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RANDOM DIRECTIONAL CONTRACTORS AND THE SOLUTION TO A SYSTEM OF RANDOM MULTIVALUED OPERATOR EQUATIONS

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Abstract

In the paper, as a generalization of random contractors, the author study the concept of random directional contractor and its applications to the solvability problem for a system of nonlinear random multivalued operator equations. As consequences, the improvement and generalization of some results in [1,2,4-13] is obtained.

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1. Introduction

AS A RANDOM generalization of contractor theory of Altman [1], Lee and Padgett [2,3] studied the concept of random contractor and its applications to the solution of nonlinear random operator equations. Reddy and Subrahmanyam [4,5,6] studied the relationship between the contractor and directional contractor theory of Altman and the fixed theorem of Matkowski [7,8], and showed some existence theorems for a system of point-valued and multivalued operator equations.

In [9,10] we investigated the relationship of random contractor and random Matkowski's fixed point theorem and obtained several existence and uniqueness theorems for the solutions to systems of nonlinear random point-valued and multivalued operator equations.

In the paper, as a generalization of random contractors, we shall study the concept of random directional contractor and its applications to the solvability problem for a system of nonlinear random multivalued operator equations. As Consequences, we obtain the improvement and generalization of some results in [1,2,4-13].

2. Preliminary definitions and lemmas

Let $(\Omega, \mathcal{A}, \mathcal{P})$ be a complete probability measure space and let $(X, \mathcal{B})$ and $(Y, \mathcal{C})$ be measurable spaces, where X and Y are separable Banach spaces and $\mathcal{B}$ and $\mathcal{C}$ are σ-algebras of subsets of X and Y, respectivily. $CB(X)$ denotes the family of all nonempty bounded closed subsets of X. For any $x \in X$, $A \subset X$, $D(x, A) = inf\{\| x - a \|: a \in A\}$. The function $H : CB(X) \times CB(X) \to [0, \infty)$ denotes the Hausdorff metric on $CB(X)$ induced by the norm of X.

A mapping $x : \Omega \to X$ is a X-valued random variable if $x^{-1}(B) = \{\omega \in \Omega : x(\omega) \in B\} \in \mathcal{A}$ for each $B \in \mathcal{B}$. A mapping $T : \Omega \to CB(X)$ is said to be a $CB(X)$-valued random variable if $T^{-1}(B) = \{\omega \in \Omega : T(\omega) \cap B \neq \emptyset\} \in \mathcal{A}$ for each $B \in \mathcal{B}$. A mapping $x : \Omega \to X$ is called a measurable selection of T if $x(\omega)$ is a X-valued random variable and $x(\omega) \in T(\omega)$ for all $\omega \in \Omega$.

New let $(X_i, \mathcal{B}_i)$ and $(Y_i, \mathcal{C}_i)$ $i = 1, \ldots, n$ be measurable spaces, where X_i and Y_i $i = 1, \ldots, n$ are separable Banach spaces and $\mathcal{B}_i$ and $\mathcal{C}_i$ are σ-algebra of subsets of X_i and Y_i $i = 1, \ldots, n$ respectively.

Definition 2.1. *A mapping $T(\omega) : \Omega \times D \subset \Omega \times X_1 \times \ldots \times X_n \to CB(Y)$ is said to be a random multivalued operator if $T(\omega)(x_1, \ldots, x_n)$ is a $CB(Y)$-valued random variable for each $(x_1, \ldots, x_n) \in D$. $T(\omega)$ is said to be almost surely (a.s.) continuous if $(x_1^m, \ldots x_n^m) \in D$, $m = 0, 1, \ldots$ and $x_i^m \to x_i^0$ $i = 1, \ldots, n$ imply $T(\omega)(x_1^m, \ldots, x_n^m) \to T(\omega)(x_1^0, \ldots, x_n^0)$ a.s. as $m \to \infty$, under the Hausdorff metric H. $T(\omega)$ is called a.s. closed if $(x_1^m, \ldots, x_n^m) \in D$, $y^m(\omega) \in T(\omega)(x_1^m, \ldots, x_n^m)$ $m = 1, 2, \ldots,$ $x_i^m \to x_i^*$ $i = 1, \ldots, n$ and $y^m(\omega) \to y^*(\omega)$ a.s. imply $(x_1^*, \ldots, x_n^*) \in D$ and $y^*(\omega) \in T(\omega)(x_1^*, \ldots, x_n^*)$*

a.s.

Remark 2.1. Obviously, if D is a closed subset of $X_1 \times \ldots \times X_n$ then $T(\omega)$ is a.s. closed whenever $T(\omega)$ is a.s. continuous.

Definition 2.2. *A random operator $\Gamma : \Omega \times Y \to X$ is said to be (a) linear if $\Gamma(\omega, \alpha y_1 + \beta y_2) = \alpha\Gamma(\omega, y_1) + \beta\Gamma(\omega, y_2)$ a.s. for all $y_1, y_2 \in Y, \alpha, \beta$, scalars, and (b) bounded if there exists a nonnegative realvalued random variable $M(\omega)$ such that for all $y_1, y_2 \in Y, \| \Gamma(\omega, y_1 - y_2) \| \leq M(\omega) \| y_1 - y_2 \|$ a.s.*

Let $L(Y, X)$ denotes the set of all bounded linear operators on a Banach space Y into a Banach space X. Obviously, $L(Y, X)$ is a Banach space with the norm given by

$$\| T \| = \sup_{\|Y\| \leq 1} \| Ty \|, \quad T \in L(Y, X) \text{ and } y \in Y.$$

Lemma 2.1. *([2,3]) Assume $\Gamma : \Omega \times Y \to L(Y, X)$ is an a.s. continuous random operator. Then $\Gamma(\omega, z(\omega))y(\omega)$ is a X-valued random variable whenever $z(\omega)$ and $y(\omega)$ are Y-valued random variables.*

Lemma 2.2. *([11]) Let $T(\omega) : \Omega \times D \subset \Omega \times X_1 \times \ldots \times X_n \to CB(Y)$ be an a.s. continuous random multivalued operator and $x_i(\omega)$ is a X_i-valued random variable, $i = 1, \ldots, n$. Then $T(\omega)(x_1(\omega), \ldots, x_n(\omega))$ is a $CB(Y)$-valued random variable.*

Lemma 2.3. *([12]) Let $S, T : \Omega \to CB(Y)$ be $CB(Y)$-valued random variables and let $u : \Omega \to Y$ be a measurable selection of S. Then for any real-valued random function $S : \Omega \to (0, \infty)$, there exists a measurable selection $v : \Omega \to Y$ of T such that*

$$\| u(\omega) - v(\omega) \| \leq H(S(\omega), T(\omega)) + S(\omega).$$

Let $(a_{i,k}(\omega))$ be a $n \times n$ matrix, where $a_{i,k} : \Omega \to [0, \infty)$, $i, k = 1, \ldots, n$ are real-valued random functions.
Define

$$a^1_{i,k}(\omega) = \begin{cases} a_{i,k}(\omega) & i \neq k \\ 1 - a_{i,k}(\omega) & i = k;\ i, k = 1, \ldots, n \end{cases} \tag{1}$$

$$(2)\qquad a_{i,k}^{l+1}(\omega)=\begin{cases} a_{1,1}^{l}(\omega)a_{i+1,k+1}^{l}(\omega)+a_{i+1,1}^{l}(\omega)a_{1,k+1}^{l}(\omega) & i\neq k\\ a_{1,1}^{l}(\omega)a_{i+1,k+1}^{l}(\omega)-a_{i+1,1}^{l}(\omega)a_{1,k+1}^{l}(\omega) & i=k\end{cases}$$

$$l=1,\dots,n-1;\ i,k=1,\dots,n-l$$

Lemma 2.4. *([9,11]) Let $a_{i,k}(\omega)$, $i,k=1,\dots,n$ be nonnegative real-valued random functions. Then the system of random inequalities*

$$(3)\qquad \sum_{k=1}^{n} a_{i,k}(\omega)\cdot r_k < r_i \quad a.s. \quad i=1,\dots,n$$

has a random positive solution $(r_1(\omega),\dots,r_n(\omega))$, (i.e. there exist random functions $r_i(\omega)>0$ a.s. $i=1,\dots,n$ such taht $\sum_{k=1}^{n} a_{i,k}(\omega)r_k(\omega) < r_i(\omega)$ a.s. $i=1,\dots,,n$) if and only if

$$(4)\qquad a_{i,i}^{l}(\omega)>0 \quad a.s. \quad i=1,\dots,n+1-l;\ \ l=1,\dots,n.$$

Remark 2.2. Lemma 2.4 is the random generalization of Matkowski's lemma in [7,8].

Suppose that $(r_1(\omega),\dots,r_n(\omega))$ is a random positive solution of the system (3). Define $q(\omega)=\max_{1\le i\le n}(r_i^{-1}(\omega)\sum_{k=1}^{n} a_{i,k}(\omega)\cdot r_k(\omega))$. Then we have

$$(5)\qquad \sum_{k=1}^{n} a_{i,k}(\omega)\cdot r_k(\omega)\le q(\omega)\cdot r_i(\omega) \quad \text{a.s.} \quad i=1,\dots,n.$$

3. Main results

In order to prove solvability theorems for the system of nonlinear random multivalued operator equations, we begin with the following:

Theorem 3.1. *Let $T_i(\omega):\Omega\times D\subset\Omega\times X_1\times\ldots\times X_n\to CB(Y_i)$, $i=1,\dots,n$, D being a vector space, be a.s. contnuous random multivalued operators, and let $\Gamma_i(\cdot,x_i):\Omega\times Y_i\to X_i$, $i=1,\dots,n$ be bounded linear random operators for $(x_1,\dots,x_n)\in D$. Suppose that the following conditions are satisfied:*

(i) For all $(x_1,\dots,x_n)\in D$

$$(6)\qquad \|\Gamma_i(\omega,x_i)\|\le B(\omega) \quad a.s.\ , \quad i=1,\dots,n$$

where $B(\omega)$ is a positive real-valued random variable, *(ii) For all* $(x_1,\dots,x_n)\in D$ *and* $y_i\in Y_i,\ \ i=1,\dots,n$

$$(\Gamma_1(\omega,x_1)y_1,\dots,\Gamma_n(\omega,x_n)y_n)\in D$$

and there exists a positive number $\varepsilon=\varepsilon(x_1,\dots,x_n;\ y_1,\dots,y_n)\le 1$ *such that*

$$\begin{aligned}&H_i(T_i(\omega)(x_1+\varepsilon\Gamma_1(\omega,x_1)y_1,\dots,x_n+\varepsilon\Gamma_n(\omega,x_n)y_n),T_i(\omega)(x_1,\dots,x_n)+\varepsilon y_n)\\ (7)\qquad &\le\varepsilon\{\sum_{k=1}^{n}b_{i,k}(\omega)\parallel y_k\parallel+\sum_{k=1}^{n}c_{i,k}(\omega)D_k(0_k,T_k(\omega)(x_1,\dots,x_n))\\ &+c(\omega)D_i(0_i,T_i(\omega)(x_1+\varepsilon\Gamma(\omega,x_1)y_1,\dots,x_n+\varepsilon\Gamma_n(\omega,x_n)y_n))\}\\ &\qquad a.s.\ \ i=1,\dots,n,\end{aligned}$$

where $b_{i,k}(\omega),c_{i,k}(\omega),\ \ i,k=1,\dots,n,$ *and* $c(\omega)$ *are nonnegative real-valued random functions and* 0_i *is the zero element of the Banach space* $Y_i,\ \ i=1,\dots,n;\ \ a_{i,k}(\omega)=b_{i,k}(\omega)+c_{i,k}(\omega),\ \ i,k=1,\dots,n$ *are such that* $a^{l}_{i,k}(\omega)$ *defined by (1) and (2) satisfy (4) and* $c(\omega)$ *satisfies*

$$(8)\ 0\le c(\omega)<min\{(1-q(\omega)),\frac{1}{\varepsilon}\{1-[1-(1-q(\omega))\varepsilon]\}e^{+(1-q(\omega))\varepsilon}\}\quad a.s.$$

where $q(\omega)=\max_{1\le i\le n}\{r_i^{-1}(\omega)\sum_{k=1}^{n}a_{i,k}(\omega)\cdot r_i(\omega)\}$ *and* $r_i(\omega),\ \ i=1,\dots,n$ *are positive random solution of (3).*

Then there exist X_i*-valued random variables* $x_i^*(\omega),\ \ i=1,\dots,n$ *such that*

$$(x_1^*(\omega),\dots,x_n^*(\omega))\in D\quad a.s.$$

and

$$0_i\in T_i(\omega)(x_1^*(\omega),\dots,x_n^*(\omega))\quad a.s.,\quad i=1,\dots,n.$$

Proof. To prove the theorem we construct well-ordered sequences of real numbers t_α and X_i-valued random variable $x_i^\alpha(\omega)$ and Y_i-valued random variable $y_i^\alpha(\omega),\ \ i=1,\dots,n$ such that $y_i^\alpha(\omega)$ is a measurable selection of $T_i(\omega)(x_1^\alpha(\omega),\dots,x_n^\alpha(\omega)),\ \ i=1,\dots,n$. Let $t_0=0$ and $(x^0,\dots,x_n^0)\in D$ be arbitrarily chosen. Letting $x_i^0(\omega)=x_i^0,\ \ i=1,\dots,n$, Lemma 2.2, $T_i(\omega)(x_1^0(\omega),\dots,x_n^0(\omega))$ is a $CB(Y_i)$-valued random variable, $i=1,\dots,n$. It follows from [13, Theorem III.6] that there exists a measurable selection y_i^0 of $T_i(\omega)(x_1^0(\omega),\dots,x_n^0(\omega)),\ \ i=1,\dots,n$. Since the set of random positive solutions of the system (3) is closed with respect to multiplication by a positive real number, without loss of generality, we can assume that

$$(9) \qquad \| y_i^0(\omega) \| \leq r_i(\omega) \quad \text{a.s.} \quad i = 1, \dots, n.$$

Suppose that t_ν, $x_i^\nu(\omega)$, $y_i^\nu(\omega) \in T_i(\omega)(x_1^\nu(\omega), \dots, x_n^\nu(\omega))$, $i = 1, \dots, n$ have been constructed for all ordinal number $\nu < \alpha$ satisfying the following conditions:

For arbitrary ordinal number $\nu < \alpha$ the following inequality holds:

$$(10) \qquad \| y_i^\nu(\omega) \| \leq e^{-(1-q(\omega))t_\nu} r_i(\omega) \quad \text{a.s.} \quad i = 1, \dots, n$$

For the first kind ordinal numbers $\nu + 1 < \alpha$ the following inequalities are satisfied:

$$(11) \quad \| x_i^{\nu+1}(\omega) - x_i^\nu(\omega) \| \leq B(\omega) e^{-(1-q(\omega))t_\nu} (t_{\nu+1} - t_\nu) r_i(\omega) \quad \text{a.s.} \quad i = 1, \dots, n;$$

$$(12) \| y_i^{\nu+1}(\omega) - y_i^\alpha(\omega) \| \leq 2(1 + q(\omega)) e^{-(1-q(\omega))t_\nu} (t_{\nu+1} - t_\nu) r_i(\omega) \text{ a.s.} \quad i = 1, \dots, n;$$

$$(13) \quad 0 < \tau_{\nu+1} = t_{\nu+1} - t_\nu \leq (1 - q(\omega))^{-1} ln(1 - q(\omega))(1 - \overline{q}(\omega))^{-1} \text{ a.s.}$$

where $\overline{q}(\omega)$ is a real-valued random variable such that $q(\omega) < \overline{q}(\omega) < 1$ a.s. and $1 - q(\omega) < ln(1 - q(\omega))(1 - \overline{q}(\omega))^{-1}$ a.s.

For second kind ordinal numbers $\nu < \alpha$ the following relations hold

$$(14) t_\nu = \lim_{\beta \nearrow \nu} t_\beta; \quad x_i^\nu(\omega) = \lim_{\beta \nearrow \nu} x_i^\beta(\omega) \quad \text{a.s.}; \quad y_i^\nu(\omega) = \lim_{\beta \nearrow \nu} y_i^\beta(\omega) \quad \text{a.s.} \quad i = 1, \dots, n.$$

Now we shall show the following estimates:

For arbitrary ordinal numbers $\nu < \alpha$ and $\lambda < \alpha$, by (11), (13) and Lemma 2.3 of [6], we have

$$\| x_i^\nu(\omega) - x_i^\lambda(\omega) \| \leq \sum_{\lambda \leq \beta < \nu} \| x_i^{\beta+1}(\omega) - x_i^\beta(\omega) \|$$

$$\leq B(\omega) r_i(\omega) \sum_{\lambda \leq \beta < \nu} e^{-(1-q(\omega))t_\beta} (t_{\beta+1} - t_\beta)$$

$$= B(\omega)r_i(\omega) \sum_{\lambda\leq\beta<\nu} e^{(1-q(\omega))(t_{\beta+1}-t_\beta)} e^{-(1-q(\omega))t_{\beta+1}}(t_{\beta+1} - t_\beta)$$

$$\leq (1 - q(\omega))(1 - \overline{q}(\omega))^{-1}B(\omega)r_i(\omega) \sum_{\lambda\leq\beta<\nu} e^{-(1-q(\omega))t_{\beta+1}}(t_{\beta+1} - t_\beta)$$

$$< (1 - q(\omega))(1 - \overline{q}(\omega))^{-1}B(\omega)r_i(\omega) \sum_{\lambda\leq\beta<\nu} \int_{t_\beta}^{t_{\beta+1}} e^{-(1-q(\omega))t}dt$$

$$(15) \qquad < (1 - q(\omega))(1 - \overline{q}(\omega))^{-1}B(\omega)r_i(\omega) \int_{t_\lambda}^{t_\nu} e^{-(1-q(\omega))t}dt \quad \text{a.s.}$$

$$i = 1, \ldots, n.$$

Proceeding as above, we can prove

$$\| y_i^\nu(\omega) - y_i^\lambda(\omega) \| \leq 2(1 - q^2(\omega))(1 - \overline{q}(\omega))^{-1}r_i(\omega) \int_{t_\lambda}^{t_\nu} e^{-(1-q(\omega))t}dt \quad \text{a.s.}$$

$$(16) \qquad i = 1, \ldots, n.$$

Suppose that α is a first kind ordinal number. If $0_i \in T_i(\omega)(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega))$ a.s., $i = 1, \ldots, n$, then the proof of the theorem is complete. New suppose $0_i \notin T_i(\omega)(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega))$ a.s., $i = 1, \ldots, n$. Since $T_i(\omega)(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega))$ is a $CB(Y_i)$-valued random variable, $i = 1, \ldots, n$, it follows from [13, Theorem III.6] that there exists a measurable selection $y_i^{\alpha-1}(\omega)$ of $T_i(\omega)(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega))$, $i = 1, \ldots, n$. By the hypothesis (ii) of the theorem, for $(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega)) \in D$ a.s. and $y_i^{\alpha-1}(\omega) \in Y_i$ $i = 1, \ldots, n$, there exists $\varepsilon \leq 1$ satisfying (7). Define $t_\alpha = t_{\alpha-1} + \varepsilon$ and let $\tau_\alpha = \varepsilon = t_\alpha - t_{\alpha-1}$. As $(1 - q(\omega)) < ln(1 - q(\omega))(1 - \overline{q}(\omega))^{-1}$ a.s., τ_α satisfies (13).

Define

$$(17) \qquad x_i^\alpha(\omega) = x_i^{\alpha-1}(\omega) - \tau_\alpha\Gamma_i(\omega, x_i^{\alpha-1}(\omega))y_i^{\alpha-1}(\omega), \;\; i = 1, \ldots, n.$$

Replacing x_i by $x_i^{\alpha-1}(\omega)$, y_i by $-y_i^{\alpha-1}(\omega)$ in (7) and using (17) we obtain

$$H_i(T_i(\omega)(x_1^\alpha(\omega), \ldots, x_n^\alpha(\omega)), T_i(\omega)(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega)) - \tau_\alpha y_i^{\alpha-1}(\omega)) \leq$$

$$\tau_\alpha\{\sum_{k=1}^n b_{i,k}(\omega) \| y_i^{\alpha-1}(\omega) \| + \sum_{k=1}^n c_{i,k}(\omega)D_k(0_k, T_k(\omega)(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega)))$$

$$(18) \qquad + c(\omega)D_i(0_i, T_i(\omega)(x_1^\alpha(\omega), \ldots, x_n^\alpha(\omega)))\} \quad \text{a.s.}, \; i = 1, \ldots, n.$$

Since $(1 - \tau_\alpha)y_i^{\alpha-1}(\omega)$ is a measurable selection of $T_i(\omega)(x_1^{\alpha-1}(\omega), \ldots, x_n^{\alpha-1}(\omega)) - \tau_\alpha y_i^{\alpha-1}(\omega)$, $i = 1, \ldots, n$, for real-valued random function $s_i(\omega) > 0$, by Lemma 2.3 there exists a measurable selection $y_i^\alpha(\omega)$ of $T_i(\omega)(x_1^\alpha(\omega),\ldots,x_n^\alpha(\omega)), i=1,\ldots,n$ such that

$$\| y_i^\alpha(\omega)-(1-\tau_\alpha)y_i^{\alpha-1}(\omega)\| \leq H_i(T_i(\omega)(x_1^\alpha(\omega),\ldots,x_n^\alpha(\omega)),T_i(\omega)(x_1^{\alpha-1}(\omega),\ldots,x_n^{\alpha-1}(\omega))-\tau_\alpha y_i^{\alpha-1}(\omega))+s_i(\omega)\quad i=1,\ldots,n. \tag{19}$$

It follows from (18) and (19) that

$$\| y_i^\alpha(\omega)-(1-\tau_\alpha)y_i^{\alpha-1}(\omega)\| \leq \tau_\alpha\{\sum_{k=1}^n b_{i,k}(\omega)\| y_k^{\alpha-1}(\omega)\| + +\sum_{k=1}^n c_{i,k}(\omega)\| y_i^{\alpha-1}(\omega)\| +c(\omega)\| y_i^\alpha(\omega)\|\}+s_i(\omega)\ \text{a.s.}\quad i=1,\ldots,n. \tag{20}$$

Therefore

$$\| y_i^\alpha(\omega)\| \leq (1-\tau_\alpha)\| y_i^{\alpha-1}(\omega)\| +\tau_\alpha\{\sum_{k=1}^n a_{i,k}(\omega)\| y_i^{\alpha-1}(\omega)\| + +c(\omega)\| y_i^\alpha(\omega)\|\}+s_i(\omega)\quad \text{a.s.}$$

From (8) and (10) and (5) we have

$$\| y_i^\alpha(\omega)\| \leq \frac{1}{1-\tau_\alpha c(\omega)}\{[1-(1-q(\omega))\tau_\alpha]e^{-(1-q(\omega))t_{\alpha-1}}r_i(\omega)+s_i(\omega)\}\quad \text{a.s. } i=1,\ldots,n. \tag{21}$$

By (20), (21), (8), (10) and (5), we also have

$$\| y_i^\alpha(\omega)-y_i^{\alpha-1}(\omega)\| \leq \tau_\alpha\| y_i^{\alpha-1}(\omega)\| +\tau_\alpha\{\sum_{k=1}^n a_{i,k}(\omega)\| y_i^{\alpha-1}(\omega)\| + c(\omega)\| y_i^\alpha(\omega)\|\}+s_i(\omega)\leq(1+q(\omega))\tau_\alpha e^{-(1-q(\omega))t_{\alpha-1}}r_i(\omega)+\frac{\tau_\alpha c(\omega)}{1-\tau_\alpha c(\omega)}\{[1-(1-q(\omega))\tau_\alpha]e^{-(1-q(\omega))t_{\alpha-1}}r_i(\omega)+s_i(\omega)\}+s_i(\omega)\leq\{1+q(\omega)+\frac{c(\omega)}{1-\tau_\alpha c(\omega)}[1-(1-q(\omega))\tau_\alpha]\}e^{-(1-q(\omega))t_{\alpha-1}}r_i(\omega)\tau_\alpha+\frac{s_i(\omega)}{1-\tau_\alpha c(\omega)}\quad \text{a.s.},\quad i=1,\ldots,n.$$

By (8) it is easy to obtain

$$\frac{c(\omega)}{1-\tau_\alpha c(\omega)}[1-(1-q(\omega))\tau_\alpha]\leq 1\quad \text{a.s.}$$

Hence we have

$$\| y_i^\alpha(\omega) - y_i^{\alpha-1}(\omega) \| \leq [2 + q(\omega)]e^{-(1-q(\omega))t_{\alpha-1}} r_i(\omega)\tau_\alpha + \frac{s_i(\omega)}{1 - \tau_\alpha c(\omega)} \tag{22}$$
$$\text{a.s.} i = 1, \dots, n.$$

Choose

$$s_i(\omega) = min\{(1 - \tau_\alpha c(\omega))r_i(\omega)e^{-(1-q(\omega))t_\alpha} - [1 - (1 - q(\omega))\tau_\alpha] \\ e^{-(1-q(\omega))t_{\alpha-1}} r_i(\omega), (1 - \tau_\alpha c(\omega))q(\omega)e^{-(1-q(\omega))t_{\alpha-1}} r_i(\omega)\tau_\alpha\}.$$

By (8), $s_i(\omega)$, $i = 1, \dots, n$ are positive real-valued random functions. From (21) and (22) we get

$$\| y_i^\alpha(\omega) \| \leq e^{-(1-q(\omega))t_\alpha} r_i(\omega) \quad \text{a.s.} \quad i = 1, \dots, n, \tag{23}$$

$$\| y_i^\alpha(\omega) - y_i^{\alpha-1}(\omega) \| \leq 2(1 + q(\omega))e^{-(1-q(\omega))t_{\alpha-1}}(t_\alpha - t_{\alpha-1})r_i(\omega) \quad \text{a.s.} \tag{24}$$
$$i = 1, \dots, n.$$

By (17) we obtain

$$\| x_i^\alpha(\omega) - x_i^{\alpha-1}(\omega) \| \leq B(\omega) \| y_i^{\alpha-1}(\omega) \| \tau_\alpha \tag{25}$$
$$\leq B(\omega)e^{-(1-q(\omega))t_{\alpha-1}}(t_\alpha - -t_{\alpha-1})r_i(\omega) \quad \text{a.s.} \quad i = 1, \dots, n.$$

Thus the conditions (10), (11), (12) and (13) are satisfied for first kind ordinal number α. Now, suppose that α is an ordinal number of second kind and put $t_\alpha = \lim_{\nu \nearrow \alpha} t_\nu$. Let ν_n be an increasing sequence converging to α. It follows from (15) that $\{x_i^{\nu_n}(\omega)\}$ is a a.s. Cauchy sequence and so is $\{x_i^\nu(\omega)\}$. Let $x_i^\nu(\omega) \to x_i^\alpha(\omega)$ a.s. $i = 1, \dots, n$. Similarly, from (16) $\{y_i^\nu(\omega)\}$ also a.s. converges to $y_i^\alpha(\omega)$. Since $T_i(\omega)$, $i = 1, \dots, n$ are a.s. closed, it follows that $(x_1^\alpha(\omega), \dots, x_n^\alpha(\omega)) \in D$ a.s. and $y_i^\alpha(\omega) \in T_i(\omega)(x_1^\alpha(\omega), \dots, x_n^\alpha(\omega))$ a.s., $i = 1, \dots, n$. If $t_\alpha = \infty$, then from (10), $y_i^\alpha(\omega) = 0_i$ a.s., $i = 1, \dots, n$. In this case, we have that $x_i^\alpha(\omega)$ is X_i-valued random variable, $i = 1, \dots, n$ such that $(x_1^\alpha(\omega), \dots, x_n^\alpha(\omega)) \in D$ a.s. and $0_i \in T_i(\omega)(x_1^\alpha(\omega), \dots, x_n^\alpha(\omega))$ a.s., $i = 1, \dots, n$. If $t_\alpha < \infty$ we proceed with the iteration till we reach a bigger ordinal number λ such that $t_\lambda = +\infty$. Clearly in the case we can find X_i-valued random variable $x_i^*(\omega)$, $i = 1, \dots, n$ such that $x_1^*(\omega), \dots, x_n^*(\omega)) \in D$ a.s. and $0_i \in T_i(\omega)(x_1^*(\omega), \dots, x_n^*(\omega))$ a.s., $i = 1, \dots, n$. Thus the proces terminates for an ordinal number α of second kind with $t_\alpha = +\infty$. The proof of the theorem is completed.

Remark 3.1. Letting $c(\omega) = 0$ in Theorem 3.1 we obtain the random generalization of the theorem 3.1 and 3.3 in [6].

Theorem 3.2. *Let $T_i(\omega) : \Omega \times D \subset \Omega \times X_1 \times \ldots \times X_n \to CB(Y_i)$, $i = 1, \ldots, n$, D being a vector space, be a.s. continuous random multivalued operators, and let $\Gamma_i(\cdot, x_i) : \Omega \times Y_i \to X_i$, $i = 1, \ldots, n$ be bounded linear random operators for $(x_1, \ldots, x_n) \in D$. Suppose that $u_i(\omega)$ is given Y_i-valued random variable, $i = 1, \ldots, n$ and the following conditions are satisfied:*

(i) for all $(x_1, \ldots, x_n) \in D$

$$\| \Gamma_i(\omega, x_i) \| \leq B(\omega) \quad a.s. \quad i = 1, \ldots, n,$$

where $B(\omega)$ is a positive real-valued random variable;

(ii) for all $(x_1, \ldots, x_n) \in D$ and $y_i \in Y_i$, $i = 1, \ldots, n$

$$\Gamma_1(\omega, x_1)y_1, \ldots, \Gamma_n(\omega, x_n)y_n \in D$$

and there exists a positive number $\varepsilon = \varepsilon(x_1, \ldots, x_n;\ y_1, \ldots, y_n) \leq 1$ such that

$$H_i(T_i(\omega)(x_1 + \varepsilon\Gamma_1(\omega, x_1)y_1, \ldots, x_n + \varepsilon\Gamma_n(\omega, x_n)y_n), T_i(\omega)(x_1, \ldots, x_n) + \varepsilon y_i) \leq \varepsilon\{\sum_{k=1}^{n} b_{i,k}(\omega) \| y_k \| + \sum_{k=1}^{n} c_{i,k}(\omega) D_k(0_k, T_k(\omega)(x_1, \ldots, x_n) - u_k(\omega)) + c(\omega) D_i(0_i, T_i(\omega)(x_1 + \varepsilon\Gamma_1(\omega, x_1)y_1, \ldots, x_n + \varepsilon\Gamma_n(\omega, x_n)y_n) - u_i(\omega)) \quad a.s. \quad i = 1, \ldots, n,$$

where $b_{i,k}$, $c_{i,k}(\omega)$, $i, k = 1, \ldots, n$ and $c(\omega)$ satisfy the hypotheses in Theorem 3.1.

Then there exist X_i-valued random variables $x_i^\star(\omega), i = 1, \ldots, n$ such that

$$(x_1^\star(\omega), \ldots, x_n^\star(\omega)) \in D \quad a.s.$$

and

$$u_i(\omega) \in T_i(\omega)(x_1^\star(\omega), \ldots, x_n^\star(\omega)) \quad a.s. \quad i = 1, \ldots, n.$$

Proof. Letting $\hat{T}(\omega)(x_1, \ldots, x_n) = T_i(\omega)(x_1, \ldots, x_n) - u_i(\omega)$, $i = 1, \ldots, n$, the all hypothesis of Theorem 3.1 are satisfied for $\hat{T}_i(\omega)$, $i = 1, \ldots, n$. It follows from Theorem 3.1 that there exist X_i-valued random variables $x_i^\star(\omega)$, $i = 1, \ldots, n$ such that

$$(x_1^\star(\omega), \ldots, x_n^\star(\omega)) \in D \quad \text{a.s.}$$

and

$$0_i(\omega) \in \hat{T}_i(\omega)(x_1^\star(\omega),\dots,x_n^\star(\omega)) - u_i(\omega) \quad \text{a.s.} \quad i = 1,\dots,n,$$

Thus

$$u_i(\omega) \in T_i(\omega)(x_1^\star(\omega),\dots,x_n^\star(\omega)) \quad \text{a.s.} \quad i = 1,\dots,n.$$

This completes the proof.

Remark 3.2. Theorem 3.2 is the improvement and random generalization of Altman's theorem 3.1 in [1, p. 72]. If for all $(x_1,\dots,x_n) \in Dy_i \in Y_i, i = 1,\dots,n,$ let $\varepsilon = \varepsilon(x_1,\dots,x_n; y_1,\dots,y_n) = 1$, in Theorem 3.1 and 3.2, we obtain the theorem 3.1 and 3.2 of [10] which extend the corresponding results in [1, 2, 4, 5].

Theorem 3.3. *Let $F_i(\omega): \Omega \times X_1 \times \dots \times X_n \to CB(X_i)$, $i = 1,\dots,n$, be a.s. continuous random multivalued operators such that for $x_i, y_i \in X_i$, $i = 1,\dots,n$*

$$\begin{aligned} &H_i(F_i(\omega)(x_1,\dots,x_n), F_i(\omega)(y_1,\dots,y_n)) \\ \le &\sum_{k=1}^{n} b_{i,k}(\omega) \parallel x_k - y_k \parallel + \sum_{k=1}^{n} c_{i,k}(\omega) D_k(y_k, F_k(\omega)(y_1,\dots,y_n)) \\ &+ c(\omega) D_i(x_i, F_i(\omega)(x_1,\dots,x_n)) \quad a.s., \quad i = 1,\dots,n, \end{aligned}$$

where $b_{i,k}(\omega)$ and $c_{i,k}(\omega), i, k = 1,\dots,n$ satisfy the hypotese in Theorem 3.1 and $c(\omega)$ satisfies $0 \le c(\omega) < 1 - q(\omega)$ a.s.

Then there exist X_i-valued random variables $x_i^\star(\omega)$, $i = 1,\dots,n$ such that

$$x_i^\star(\omega) \in F_i(\omega)(x_1^\star(\omega),\dots,x_n^\star(\omega)) \quad a.s., \quad i = 1,\dots,n.$$

Proof. In Theorem 3.1 choose $Y_i = X_i$, $i = 1,\dots,n$, $D = X_1 \times \dots \times X_n$, $\Gamma_i(\omega, x_i) = I_i(\omega)$ for all $x_i \in X_i$, $i = 1,\dots,n$, $T_i(\omega)(x_1,\dots,x_n) = x_i - F_i(\omega)(x_1,\dots,x_n)$, $i = 1,\dots,n$ and $\varepsilon = \varepsilon(x_1,\dots,x_n;\ y_1,\dots,y_n) = 1$ for all $x_i, y_i \in X_i$, $i = 1,\dots,n$. Then from Theorem 3.1 it follows that there exist X_i-valued random variables $x_i^\star(\omega)$, $i = 1,\dots,n$ such that

$$x_i^\star(\omega) \in F_i(\omega)(x_1^\star(\omega),\dots,x_n^\star(\omega)) \quad \text{a.s.}, \quad i = 1,\dots,n.$$

Remark 3.3. Theorem 3.3 is the improvement and random generalization of Theorem 3.2 of [6], Theorem of Czerwik [14] and Matkowski's fixed point theorem in [7, 8].

Theorem 3.4. *Let $x_i^0 \in X_i$, $i = 1, \ldots, n$, $S_i = S_i(x_i, r_i) = \{x_i \in X_i : \| x_i - x_i^0 \| < r_i\}$ where r_i is given positive real number, $i =, \ldots, n$, and let $T_i(\omega) : \Omega \times \bar{S}_1 \times \ldots \times \bar{S} \to CB(Y_i)$, $i =, \ldots, n$ be a.s. closed and a.s. continuous random multivalued operators, where $\bar{S}_i$ is the closure of S_i, $i =, \ldots, n$. Suppose that there exist bounded linear random operators $\Gamma_i(\cdot, x_i) : \Omega \times Y_i \to X_i$ for $x_i \in S_i$, $i = 1, \ldots, n$ such that the following conditions are satisfied:*

(i) for all $x_i \in S_i$, $i = 1, \ldots, n$

$$\| \Gamma_i(\omega, x_i) \| \leq B(\omega) \quad a.s., \ i = 1, \ldots, n,$$

where $B(\omega)$ is positive real-valued random variable.

(ii) for all $x_i \in S_i$ and $y_i \in Y_i$, $i = 1, \ldots, n$, there exists a positive number $\varepsilon = \varepsilon(x_1, \ldots, x_n; y_1, \ldots, y_n) \leq 1$ such that the inequalities in (7) hold, where $b_{i,k}(\omega)$, $c_{i,k}(\omega)$, $i, k = 1, \ldots, n$ and $c(\omega)$ satisfy the hypotheses in Theorem 3.1.

(iii) $B(\omega)(1 - \bar{q}(\omega))^{-1} r_i(omega)(\omega) \leq r_i$ a.s., $i = 1, \ldots, n$, where $\bar{q}(\omega)$ is a positive real-valued random variable such that $q(\omega) < \bar{q}(\omega) < 1$ a.s. and $1 - q(\omega) < ln(1 - q(\omega))(1 - \bar{q}(\omega))^{-1}$ a.s., $q(\omega) = \max_{1 \leq i \leq n}\{r_i^{-1}(\omega) \sum_{k=1}^{n} a_{i,k}(\omega) r_k(\omega)\}$ and $r_i(\omega)$, $i = 1, \ldots, n$ are the positive random solution of (2.3).

Then there exist X_i-valued random variables $x_i^*(\omega)$, $i = 1, \ldots, n$, such that $x_i^*(\omega) \in \bar{S}_i$ a.s. $i = 1, \ldots, n$ and

$$0_i(\omega) \in T_i(\omega)(x_1^*(\omega), \ldots, x_n^*(\omega)) \quad a.s. \quad i = 1, \ldots, n,$$

Proof. In the same way in the proof of Theorem 3.1, we can construct well -ordered sequences of real numbers $t_\alpha (t_0 = 0)$ and X_i-valued random variables $x_i^\alpha(\omega)$ and Y_i-valued random variables $y_i^\alpha(\omega)$, $i = 1, \ldots, n$ such that $y_i^\alpha(\omega)$ is a measurable selection of $T_i(\omega)(x_1^\alpha(\omega), \ldots, x_n^\alpha(\omega))$, $i = 1, \ldots, n$. They satisfy the conditions (9)-(14). Then using the same argument as in the proof of Theorem 3.1, we obtain for arbitrary $\nu < \alpha$ and $\lambda < \alpha$

$$\| x_i^\nu(\omega) - x_i^\lambda(\omega) \| < (1 - q(\omega))(1 - \bar{q}(\omega))^{-1} B(\omega) r_i(\omega) \int_{t_\lambda}^{t_\nu} e^{-(1-q(\omega))t} dt \quad \text{a.s.}$$

$$i = 1, \ldots, n.$$

Hence, we obtain, by the hypothesis (iii),

$$\| x_i^\nu(\omega) - x_i^0 \| < (1 - q(\omega))(1 - \bar{q}(\omega))^{-1} B(\omega) r_i(\omega) \int_0^\infty e^{-(1-q(\omega))t} dt$$
$$\leq B(\omega)(1 - \bar{q}(\omega))^{-1} r_i(\omega) \leq r_i \quad \text{a.s.} \quad i = 1, \ldots, n.$$

Thus all $x_i^\nu(\omega) \in S_i$ a.s., $i = 1, \ldots, n$. The further reasoning is exactly the same as in the proof of Theorem 3.1.

Remark 3.4. Theorem 3.4 is the improvement and random generalization of Altman's local existence theorem 6.1 in [1, p. 83].

Remark 3.5 For Theorem 3.1 with $T_i(\omega)$ $i = 1, \ldots, n$ being random single-valued operators, we can also apply it to prove an existence theorem for the system of random nonlinear evolution equations and hence we can also generalize the corresponding results of Altman in [1, chapter 4, section 5]. Here, we omit.

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REZIME

SLUČAJNI KONTRAKTORSKI PRAVCI I REŠENJE SISTEMA SLUČAJNIH VIŠEZNAČNIH OPERATORSKIH JEDNAČINA

U ovom radu autor je posmatrao koncept slučajnih kontraktorskih pravaca i njegovu primenu na rešavanje sistema nelinearnih slučajnih višeznačnih operatorskih jednačina.

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