Univ. u Novom Sadu
Zb. Rad. Prirod.–Mat. Fak.
Ser. Mat. 23, 1 (1993), 1 - 11

Review of Research
Faculty of Science
Mathematics Series

RELATION BETWEEN QUASIASYMPTOTICS, EQUIVALENCE AT INFINITY AND S–ASYMPTOTICS

Bogoljub Stanković
Institute of Mathematics, University of Novi Sad,
Trg Dositeja Obradovića 4, 21000 Novi Sad,Yugoslavia

Abstract

Three theorems have been proved relating the $S-$asymptotics with the quasiasymptoics and the equivalence at infinity. The obtained results include the known and the new ones, proved in a unique way.

AMS Mathematics Subject Classifications (1991): 46F12
Key words and phrases: Asymptotic behaviour of distributions, quasi-asymptotics, equivalence at infinity, $S-$asymptotics.

1. Introduction

In the last twenty years many aspects of asymptotic behaviour of distributions have been elaborated and applied. Between them are quasi-asymptotics [2], equivalence at infinity [3] and $S-$asymptotics (shift asymptotics) [5]. In the following, for the quasiasymptotics we shall refer to the book [9] and for the $S-$asymptotics to the book [6].

The first theorem which gives a relation between the quasi-asymptotics and the $S-$asymptotics has been proved in [2]. This theorem asserts: "If $f \in \mathcal{S}'_+$ and has the $S-$asymptotics related to h^α, $\alpha > -1$, $h \to \infty$, then f has the quasiasymptotics of order α, as well. Later on, this theorem has been improved (see [9], p. 74). In case $\alpha \le -1$ the situation is a little different. In [4] one can find precise results for $\alpha = -1$, but for $\alpha < -1$ the obtained result includes the quasiasymptotic behaviour when the limit equels zero, as well.

Our aim in this paper is to give relations between the S-asymptotics and other two asymptotic behaviour with a unique method based only on forthcoming relation (6).

2. Notion and definitions

D' is the space of Schwartz distributions. $\mathcal{S}'_+$ is the space of tempered distributions with the support in $[0,\infty)$. The class of distributions f_α, $\alpha \in R$, belonging to $\mathcal{S}'_+$, are defined in the following way:

$$f_\alpha(t) = \begin{cases} \theta(t)t^{\alpha-1}/\Gamma(\alpha), & \alpha > 0 \\ f_{\alpha+m}^{(m)}(t), & \alpha \le 0,\ \alpha + m > 0, \end{cases} \tag{1}$$

where $\theta(t) = 1,\ t \ge 0$; $\theta(t) = 0,\ t < 0$; the derivative is in the distributional sense. (see [9], pp. 27 and 36).

By $f^{(-m)}$ we denote: $f^{(-m)} = f_m * f$ ($*$ is the sign of the convolution);

$$f_{-n} = \delta^{(n)},\quad n \ge N;\quad f_0 = \delta \quad \text{and} \quad f_p * f_q = f_{p+q}.$$

In the following we shall use the class of slowly varying functions. A function $L \in L_{loc}$ is a slowly varying function at infinity if $L(x) > 0,\ x \ge a > 0$ and if

$$\lim_{h\to\infty} L(ht)/L(h) = 1,\quad t > 0. \tag{2}$$

In a similar way we can define a slowly varying function at zero.

We know ([1], p. 16) that if $L_2(x) \to \infty,\ x \to \infty$ and L_1, L_2 are slowly varying, then $L_1(L_2)$ is slowly varying, as well. Hence, for $x > -\infty$

$$\lim_{h\to\infty} L(x+h)/L(h) = \lim_{u\to\infty} L(\ell n ut)/L(\ell n u) = 1,\quad t > 0. \tag{3}$$

For any slowly varying function L we can construct another slowly varying function L_1 such that $L_1 \in C^\infty_{(0,\infty)}$ and

$$\lim_{h\to\infty} L_1(h)/L(h) = 1. \tag{4}$$

Let w be a positive smooth function with the support in $[-b,b]$, $b>0$, and $\int_{-\infty}^{\infty} w(t)dt = 1$. Then L_1 can be:

$$L_1(x) = \int_{-\infty}^{\infty} w(x-t)L(t)dt = \int_{-b}^{b} w(-y)L(y+x)dy.$$

By the property of L given in (3) it follows (4). Hence, if we know that a limit of the form: $\lim\limits_{k\to\infty} f(kx)/L(k)$ exists, then the limit: $\lim_{k\to\infty} f(kx)/L_1(k)$ exists, as well and the two limits are equal.

Definition 1. *A distribution $f \in S'_+$ has the quasiasymptotics in S' of order α if for every $\varphi \in S$ there exists the limit*

$$\lim_{k\to\infty} < f(kt)/k^\alpha L(k),\ \varphi(t) > = < C f_{\alpha+1}, \varphi >,\quad C \neq 0.$$

We shall write in short: $f \overset{q}{\sim} k^\alpha L(k) \cdot C f_{\alpha+1}$.

One can give a more general definition of the quasiasymptotics in D'.

Definition 1'. *A distribution $T \in D'$ has the quasiasymptotics in D' related to a locally integrable and positive function $c(k)$, $k>0$ with the limit $U \in D'$, $U \neq 0$, if for every $\varphi \in D$ there exists*

$$\lim_{k\to\infty} < f(kx)/c(k), \varphi(x) > = < U, \varphi > .$$

Definition 2. *The distribution T is equivalent at infinity with Ax^α, $A \neq 0$, $\alpha \in R$, if there exist $m \in N_0$ $(m+\alpha>0)$, $a>1$ and a continuous function F or R such that $T = D^m F$ on $[a,\infty)$ and $F(x) \sim AC_{\alpha,m}x^{\alpha+m}$, as $x \to \infty$.*

A.Takači in [8] gave a generalization of this definition replacing function x^α by $x^\alpha L(x)$, $\alpha \notin (-N)$. If T is a regular distribution, then α can be negative integer and $T(x) \sim Ax^{-n}L(x)$, $x \to \infty$ in R. This generalization of Definition 2 we shall denote by Definition 2'.

Definition 3. *A distribution $T \in D'$ has the $S-$asymptotics related to $c(h)>0$, $h>0$, with the limit $U \neq 0$, $U \in D'$ if for every $\phi \in D$*

$$\lim_{h\to\infty} < T(t+h)/c(h),\ \phi(t) > = < U, \phi > .$$

We shall write in short: $T(x+h) \overset{s}{\sim} c(h) \cdot U$ *in* D'.

3. Relation between the quasiasymptotics and $S-$asymptotics

It is not easy to compare the quasiasymptotics and the $S-$asymptotics because the $S-$asymptotics is a local property, while the quasiasymptotics is in general a global property. We have also to fix the space of distributions and the class of functions which is used as the measure of the quasiasymptotic behaviour. One can give examples of distributions which have the quasiasymptotics and have no $S-$asymptotics. Such an one is the regular distribution $\theta(t)$ sint (see [6], p. 91). Or, on the contrary, the regular distribution $\theta(t)\exp(-t)$ has the $S-$asymptotics related to $\exp(-h)$, but it has no quasiasymptotics in $\mathcal{S}'$: for $\varphi_n(t) = \eta(t)t^n \exp(-t) \in \mathcal{S}$, where $\eta(t) \in C_0^\infty, \eta(t) = 1,\ t \geq 0$;

$$\begin{aligned} < \theta(kt)\exp(-kt), \varphi_n(t) > &= \int_0^\infty t^n \exp((-k-1)t)dt \\ &= (k+1)^{-n-1} \int_0^\infty t^n e^{-t} dt \\ &= (k+1)^{-n-1} < \theta(t), \varphi_n(t) > . \end{aligned}$$

We see that the behaviour of this expression depends on $\varphi_n,\ n \in N$.

Since the quasiasymptotics is defined and elaborated for distributions belonging to $\mathcal{S}'_+$ and the $S-$asymptotics for D', we have to choose the space $\mathcal{S}'_+$ in which we can compare these two asymptotics. The choice of the space $\mathcal{S}'_+$ involves the class of functions which measures the asymptotic behaviour (see [9], p. 66). In fact it is the class of regularly varying functions [1] which has the form $h^\alpha L(h),\ \alpha \in R$ and L is a slowly varying function.

Before we are going on our theorems we have to analyse the operator I : $L_{loc}(0,\infty) \to C(0,\infty)$. If $f \in L_{loc}(0,\infty)$ and supp$f \subset [a,\infty),\ a > 0$, then

$$(If)(x) = f^{(-1)}(x) = \begin{cases} \int_a^x f(t), & x > a \\ 0, & 0 < x \leq a \end{cases} .$$

For $k \in N$, I^k will be the $k-$times ireteted $I, I^k = f^{(-k)} = f_k * f,\ I^0 f = f$, where f_k is given by (1).

In the following the letter L we shall use only for a slowly varying function.

Lemma 1. *The operator I has the following properties: For $f(t) = t^\alpha L(t),\ t \geq a;\ f(t) = 0,\ 0 \leq t < a;\ a > 0$*

1) $D^k I^k f = f$

2) If $\alpha > -1$, *then* $(I^k f)(x) = x^{\alpha+k} L_{k,\alpha}(x)$, $x \geq a$, *where* $L_{k,\alpha}$ *is a slowly varying function and* $L_{k,\alpha}(x)/L(x) \to ((\alpha+1)...(\alpha+k))^{-1}$, $x \to \infty$.

3) If $-k < \alpha < -1$, *then* $(I^k f)(x) \sim x^{k-1}/\Gamma(k)$.

4) If $-k < \alpha = -1$, *then* $(I^k f)(x) \sim x^{k-1} L\hat{}(x)/\Gamma(k)$, *where* $L\hat{}(x)/L(x) \to \infty$.

Proof. 1) is trivial. 2) By Proposition 1.5.8 in [1] we have $If \sim x^{\alpha+1}L(x)/(\alpha+1)$, $x \to \infty$, and by Theorem 1.4.1. in [1], $L_{1,\alpha}$ is a slowly varying function such that $L_{1,\alpha}/L(x) \to (\alpha+1)^{-1}$. The property of I^k, $k > 1$, follows by iteration from $L_{k,\alpha}(x)/L(x) = L_{k,\alpha}(x)/L_{k-1,\alpha}(x)...L_{1,\alpha}(x)/L(x)$, $x \geq a$.

3) and 4). By the definition of I^k

$$\begin{aligned} I^k(f)(x) &= \int_a^x \frac{(x-t)^{k-1}}{\Gamma(k)} t^\alpha L(t)dt \\ &= \frac{1}{\Gamma(k)} \sum_{j=0}^{k-1} \binom{k-1}{j} (-1)^{k-1-j} x^j \int_a^x t^{k-1-j+\alpha} L(t)dt. \end{aligned}$$

If $-k < \alpha < -1$, then

$$(I^k f)(x) \sim x^{k-1} \int_a^\infty t^\alpha L(t)dt/\Gamma(k).$$

If $-k < \alpha = -1$, then

$$(I^k f)(x) \sim \frac{1}{\Gamma(k)} x^{k-1} \int_a^x t^{-1} L(t)dt = \frac{1}{\Gamma(k)} x^{k-1} L\hat{}(x),$$

where $L\hat{}(x)/L(x) \to \infty$, $x \to \infty$.

noindent**Remark.** If $t^{-1}L(t) \in L(a,\infty)$, then $L\hat{}(x) \to C$, $x \to \infty$. If $L\hat{}(x) \to \infty, x \to \infty$, then $L\hat{}$ is a slowly varying function.

Theorem 1. *Suppose that* $T \in \mathcal{S}'_+$ *and that* T *has the* $S-$*asymptotics related to* $h^\alpha L(h)$. T *has also the quasiasymptotics related to* $h^\beta L'(h)$: *If* $\alpha > -1$, *then* $\alpha = \beta$, $L' = L$; *if* $\alpha = -1$ *and* $L\hat{}(x) \to \infty$, $x \to \infty$, *then* $\beta = -1$, $L' = L\hat{}$; *if* $\alpha < -1$ *or* $\alpha = -1$ *and* $L\hat{}(x) < \infty$, *then* $\beta = -1$, $L' = const.$, *but the limit can be zero.* $L\hat{}(x) = \int_a^x t^{-1}L(t)dt$.

Proof. First we shall prove that a necessary and sufficient condition that a distribution $T \in \mathcal{S}'_+$ has the $S-$asymptotics related to $h^\alpha L(h)$, $\alpha \in R$ is the existence of continuous and bounded functions E_i on (a,∞), $a > 0$, $i = 0,1,...,m$ such that $E_i(x+h) \to C_i$, when $h \to \infty$, $x \in (a,b)$, $a < b$, $C_0 \neq 0$ and that $T^{(-m')} = f_{m'} * T$ is of the form

$$(5) \qquad T^{(-m')}(x) = \sum_{i=0}^{m} I^{m'-i}(t^\alpha L(t)E_i(t))(x), \quad x \geq a > 0,$$

where $m' \geq \max(m, -\alpha)$.

By Theorem and its Consequence 3 in [7] a necessary condition that a distribution T has the $S-$asymptotics related to $h^\alpha L(h)$, $h \to \infty$, is that T has the form:

$$(6) \qquad T(x) = \sum_{i=0}^{m} D^i(x^\alpha L(x)E_i(x)), \quad x \geq a > 0$$

$$= \sum_{i=0}^{m} D^{m'} I^{m'-i}(t^\alpha L(t)E_i(t))(x),$$

where E_i have the properties we seek. Using the convolution by $f_{m'}$, from (6) it follows (5). By properties of the $S-$asymptotics we have:

$$\lim_{h\to\infty} D^i(x+h)^\alpha L(x+h)E_i(x+h)/h^\alpha L(h) = 0, \quad i = 1,...,m.$$

Now, it is easy to see that a distribution T, of the form (6), has the $S-$asymptotics related to $h^\alpha L(h)$ and that $C_0 \neq 0$.

We can also suppose that L is a continuous function on (a,∞) without any restriction of generality (see properties of L in 2.).

If $C_i \neq 0$, we denote bu a_i such a number that $C_i E_i(x) > 0$, $x \geq a_i$; we can now fix $a > 1$, $a \geq \max(a_i,\ i = 1,...,m)$. Let $M_i = \sup(|E_i(x)|,\ x \geq a > 0)$.

From relation (6) it follows:

$$(7) \qquad T^{(-m')}(x) = \sum_{i=0}^{m} I^{m'-i}(t^\alpha L(t)E_i(t))(x), \quad x \geq a,\ C_0 \neq 0.$$

We shall now find the asymptotic behaviour of $T^{(-m')}$. If $C_i \neq 0$, then E_i is a slowly varying function for $x \in [a, \infty)$. If $C_i = 0$, then

$$| \int_a^x t^\alpha L(t) E_i(t) dt | \leq M_i \int_a^x t^\alpha L(t) dt, \quad x \geq a.$$

By the properties of the operator I^k the asymptotic behaviour of the function $T^{(-m')}$ is predetermined by $I^{m'}(t^\alpha L(t) E_0(t))(x)$, $x \geq a$. By Lemma 1 it is easy to establish the asymptotic behaviour of $T^{(-m')}$ given by (5). We can come up to the asymptotic behaviour of T using relation (5) and Theorem 2 in [9]. By this Theorem a necessary and sufficient condition that $T \in \mathcal{S}'_+$ has the quasiasymptotics related to $t^\alpha L(t)$ is the existence of an $m, m+\alpha > -1$, such that $T^{(-m)}(x)/(x^{m+\alpha} L(x)) \to C \neq 0$, $x \to \infty$.

It is important to remark that relation (7) gives $T^{(-m')}$ only in an interval (a, ∞), $a > 0$. In such a way, we can use directly the mentioned Theorem and relation (5) only in cases in which the quasiasymptotics is a local property. That happens if $\alpha > -1$, or $\alpha = -1$ and $L^\wedge(x) \to \infty$, $x \to \infty$, which gives the first part of our Theorem.

In case $\alpha < -1$, or $\alpha = -1$ and $L^\wedge(x) < \infty$, $x \geq a > 0$, the quasiasymptotics is no more a local property. Knowing a distribution T only in a neighbourhood of infinity, we can not give a precise result about the quasiasymptotic behaviour of T. However something can be said.

Let $T \in \mathcal{S}'_+$ and η be such a smooth function that $\eta(t) = 1$, $t \geq a$, and $\eta(t) = 0$, $t < a'$, where $0 < a' < a$. Then $T = (1-\eta)T + \eta T$. The distribution $(1-\eta)T \in E'_+$. We know ([6], p. 32) that there exists a $r \in N_0$ such that $(1-\eta)T \overset{q}{\sim} k^{-r-1} \cdot \delta^{(r)}$. The function $(\eta T)^{(-m')}$ has the same asymptotic behaviour as $T^{(-m')}$. Then the quasiasymptotics of ηT can be obtained from (5) and the mentioned Theorem 2 in [9].

Let us start from the following relation:

$$\begin{aligned} < T(kt), \phi(t) \; &= \; < (1 - \eta(kt) T(kt), \phi(t) > \\ &+ \; < \eta(kt) T(kt), \phi(t). \end{aligned}$$

The asymptotic behaviour of the first addend, when $k \to \infty$, is given by k^{-1-r}. For the second adeend, it depends on α. If $\alpha < -1$ or $\alpha = -1$ and $L^\wedge(x) < \infty$, $x \geq a > 0$, then it behaves as h^{-1}, or the limit in Definition 1 can be zero.

Corollary 1. *1) If $T \in \mathcal{S}'_+$ has the quasiasymptotics related to $h^\beta L'(h)$, a* *necessary and sufficient condition that T has the $S-$asymptotics related to $h^\alpha L(h)$ is that $T^{(-m')}$ has the form given by relation (7).*

2) If $T \in \mathcal{S}'_+$ has the quasiasymptotics related to h^{-1}, then T must not belong to $\mathcal{E}'_+$.

At the end, as an illustration of the case $\alpha < -1$ in Theorem 1 we can use the distribution $T = \delta^{(r)} + (x^\alpha L(h))_+$; The quasiasymptotics depends not only on α but on r too.

We can compare the quasiasymptotics with the $S-$asymptotics not only for elements belonging to $\mathcal{S}'_+$ but to D', as well. In this case we shall use Definition 1' instead of Definition 1. We know that the general form of the funcrion c, related to which we can measure the $S-$asymptotics, is $c(h) = \exp(\alpha h)L(\exp h), \alpha \in R$.

Theorem 2. *If $T \in D'$ has the $S-$asymptotics related to $c(h) = \exp(\alpha h) \cdot L(\exp h)$ with $\alpha \neq 0$, then T can not have the quasiasymptotics.*

Proof. By the Structural theorem and consequences of it (see [7]), for the interval (a, ∞) there exist numerical functions E_i, $i = 0, 1, ..., m$, continuous and bounded on (a, ∞) such that $\lim_{x\to\infty} E_i(x) = C_i$ and the restriction of the distribution T on (a, ∞) has the form:

$$T(x) = \sum_{i=0}^{m} D^i(\exp(\alpha x)L(\exp x)E_i(x)), \quad \sum_{i=0}^{m} C_i\alpha^i \neq 0,$$

$$= D^m \sum_{i=0}^{m} I^{m-i}(\exp(\alpha t)L(\exp t)E_i(t))(x),$$

where

$$I(\exp(\alpha t)L(\exp t)E_i(t))(x) = I(t^{\alpha-1}L(t)E_i(\ell n t))(e^x), \quad \alpha \neq 0.$$

It is easy to see that

$$I^k(\exp(\alpha t)L(\exp t))(x) = \exp(\alpha x)L'_{k,\alpha-1}(e^x),$$

where $L'_{k,\alpha-1}(x)/L(x) \to \alpha^{-k}$, $x \to \infty$. Now, using the same method of proceeding as in the proof of Theorem 1, we have for $x \in (a, \infty)$:

$$T(x) = D^m[e^{\alpha x}\sum_{i=0}^{m} L_{m-i,\alpha-1}(e^x)E_i(x)] = D^m[e^{\alpha x}\Sigma\rho(e^x)],$$

where

$$\Sigma_0(e^x) \sim \alpha^{-m} \sum_{i=0}^{m} C_i \alpha^i L(x) \neq 0, \quad x \to \infty.$$

By Definition 1' we have

$$< T(kx)/c(k), \phi(x) >= \frac{(-1)^m}{k^m c(k)} \int_a^b \exp(\alpha k x) \Sigma_0(e^x) \phi^{(m)}(x) dx.$$

By the mean value theorem for integrals, there exist d' and d'' such that

$$< T(kx)/c(k), \phi(x) >= \begin{cases} \frac{(-1)^m}{k^m c(k)} e^{\alpha k a} \int_a^{d'} \Sigma_0(\exp kx) \phi^{(m)}(x) dx, \alpha < 0 \\ \frac{(-1)^m}{k^m c(k)} e^{\alpha k b} \int_{d''}^{b} \Sigma_0(\exp kx) \phi^{(m)}(x) dx, \alpha > 0. \end{cases}$$

Or

$$< T(kx)/c(k), \phi(x) >\sim \begin{cases} A k^{-m} e^{\alpha k a} L(e^k)/c(k) \int_a^{d'} \phi^{(m)}(x) dx, \alpha < 0, A \neq 0 \\ B k^{-m} e^{\alpha k b} L(e^k)/c(k) \int_{d''}^{b} \phi^{(m)}(x) dx, \alpha > 0, B \neq 0. \end{cases}$$

The factors $\exp(\alpha k a)$ or $\exp(\alpha k b)$ shows that the asymptotical behaviour of $< T(kx)/c(k), \phi(x) >$ depends on the support of ϕ and therefore it is not possible to find a $c(h)$ for the quasiasymptotics of T.

4. Relation between the equivalence at infinity and $S-$ asymptotics

Equivalence at infinity has been defined for elements of D' just as the $S-$ asymptotics. The restriction is in the function $x^\alpha L(x)$, $x > 0$, $\alpha \notin (-N)$, which measures the equivalence at infinity. In case regular distributions, the difference between these two definitions of asymptotic behaviour follows from the fact that the $S-$ asymptotics preserves the usual asymptotics of numerical functions (see [6], p. 78), but equivalence at infinity can give inexpected results. Let us see the next example:

$$F(x) = x^{-1/2} + e^x cose^x = D(2x^{1/2} + sine^x).$$

By definition, F is equivalent at infinity with $x^{-1/2}$. This function F has no $S-$ asymptotics.

Theorem 3. *If $T \in D'$ and has the $S-$ asymptotics related to $h^\alpha L(h)$, $h > 0$, $\alpha > -1$ then it is equivalent at infinity with $Ah^\alpha L(h)$, $h > 0$, $A \neq 0$.*

Proof. By relation (6)

$$T(x) = D^{m'} \sum_{i=0}^{m} I^{m'-i}((t^\alpha L(t)E_i(t))(x), \; x \geq a > 0, \; C_0 \neq 0. \tag{8}$$

The function F given by the sum in (9) has the property: If $\alpha > -1$, then

$$F(x) = \sum_{i=0}^{m} I^{m'-i}(t^\alpha L(t)E_i(t))(x) \sim Cx^{\alpha+m'}L(x), \; x \to \infty, \; C \neq 0.$$

(see Lemma 1 assertion 2)).

We proved that $T = D^{m'}F(x)$, where $F(x) \sim Cx^{\alpha+m'}$, $C \neq 0$. By Definition 2' there exists $A \neq 0$ such that T is equivalent at infinity with $Ax^\alpha L(x)$.

References

[1] Bingham N.H., Goldie C.M., Teugels J.L., Regular variation, Cambridge University Press, 1987.

[2] Drožžinov Yu.N., Zavjalov B.I., Quasiasymptotic of generalized functions and Tauberian theorems in the complex domain, Mat. Sb. T. 102 (1977), 372-390. (in Russian).

[3] Lavoine J., Misra O.P., Théorèmes Abéliens pour la transformation de Stieltjes des distributions, C. R. Acad. Sci. Paris, Ser. A 179 (1974), 99-102.

[4] Pilipović S., The translation asymptotic and the quasiasymptotic behaviour of distributions, Acta Math. Hung. 55(3-4) (1990), 239-243.

[5] Pilipović S., Stanković B., S-asymptotic of a distribution, Pliska Studia Math. Bulgar., 10 (1989), 147-156.

[6] Pilipović S., Stanković B., Takači A., Asymptotic Behaviour and Stieltjes Transformation of Distributions, Teubner, Leipzig 1990.

[7] Stanković B., A structural theorem for distributions having S-asymptotic, Publ. Inst. Math. (Beograd) (N.S.) T. 45 (1989), 35-40.

[8] Takači A., On the equivalence of distributions at infinity, Univ. u Novom Sadu, Zb. Rad. Prirod. Mat. Fak., Ser. Mat. 15, 1 (1985), 175-187.

[9] Vladimirov V.S., Drožžinov Yu.N., Zavjalov B.I., Multi - dimensional Tauberian Theorems for Generalized Functions, Nauka, Moscow, 1986 (in Russian).

REZIME

VEZA KVAZIASIMPTOTIKE, EKVIVALENCIJE U BESKONAČNOSTI I S - ASIMPTOTIKE

U radu su dokazane tri teoreme. U prvoj je dokazano da ako distribucija $T \in \mathcal{S'}_+$ ima $S-$ asimptotiku u odnosu na funkciju $h^\alpha L(h)$. Gde je L sporo-promenljiva funkcija, tada ona ima i kvaziasimptotiku u odnosu na funkciju $h^\beta L'(h)$ za odredjene β i L'.

U drugoj teoremi dokazano je da distribucija koja ima $S-$ asimptotiku u odnosu na $e^{\alpha h} L(e^h)$ nema kvaziasimptotiku ako je $\alpha \neq 0$.

U trećoj teoremi dokazano je da ako distribucija ima $S-$ asimptotiku u odnosu na $h^\alpha L(h)$, $\alpha > -1$, ona je ekvivalentna u beskonačnosti sa $Ax^\alpha L(x)$.

Najzad dat je i potreban i dovoljan uslov da iz kvaziasimptotike ili ekvivalencije u beskonačnosti sledi $S-$ asimptotika.

Neki od dokazanih rezultata su već bili poznati, dokazani na različite načine. U ovom radu svi rezultati se dobijaju iz jedne jedine relacije (6).

Received by the editors April 9, 1991