi_n, \phi) =$$

$$= \int_I [\int_B E^{\mathcal{U}}[X_k(\omega, t)dP_{\mathcal{U}}(\omega)]\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} [\int_B E^{\mathcal{U}}(c_n)dP_{\mathcal{U}}(\omega)](\psi_n, \phi) =$$

$$= \int_B [\int_I E^{\mathcal{U}}[X_k(\omega, t)]\mathcal{R}^{k+p+s}\phi(t)dt + \sum_{n\in\Lambda} E^{\mathcal{U}}(c_n)(\psi_n, \phi)]dP_{\mathcal{U}}(\omega). \ \square$$

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REZIME

OČEKIVANJE UOPŠTENOG SLUČAJNOG PROCESA NA PROSTORU ZEMANIANA $\mathcal{A}$

Date su reprezentacione teoreme za matematičko očekivanje i uslovno matematičko očekivanje uopštenog slučajnog procesa na prostoru Zemaniana $\mathcal{A}$.

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