Univ. u Novom Sadu
Zb. Rad. Prirod.–Mat. Fak.
Ser. Mat. 23, 2 (1993), 23 - 33

Review of Research
Faculty of Science
Mathematics Series

FIXED POINTS FOR THREE MAPPINGS

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Abstract

In this paper we discuss some common fixed point theorems for three self mappings on a quasi-gauge space which extend the results for a metric space in [1], [2], [4] and [5].

AMS Mathematics Subject Classification (1991): 54H25
Key words and phrases: fixed point, quasi-gauge space.

1. Introduction

In this paper we discuss some common fixed point theorems for three self mappings on a quasi-gauge space which extend the results for a metric space in [1], [2], [4] and [5]. We need the concepts of quasi-gauge space, P-Cauchy sequence, Sequential completeness as in [6] and [8].

A quasi-pseudometric on a set X is a non-negative real valued function on $X \times X$ such that for any x, y, z in X .
$p(x,x) = 0$ and $p(x,y) \leq p(x,z) + p(z,y)$.

A quasi-gauge structure for a topological space (X,T) is a family P of quasi-pseudometrics on X such that T has as a subbase family $\{B(x,p,\varepsilon) : x \in X,\ \ p \in P,\ \ \varepsilon > 0\}$ where $B(x,p,\varepsilon)$ is the set $\{y \in X : p(x,y) < \varepsilon\}$.

If a topological space has a quasi-gauge structure, it is called a quasi-gauge space.

The sequence $\{x_n\}$ in a quasi-gauge space is called left (right) P-Cauchy sequence if for each $p \in P$ and each $\varepsilon > 0$ there is a point in X and an integer k such that $p(x, x_m) < \varepsilon$, $(p(x_m, x) < \varepsilon)$ for all $m \geq k$. (x and k may depend upon ε and p.)

A quasi-gauge space is left (right) sequentially complete if every left (right) P-Cauchy sequence in X converges to some element of X.

We prove the following result.

Theorem 1. *Let T and I be commuting mappings and let T and J be commuting mappings of a left (right) sequentially complete quasi-gauge T_0 space satisfying the inequality for each p in P.*

$$(1) \qquad p(Tx, Ty) \leq C \max \left\{ \begin{array}{l} p(Ix, Jy), \ p(Ix, Tx), \ p(Jy, Ty), \\ p(Ix, Ty), \ p(Jy, Tx) \end{array} \right\}$$

for all x, y in X where $0 \leq C < 1$.
Suppose that for all x in X, there exists an y in X such that

$$Tx = Iy = Jy.$$

If T is continuous and whenever $Tx_n \to x$ implies $p(Tx_n, x) \to 0$ as $n \to \infty$ for each p in P, then T, I and J have a unique common fixed point z.

Proof. Let x_0 be an arbitrary point in X, define a sequence $\{x_n\}$ inductively by choosing

$$Tx_{n-1} = Ix_n = Jx_n, \quad n = 1, 2, ...$$

Let us now suppose that set of real numbers $\{p(Tx_n, Tx_1), p(Tx_1, Tx_n)\}$ is unbounded. Then there exists an integer n such that

$$(1-C)\max\{p(Tx_n, Tx_1), p(Tx_1, Tx_n)\} > C\max\{p(Tx_1, Tx_0), p(Tx_0, Tx_1)\}$$

$$(2) \qquad \max \left\{ \begin{array}{c} p(Tx_n, Tx_1), \\ p(Tx_1, Tx_n) \end{array} \right\} > \max \left\{ \begin{array}{c} p(Tx_r, Tx_0), p(Tx_0, Tx_r) \\ 0 \leq r < n \end{array} \right\}$$

These inequalities imply that for $r = 1, 2, ... n$.

$$\begin{aligned} C \max \left\{ \begin{array}{c} p(Tx_r, Tx_0), \\ p(Tx_0, Tx_r) \end{array} \right\} &\leq C \max \left\{ \begin{array}{c} p(Tx_r, Tx_1) + p(Tx_1, Tx_0), \\ p(Tx_0, Tx_1) + p(Tx_1, Tx_r) \end{array} \right\} \\ &< \max\{p(Tx_n, Tx_1), p(Tx_1, Tx_n)\} \end{aligned}$$

and so

$$(3) \qquad \max\left\{\begin{array}{c} p(Tx_n, Tx_1), \\ p(Tx_1, Tx_n) \end{array}\right\} > C\max\left\{\begin{array}{c} p(Tx_r, Tx_0), p(Tx_0, Tx_r) \\ 0 \le r < n \end{array}\right\}$$

We now prove by induction that

$$\max\left\{\begin{array}{c} p(Tx_n, Tx_1), \\ p(Tx_1, Tx_n) \end{array}\right\} \le C^k \max\{p(Tx_r, Tx_s) : 1 \le r, s \le r\}$$

for $k = 1, 2, \ldots$ Using inequality (1) we have

$$\begin{aligned} p(Tx_n, Tx_1) &\le C\max\left\{\begin{array}{l} p(Ix_n, Jx_1),\ p(Ix_n, Tx_n),\ p(Jx_1, Tx_1), \\ p(Ix_n, Tx_1),\ p(Jx_1, Tx_n) \end{array}\right\} \\ &= C\max\left\{\begin{array}{l} p(Tx_{n-1}, Tx_0),\ p(Tx_{n-1}, Tx_n),\ p(Tx_0, Tx_1), \\ p(Tx_{n-1}, Tx_1),\ p(Tx_0, Tx_n) \end{array}\right\} \end{aligned}$$

$$\begin{aligned} p(Tx_1, Tx_n) &\le C\max\left\{\begin{array}{l} p(Ix_1, Jx_n),\ p(Ix_1, Tx_1),\ p(Jx_n, Tx_n), \\ p(Ix_1, Tx_n),\ p(Jx_n, Tx_n) \end{array}\right\} \\ &\le C\max\left\{\begin{array}{l} p(Tx_0, Tx_{n-1}),\ p(Tx_0, Tx_1),\ p(Tx_{n-1}, Tx_n), \\ p(Tx_0, Tx_n),\ p(Tx_{n-1}, Tx_1) \end{array}\right\} \end{aligned}$$

These inequalities further reduce to

$$\max\{p(Tx_n, Tx_1), p(Tx_1, Tx_n)\} \le Cp(Tx_{n-1}, Tx_n),$$

on using inequalities (2) and (3). Thus inequality holds for $k = 1$. Assume that the inequality holds for some k. Then

$$\begin{aligned} \max\left\{\begin{array}{c} p(Tx_n, Tx_1), \\ p(Tx_1, Tx_n) \end{array}\right\} &\le C^k \max\left\{\begin{array}{l} p(Tx_r, Tx_s) \\ 1 \le r, s \le n \end{array}\right\} \\ &\le C^{k+1}\max\left\{\begin{array}{c} p(Ix_r, Jx_s),\ p(Ix_r, Tx_r), \\ p(Jx_s, Tx_s),\ p(Ix_r, Tx_s), \\ p(Jx_s, Tx_r) :\ 1 \le r, s \le n \end{array}\right\} \\ &\le C^{k+1}\max\left\{\begin{array}{c} p(Tx_{r-1}, Tx_{s-1}),\ p(Tx_{r-1}, Tx_r), \\ p(Tx_{s-1}, Tx_s),\ p(Tx_{r-1}, Tx_s), \\ p(Tx_{s-1}, Tx_r) :\ 1 \le r, s \le n \end{array}\right\} \end{aligned}$$

On using inequality (3), this reduces to

$$(4) \qquad \max\left\{\begin{array}{c} p(Tx_n, Tx_1), \\ p(Tx_1, Tx_n) \end{array}\right\} \le C^{k+1} \max\{p(Tx_r, Tx_s) : 1 \le r, s \le r\}$$

Inequality follows by induction.
Letting k tend to infinity in inequality (4) it now follows that

$$\max\{p(Tx_n, Tx_1), p(Tx_1, Tx_n)\} = 0$$

giving a contraction to the assumption that the set of real numbers $\{p(Tx_n, Tx_1), p(Tx_1, Tx_n)\}$ is unbounded.
It now follows that

$$\begin{aligned} M_p &= \sup\{p(Tx_r, Tx_s) : \; r, s = 0, 1, ...\} \\ &\leq \sup\{p(Tx_r, Tx_1) + p(Tx_1, Tx_s) : \; r, s = 0, 1, ...\} \end{aligned}$$

is finite.
Now for arbitrary $\varepsilon > 0$ choose an integer N_p such that $C^{N_p} M_p < \varepsilon$ for each p in P.

$$\begin{aligned} p(Tx_m, Tx_{N_p+1}) &\leq C \max \left\{ \begin{array}{c} p(Tx_{m-1}, Tx_{N_p}), \;\; p(Tx_{m-1}, Tx_m), \\ p(Tx_{N_p}, Tx_{N_p+1}), \;\; p(Tx_{m-1}, Tx_{N_p+1}), \\ p(Tx_{N_p}), Tx_m) \end{array} \right\} \\ &\leq C \max \left\{ \begin{array}{c} p(Tx_r, Tx_s), \;\; p(Tx_r, Tx_{r'}), \\ p(Tx_s, Tx_{s'}), \;\; p(Tx_s, Tx_r), \\ m-1 \leq r, \;\; r' \leq m, \\ N_p \leq s, \;\; s' \leq N_p + 1 \end{array} \right\} \\ &\leq C^2 \max \left\{ \begin{array}{c} p(Tx_r, Tx_s), \;\; p(Tx_r, Tx_{r'}), \\ p(Tx_s, Tx_{s'}), \;\; p(Tx_s, Tx_r), \\ m-2 \leq r, \;\; r' \leq m, \\ N_p - 1 \leq s, \;\; s' \leq N_p + 1 \end{array} \right\} \\ &\;\; \vdots \\ &\leq C^{N_p} \max \left\{ \begin{array}{c} p(Tx_r, Tx_s), \;\; p(Tx_r, Tx_{r'}), \\ p(Tx_s, Tx_{s'}), \;\; p(Tx_s, Tx_r), \\ m - N_p \leq r, \;\; r' \leq m, \\ 1 \leq s, \;\; s' \leq N_p + 1 \end{array} \right\} \\ &\leq C^{N_p} M_p < \varepsilon \end{aligned}$$

Similarly we can show that

$$p(Tx_{N_p+1}, Tx_m) < \varepsilon.$$

Hence $\{Tx_n\}$ is both left and right P-Cauchy sequence in a left (right) sequentially complete quasi-gauge space. So $\{Tx_n\} = \{Ix_{n+1}\} = \{Jx_{n+1}\}$

converges to some z in X.
Since T is continuous $\{T^2x_n\} = \{TIx_{n+1}\} = \{TJx_{n+1}\}$ converges to Tz.

$$\begin{aligned} p(Tz,z) &\leq p(Tz,T^2x_n) + p(T^2x_n,Tx_n) + p(Tx_n,z) \\ &\leq p(Tz,T^2x_n) + C\max\left\{\begin{array}{c} p(ITx_n,Tx_n), \\ p(ITx_n,T^2x_n),\ \ p(Jx_n,Tx_0), \\ p(ITx_n,Tx_n),\ \ p(Jx_n,T^2x_n) \end{array}\right\} \end{aligned}$$

Letting n tend to infinity since T has the property whenever $Tx_n \to x$, $p(Tx_n,x) \to 0$

$$p(Tz,z) \leq C\max\{p(Tz,z),p(z,Tz)\}.$$

Similarly on using inequality (1) for $p(Tx_n,T^2x_n)$ and letting n tend to infinity

$$p(z,Tz) \leq C\max\{p(z,Tz),p(Tz,z)\}.$$

Since $C < 1$ from these inequalities $p(z,Tz) = p(Tz,z) = 0$ for all p in P.
There must exists ω in X such that

$$z = Tz = J\omega = I\omega.$$

Then on using inequality (1) we have

$$p(Tx_n,T\omega) \leq C\max\left\{\begin{array}{c} p(Ix_n,J\omega),\ \ p(Ix_n,Tx_n), \\ p(J\omega,T\omega),\ \ p(Ix_n,T\omega), \\ p(J\omega,Tx_n) \end{array}\right\}$$

$$\begin{aligned} p(z,T\omega) &\leq p(z,Tx_n) + p(Tx_n,T\omega) \\ &\leq p(z,Tx_n) + C\max\left\{\begin{array}{c} p(Tx_{n-1},z),\ \ p(Tx_{n-1},Tx_n), \\ p(z,T\omega),\ \ p(Tx_{n-1},T\omega), \\ p(z,Tx_n) \end{array}\right\} \end{aligned}$$

on letting n tend to infinity

$$\begin{aligned} p(z,T\omega) &\leq Cp(z,T\omega) \\ p(T\omega,z) &\leq p(T\omega,Tx_n) + p(Tx_n,z) \\ &\leq C\max\left\{\begin{array}{c} p(I\omega,Jx_n),\ \ p(I\omega,T\omega) \\ p(Jx_n,Tx_n),\ \ p(I\omega,Tx_n),\ \ p(Jx_n,T\omega) \end{array}\right\} + p(Tx_n,z) \\ &\leq C\max\left\{\begin{array}{c} p(z,Tx_{n-1}),\ \ p(z,T\omega),\ \ p(Tx_{n-1},Tx_n), \\ p(z,Tx_n), \\ p(z,T\omega) + p(Tx_{n-1},Tx_n) + p(Tx_n,z) \end{array}\right\} + p(Tx_n,z) \end{aligned}$$

Letting n tend to infinity

$$p(T\omega, z) \leq Cp(z, T\omega).$$

Since $C < 1$ from these inequalities

$$p(z, T\omega) = p(T\omega, z) = 0, \quad p \in P.$$

Hence

$$z = T\omega = I\omega = J\omega$$

$$Jz = JT\omega = TJ\omega = Tz = z$$

$$Iz = IT\omega = TI\omega = Tz = z$$

Thus z is the common fixed point of T, I and J.

Now suppose that T, I and J have another fixed point z'. Then

$$\begin{aligned} p(z,z') = p(Tz,Tz') &\leq C \max \left\{ \begin{array}{c} p(Iz,Jz'),\ p(Iz,Tz), \\ p(Jz',Tz'),\ p(Iz,Tz'), \\ p(Jz',Tz) \end{array} \right\} \\ &\leq C \max\{p(z,z'),\ p(z',z)\}. \end{aligned}$$

Similarly

$$p(z', z) \leq C \max\{p(z', z),\ p(z, z')\}.$$

So $p(z, z') = p(z', z) = 0$ for all p in P, uniqueness follows from this.

We now note that though it is not necessary for the mapping T to be continuous in Theorem 1 of [2], it is certainly necessary for the mapping T to be continuous, moreover T should satisfy the property that whenever Tx_n converges to x. $p(Tx_n, x)$ should also converges to zero for each p in P, in this theorem.

To see this let $X = [0, 1]$, (X, P) be quasi-gauge left sequentially complete T_0 space where P is formed by the quasi-pseudometric

$$p(x,y) = \begin{cases} x - y & \text{if } x \geq y \\ 0 & \text{if } x \leq y < 1/2 \\ 1 & \text{otherwise} \end{cases}$$

Define the mapping T by

$$Tx = \begin{cases} \frac{1+x}{3} & \text{if } x < \frac{1}{2} \\ \frac{1}{3} & \text{if } x \geq \frac{1}{2} \end{cases}$$

Choose J and I to be identity mapping. T, I and J satisfy all the conditions in the theorem with $C = 1/2$ except that whenever $Tx_n \to x$, $p(Tx_n, x) \to 0$ for all p in P. Hence T, I and J have no common fixed point.

The following example shows the necessity of the continuity of T.

Example. Let $X = [0,1]$ with the quasi-gauge structure P formed by the quasi-pseudometric

$$p(x,y) = \begin{cases} x - y & \text{if } x \geq y \\ \frac{y-x}{2} & \text{if } y \geq x \end{cases}$$

(X, P) is a left and right sequentially complete quasi-gauge T_2 space with the property that whenever $x_n \to x$, $p(x_n, x) \to 0$. Define the continuous mapping I by

$$Ix = \begin{cases} x & \text{if } x < \frac{1}{3} \\ \frac{1}{3} & \text{if } x \geq \frac{1}{3} \end{cases}$$

J and T by

$$Jx = \begin{cases} x & \text{if } x < \frac{1}{3} \\ 1 & \text{if } x \geq \frac{1}{3} \end{cases}$$

$$Tx = \begin{cases} \frac{1+x}{4} & \text{if } x < \frac{1}{3} \\ \frac{1}{4} & \text{if } x \geq \frac{1}{3} \end{cases}$$

satisfies all the conditions of the theorem with $C = 1/2$ except that T is continuous. Hence they do not have a common fixed poit. Now we will prove a common fixed point theorem, in which it is not necessary for T to be continuous.

Theorem 2. *Let TT and I be commuting mappings and let T and J be commuting mappings of a left (right) sequentially complete quasi-gauge T_0 space (X, P) satisfying the inequality for each p in P*

$$(5) \max \left\{ \begin{array}{c} p(Tx, Ty), \\ p(Ty, Tx) \end{array} \right\} \leq C \max \left\{ \begin{array}{c} p(Ix, Jy), \ p(Ix, Tx), \ p(Jy, Ty), \\ p(Ix, Ty), \ p(Jy, Tx) \end{array} \right\}$$

for all x, y in X where $0 \leq C < 1$. If for each x in X, there exists an y in X such that

$$Tx = Iy = Jy$$

and if one of T, I and J is continuous with the property that whenever, for example, $Ix_n \to x$, $p(Ix_n, x) \to 0$ for each p in P. Then T, I and J have a unique common fixed point.

Proof. Let x_0 be an arbitrary point in X. Define the sequence $\{x_n\}$ inductively as in the proof of Theorem 1 by choosing x_{n+1} such that

$$Tx_n = Ix_{n+1} = Jx_{n+1}, \quad n = 0, 1, 2, \ldots$$

then $\{Tx_n\} = \{Ix_{n+1}\} = \{Jx_{n+1}\}$ is both left and right P-Cauchy sequence. Argument for this runs almost in the same lines as in the proof of Theorem 1. So we will omit the details.

Since $\{Tx_n\} = \{Ix_{n+1}\} = \{Jx_{n+1}\}$ is the left and right P-Cauchy sequence in a left (right) sequentially complete quasi-gauge space (X, P) has a limit z in X.

We will now suppose that the mapping I is continuous and for each p in P, $p(Ix_n, x) \to 0$ whenever $Ix_n \to x$. Then the sequence $\{ITx_n\} = \{TIx_{n+1}\} = \{I^2x_{n+1}\}$ converges to the limit Iz.

Using the inequality (5) we have

$$\max\left\{\begin{array}{c} p(TIx_n, Tx_n), \\ p(Tx_n, Tx_n) \end{array}\right\} \le C \max\left\{\begin{array}{c} p(I^2x_n, Jx_n), \; p(I^2x_n, TIx_n), \\ p(Jx_n, Tx_n), \; p(I^2x_n, Tx_n), \\ p(Jx_n, TIx_n) \end{array}\right\}$$

$$\begin{aligned} p(z, Iz) &\le p(z, Tx_n) + p(Tx_n, TIx_n) + p(TIx_n, Iz) \\ &\le p(z, Tx_n) + C \max\left\{\begin{array}{c} p(I^2x_n, Jx_n), \; p(I^2x_n, TIx_n), \\ p(Jx_n, Tx_n), \; p(I^2x_n, Tx_n), \\ p(Jx_n, TIx_n) \end{array}\right\} + \\ &\quad + p(TIx_n, Iz) \end{aligned}$$

Letting n tend to infinity we have

$$p(z, Iz) \le C \max\{p(z, Iz), p(Iz, z)\}.$$

Similarly we will get

$$p(Iz, z) \le C \max\{p(z, Iz), p(Iz, z)\}.$$

Since $C < 1$, $p(z, Iz) = p(Iz, z) = 0$ for all p in P. So $z = Iz$.

Using inequality (5) again we have

$$\max\left\{\begin{array}{c} p(Tz, Tx_n), \\ p(Tx_n, Tz) \end{array}\right\} \le C \max\left\{\begin{array}{c} p(Iz, Jx_n), \; p(Iz, Tz), \; p(Jx_n, Tx_n), \\ p(Iz, Tx_n), \; p(Jx_n, Tz) \end{array}\right\}$$

$$\begin{aligned} p(z,Tz) &\leq p(z,Tx_n)+p(Tx_n,Tz) \\ &\leq p(z,Tx_n)+C\max\left\{\begin{array}{c} p(Iz,Jx_n),\ \ p(Iz,Tz), \\ p(Jx_n,Tx_n),\ \ p(Tz,Tx_n), \\ p(Jx_n,Tz) \end{array}\right\} \end{aligned}$$

Then by letting n tend to infinity

$$p(z,Tz)\leq Cp(z,Tz).$$

Similarly $p(Tz,z)\leq Cp(z,Tz)$.
Since $C<1$, $Tz=z$.
Then there exists a point ω in X such that

$$Tz=z=I\omega=J\omega.$$

On using inequality (5) we have

$$\begin{aligned} \max\left\{\begin{array}{c} p(z,T\omega), \\ p(T\omega,z) \end{array}\right\} &\leq C\max\left\{\begin{array}{c} p(Iz,J\omega),\ \ p(Iz,Tz),\ \ p(J\omega,T\omega), \\ p(Iz,T\omega),\ \ p(J\omega,Tz) \end{array}\right\} \\ &\leq Cp(z,T\omega) \end{aligned}$$

for each p in P, then it follows that

$$z=T\omega.$$

Thus $Jz=JT\omega=TJ\omega=Tz=z$ and we have proved that z is the common fixed point of T, I and J.

If the mapping J is continuous instead of I, then the proof that T, I and J have a common fixed point is of course similar.

If the mapping T is continuous the result follows from Theorem 1. The proof of uniqueness is the same as that in Theorem 1. Theorem 1 of [2] becomes a special case.

Corollary 1. *Let T and I be commuting mappings of a sequentially complete quasi-gauge T_0 space satisfy the inequality for each p in P*

$$\max\left\{\begin{array}{c} p(Tx,Ty), \\ p(Ty,Tx) \end{array}\right\}\leq C\max\left\{\begin{array}{c} p(Ix,Iy),\ \ p(Ix,Tx),\ \ p(Iy,Ty), \\ p(Ix,Ty),\ \ p(Iy,Tx) \end{array}\right\}$$

for all x, y in X where $0\leq C<1$. If the range of T is contained in the range of I and if I is continuous and whenever $x_n\to x$, $p(Ix_n,x)\to 0$ for all p in P, then T and I have a unique common fixed point.

Proof. When $I = J$ in Theorem 2 the condition that for each x in X there exists an y in X such that

$$Tx = Iy = Jy$$

reduces to the range of T is contained in the range of I. Then the result follows immediately from the theorem. This result for complete metric space was proved in [5]. Next corollary also follows similarly from Theorem 1 and for complete metric space it was given in [2] and for bounded metric space in [5].

Corollary 2. *Let T and I be commuting mappings of a left (right) sequentially complete quasi-gauge T_0·space satisfying the inequality for each p in P*

$$p(Tx,Ty) \leq C \max \left\{ \begin{array}{c} p(Ix,Iy),\ p(Ix,Tx),\ p(Iy,Ty), \\ p(Ix,Ty),\ p(Iy,Tx) \end{array} \right\}$$

for all x, y in X where $0 \leq C < 1$. If the range of T is contained in the range of I and if T is continuous, whenever $x_n \to x$, $p(Tx_n, x) \to 0$ for all p in P, then T and I have a unique common fixed point.

When the mapping I in Corollary 2 is the identity mapping we have the following result which will be a generalization of theorem in [1] and the theorem Ćirić [4].

Corollary 3. *Let T be a mapping on a T-orbitally complete quasi-gauge T_0 space satisfying the inequality that for each p in P*

$$\max \left\{ \begin{array}{c} p(Tx,Ty), \\ p(Ty,Tx) \end{array} \right\} \leq C \max \left\{ \begin{array}{c} p(x,y),\ p(x,Tx),\ p(y,Ty), \\ p(x,Ty),\ p(y,Tx) \end{array} \right\}$$

for all x, y in X where $0 \leq C < 1$. Then T has a unique fixed point.

It can be noted that when $I = J$ identity map $\{x_n : \ n = 0,1,2,...\}$ which we choose in Theorem 2 is nothing but x_0, Tx_0, T^2x_0, ... So sequential completeness can be replaced by T-orbital completeness.

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REZIME

NEPOKRETNE TAČKE ZA TRI PRESLIKAVANJA

U radu su razmatrane teoreme o zajedničkoj fiksnoj tački za tri zasebna preslikavanja u kvazi-metričkom prostoru, koje proširuju rezultate metričkog prostora iz radova [1], [2], [4] and [5].

Received by the editors October 23, 1989